Roll No. Total No. of Pages : 02

Total No. of Questions : 09

B.Tech.(ME) (2011 Onwards) (Sem.–5)

MATHEMATICS-III

Subject Code : BTAM-500

Paper ID : [A2127]

Time : 3 Hrs. Max. Marks : 60

INSTRUCTION TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt ANY FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students have to attempt ANY TWO questions.

SECTION-A

1. Write briefly :

(a) Find the Fourier series of $f(x)=\begin{cases}\frac{1}{2}+x, & -\frac{1}{2}<x<0\\ \frac{1}{2}-x, & 0<x<\frac{1}{2}\end{cases}$.

(b) Find Laplace transform of $(1 + te^{-t})^3$.

(c) Find inverse Laplace transform of $\tan^{-1}\frac{2}{s}$.

(d) Evaluate $\int_0^1 \sqrt[3]{x\ln\left(\frac{1}{x}\right)}\,dx$.

(e) Evaluate $\int x^2 J_1(x)\,dx$.

(f) By eliminating arbitrary function, form a partial differential equation from $z = x^n f\left(\frac{y}{x}\right)$.

(g) Solve the given partial differential equation $p - q = \ln(x + y)$.

(h) Show that imaginary part of an analytic function is harmonic.

(i) Find the orthogonal trajectories of the family of curves $x^3 y - x y^3 = c =$ constant.

(j) State Cauchy's integral formula.

SECTION-B

2. Find the Fourier series of $f(x) = |\cos x|$ in the interval $(-\pi, \pi)$.

3. Using Laplace transform, solve $y'' + 4y = u(t-2)$, $y(0) = 0$, $y'(0) = 1$, where $u(t)$ is a unit step function.

4. Using Frobenius method, find the general solution of

   $8x^2 y'' + 10\,xy' - (1+x)\,y = 0.$

5. Solve given partial differential equation $(2D_x^2 + 5D_xD_y + 2D_y^2)\,z = 0$.

6. Verify that $u = 3xy^2 - x^3$ is harmonic and find its conjugate harmonic function.

SECTION-C

7. (a) Use Laplace Transform to solve given system of simultaneous differential equations $\frac{dx}{dt} - y = e^t, \frac{dy}{dt} + x = \sin t$, where $x(0) = 1$ and $y(0) = 0$.

   (b) For Legendre polynomials $P_n(x)$ show that $\int_{-1}^{1} P_m(x)\,P_n(x)\,dx = \frac{2}{2n+1}$ when $m = n$.

8. (a) State and prove Convolution for Laplace transform.

   (b) A bar of 30 cm length has its ends kept at 20° and 80° respectively until steady-state condition prevail. The temperature at each end is then suddenly reduced to 0° and maintained thereafter, Find the temperature in bar.

9. (a) Find Laurent series of $\frac{z}{(1+z)(z+2)}$ about $z_0 = -2$.

   (b) Using Residue theorem, evaluate $\int_C \frac{\tan z\,dz}{(z^2-1)}, C : |z| = \frac{3}{2}$.