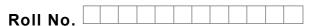


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B.Tech.(ME) (2011 Onwards) (Sem.–5) MATHEMATICS-III Subject Code : BTAM-500 Paper ID : [A2127]

Time: 3 Hrs.

Max. Marks : 60

INSTRUCTION TO CANDIDATES :

- 1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
- 2. SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt ANY FOUR questions.
- 3. SECTION-C contains THREE questions carrying TEN marks each and students have to attempt ANY TWO questions.

SECTION-A

- 1. Write briefly :
 - (a) Find the Fourier series of $f(x) = \begin{cases} \frac{1}{2} + x, & -\frac{1}{2} < x < 0\\ \frac{1}{2} x, & 0 < x < \frac{1}{2} \end{cases}$.
 - (b) Find Laplace transform of $(1 + te^{-t})^2$
 - (c) Find inverse Laplace transform of $\tan^{-1}\frac{2}{r}$
 - (d) Evaluate $\int_{0}^{1} \sqrt[3]{x \ln(\frac{1}{x})} dx$.
 - (e) Evaluate $\int x^2 J_1(x) dx$.
 - (f) By eliminating arbitrary function, form a partial differential equation from

$$z = x^n f\left(\frac{y}{x}\right).$$

- (g) Solve the given partial differential equation $p q = \ln (x + y)$.
- (h) Show that imaginary part of an analytic function is harmonic.
- (i) Find the orthogonal trajectories of the family of curves $x^3 y x y^3 = c = \text{constant}$.
- (j) State Cauchy's integral formula.



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SECTION-B

- 2. Find the Fourier series of $f(x) = |\cos x|$ in the interval $(-\pi, \pi)$.
- 3. Using Laplace transform, solve y'' + 4y = u(t 2), y(0) = 0, y'(0) = 1, where u(t) is a unit step function.
- 4. Using Frobenius method, find the general solution of

 $8x^2y'' + 10 xy' - (1 + x) y = 0.$

- 5. Solve given partial differential equation $(2D_x^2 + 5D_xD_y + 2D_y^2) z = 0$.
- 6. Verify that $u = 3xy^2 x^3$ is harmonic and find its conjugate harmonic function.

SECTION-C

- 7. (a) Use Laplace Transform to solve given system of simultaneous differential equations $\frac{dx}{dt} - y = e^t, \frac{dy}{dt} + x = \sin t, \text{ where } x(0) = 1 \text{ and } y(0) = 0.$
 - (b) For Legendre polynomials $P_n(x)$ show that $\int_{-1}^{1} P_m(x) P_n(x) dx = \frac{2}{2n+1}$ when m = n.
- 8. (a) State and prove Convolution for Laplace transform.
 - (b) A bar of 30 cm length has its ends kept at 20° and 80° respectively until steady-state condition prevail. The temperature at each end is then suddenly reduced to 0° and maintained thereafter, Find the temperature in bar.

9. (a) Find Laurent series of
$$\frac{z}{(1+z)(z+2)}$$
 about $z_0 = -2$.

(b) Using Residue theorem, evaluate
$$\int_{C} \frac{\tan z \, dz}{(z^2 - 1)}, C : |z| = \frac{3}{2}.$$