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Total No. of Questions : 07

B.Sc. (Computer Science) (2013 & Onwards) (Sem.–3)

SOLID GEOMETRY

Subject Code : BCS-301

Paper ID : [A3135]

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains SIX questions carrying TEN marks each and students has to attempt any FOUR questions.

SECTION–A

1. Write briefly :

(a) Define an enveloping cone and an enveloping cylinder.

(b) Define power of a point with respect to a sphere.

(c) Define the polar plane of a point with respect to a sphere.

(d) Find the equation of the enveloping cone of the sphere $x^2 + y^2 + z^2 + 2x - 2y = 2$ with its vertex at (1, 1, 1).

(e) Write the condition for three planes to intersect along a line.

(f) Define a parabolic and a hyperbolic cylinder.

(g) Define radical plane of two spheres.

(h) Find the equation of the sphere which passes through the points (1, 2, 3), (–1, 1, 4), (0, 3, 3) and (1, 3, 2).

(i) Define exterior and interior points of a sphere.

(j) Find the equation of the right circular cone whose vertex is (2, –3, 5), semi–vertical angle is 30° and axis makes equal angles with coordinate axes.

SECTION–B

2. (a) Find the equation of the enveloping cylinder of the sphere

$$x^2 + y^2 + z^2 + 2x + 2y + 2z + 2 = 0,$$

whose generators are parallel to the line $x = -y = z$.

(b) Find the equation of the cone with vertex (5, 4, 3) and guiding curve $3x^2 + 2y^2 = 6$, $y + z = 0$.

3. (a) Define a great circle. Prove that the plane $x + 2y + 2z = 15$ cuts the sphere

$$x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$$

in a circle. Find the centre and radius of the circle. Also find the equation of the sphere which has this circle for one of the great circles.

(b) Find the equation of the right circular cylinder of radius 2 and whose axis is

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{6}.$$

4. (a) Prove that the equation of a cone, whose vertex is the origin, is homogeneous in x, y, z and conversely every homogeneous equation in x, y, z represents a cone whose vertex is origin.

(b) Find the points of intersection of the line

$$\frac{x-8}{4} = \frac{y}{1} = 1 - z$$

and the sphere $x^2 + y^2 + z^2 - 4x + 6y - 2z + 5 = 0$.

5. Find the equation of the tangent plane at the point P (x_1, y_1, z_1) of the cone

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0.$$

6. (a) Define a reciprocal cone. Prove that the cones $ax^2 + by^2 + cz^2 = 0$ and $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$ are reciprocal cones.

(b) Prove that the two spheres

$$S' = x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$$

and $S'' = x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$

will cut orthogonally iff $2(u_1u_2 + v_1v_2 + w_1w_2) = d_1 + d_2$.

7. Define a cone, its vertex, generator and guiding curve. If the section of a cone, whose vertex is P and guiding curve, is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$ by the plane $x = 0$ is a rectangular hyperbola, then find the locus of P.