Roll No. ☐☐☐☐☐☐☐☐☐☐☐   Total No. of Pages : 02

Total No. of Questions : 09

B.Tech (ME)   (Sem.–4)
MATHEMATIC-III
Subject Code : AM-201
Paper ID : [A0865]

Time : 3 Hrs.   Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :
1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

1. Write briefly :

    a)   State Cauchy Integral theorem and Cauchy Residue theorem.

    b)   Define bilinear transformation and prove that it passes circles into circles.

    c)   Find the general solution of $4\dfrac{\partial^2 z}{\partial x^2} + 12\dfrac{\partial^2 z}{\partial x \partial y} + 9\dfrac{\partial^2 z}{\partial y^2} = 0$.

    d)   Find the Laurent's series of $f(z) = \dfrac{1}{(z+1)(z+3)}$ for the region $1 < |z| < 3$.

    e)   Find the residue of $f(z) = \dfrac{ze^{iz}}{z^2 + 1}$ at each of its pole.

    f)   Form a partial differential equation from $z = f\left(\dfrac{y}{x}\right)$.

    g)   Write C-R equation in cartesian and polar form.

    h)   Is the function $u(x, y) = 2xy + 3xy^2 - 2y^3$, a harmonic function?

    i)   Find the analytic function, whose imaginary part is $e^x \cos y$.

    j)   State the sufficient conditions for the existence of Laplace Transforamtion.

SECTION-B

2. Evaluate $\int_0^\infty \frac{\sin t}{t}\, dt$ using Laplace Transformation.

3. An elastic sting of length L cm which is fastened at its end x = 0 and x = L is picked up at it center point $x = L/2$ to a height of L/2 and released from rest. Find the displacement of the string.

4. Solve $(D^2 + DD' - 2D'^2)z = (y-1)x$.

5. Using Cauchy integral formula, evaluate $\oint \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\, dz$, over a circle $|z| = 3$.

6. Express $f(x) = x^4 + 2x^3 - 6x^2 + 5x - 3$ in terms of Legendre polynomials.

SECTION-C

7. (a) Evaluate $\oint \frac{12z - 7}{(z-1)^2(2z+3)}\, dz$, over a circle, $|z + i| = \sqrt{3}$.

   (b) Using Laplace Transformation, solve $\frac{d^2 y}{dt^2} + 2\frac{dy}{dt} - 2y = 3\cos 3t - 11\sin 3t$ with $y(0) = 0, y'(0) = 0$.

8. (a) Using Frobenius method, solve $(1 - x^2)\frac{d^2 y}{dx^2} - 2x\frac{dy}{dx} + 6y = 0$, and about $x = 0$.

   (b) Find the solution of $\left(D_x^2 + 2D_x D_y + D_y^2\right)z = e^{x-y}$.

9. Obtain Fourier series for function $f(x)$ given by $f(x) = \begin{cases} 2+x, & -2 \le x \le 0 \\ 2-x, & 0 \le x \le 2 \end{cases}$, where $f(x + 4) = f(x)$.