Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 09

B.Tech.(Petroleum Refinary Engg.) (2013 Onwards) (Sem.–3)

ENGINEERING MATHEMATICS – III

Subject Code : BTAM-201

Paper ID : [A3258]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students has to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students has to attempt any TWO questions.**

SECTION–A

1. Solve :

(a) Find Fourier series of $f(x) = x$ on the interval $[-\pi, \pi]$.

(b) Find inverse Laplace transform of $\dfrac{s+3}{(s-1)(s+2)}$.

(c) Find inverse Laplace transform of $\dfrac{e^{-3s}}{(s+2)}$.

(d) Find Laplace transform of $t\int_0^t e^{-2u}\cos 3u\,du$.

(e) Express $P(x) = 3P_3(x) + 2P_2(x) + 4P_1(x) + 5P_0(x)$ as a polynomial in x, where $P_n(x)$ is Legendre's polynomial of order n.

(f) For Legendre's polynomial $P_n(x)$ of order n, show that $P'_n(1) = \dfrac{n(n+1)}{2}$.

(g) Eliminate arbitrary constants a and b from $z = ax + by + a^2 b^2$, to obtain the partial differential equation governing it.

(h) Form the general solution of Lagrange's equation $2yzp + zxq = 3xy$.

(i) If $\lim_{z \to z_0} f(z)$ exists then show it is unique.

(j) Show that if $f(z)$ is analytic and Re $f(z)$ is constant then show that $f(z)$ is also constant.

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SECTION–B

2. Find a Fourier cosine and sine series of the function $f(x) = 1$ over the interval [0, 2].

3. Solve the initial value problem $y'' + 2y' - 3y = 3$, $y(0) = 4$, $y'(0) = -7$.

4. Find two linearly independent solutions of the equation

   $2x^2 y'' + xy' - (x^2 + 1)y = 0$ using the Frobenius method.

5. Find the solution of homogeneous partial differential equation

$$[2D^2 + 5DD' + 3(D')^2 + D + D']z = 0.$$

6. Show that the function $u(x, y) = 2x + y^3 - 3x^2y$ is harmonic. Find its conjugate harmonic function $v(x, y)$ and the corresponding analytic function $f(z)$.

SECTION–C

7. (i) Find the Laplace transform of the periodic function $f(t) = t$ over the interval $[0, a]$ and $f(t + a) = f(t)$.

   (ii) Show that $\int_{-1}^{1} P_n(x) P_m(x)\, dx = 0$ if $n \neq m$.

8. (i) Find the Power series solution about $x = 0$ of the given differential equation

$$\frac{d^2y}{dx^2} - 4y = 0.$$

   (ii) Using Method of Separation of Variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given is $u(x, 0) = 6e^{-3x}$.

9. (i) Evaluate $\oint_C \frac{dz}{z(z^2+4)}$, $C : |z| = 1$.

   (ii) Find all possible Taylor's and Laurent series expansions of the function

$$f(z) = \frac{1}{(z+1)(z+2)^2} \quad \text{about the point } z = 1.$$

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