Roll No. | **Total No. of Pages : 02**

Total No. of Questions : 07

B.Sc. (Computer Science) (2013 & Onwards) (Sem.–3)

SEQUENCE SERIES AND CALCULUS

Subject Code : BCS-302

Paper ID : [A3136]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains SIX questions carrying TEN marks each and students have to attempt ANY FOUR questions.**

SECTION-A

1. Write briefly :

a) Test whether the sequence $\{a_n\}$ where $a_n = ln\left(1+\frac{1}{n}\right)^n$ is convergent or divergent?

b) Using non-decreasing sequence theorem test the convergence of sequence $\{a_n\}$ where $a_n = \frac{3n+1}{n+1}$.

c) Discuss the convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n n}{n^2+1}$.

d) State Cauchy's condensation test.

e) Differentiate between conditional and absolute convergence of an alternating series.

f) Define upper integral of a bounded functions on $[a, b]$.

g) Determine whether $\int_1^{\infty} \frac{dx}{x^2}$ converges or not ?

h) Define absolute convergence of an improper integral.

i) Evaluate the integral $\int_1^{\infty} x^3 e^{-x} dx$ by expressing it in terms of gamma function

j) Prove that beta function is symmetric.

SECTION-B

2. State and explain the Cauchy's convergence criterion. (10)

3. a) State and prove integral test for testing the convergence/ divergence of a positive term infinite series. (5)

   b) Examine for convergence and absolute convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n n}{n^2+1}$. (5)

4. Discuss the convergence /divergence of the following infinite series :

$$\left(\frac{1}{3}\right)^2 + \left(\frac{1.4}{3.6}\right)^2 + \left(\frac{1.4.7}{3.6.9}\right)^2 + \ldots - \infty .$$ (10)

5. Prove that if $f : [a, b] \to R$ is continuous on $[a, b]$, then f is Riemann-integrable on $[a, b]$. (10)

6. a) If $c \in (a, b)$ and $f : [a, b] \to R$ is Riemann-integrable on $[a, c]$ and on $[c, b]$, then f is Riemann-integrable on $[a, b]$. (6)

   b) Discuss the convergence of the integral $\int_0^{\infty} \sin u^2 du$. (4)

7. Prove that $B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$, where $B(x, y)$ represents beta function and $\Gamma(x)$ represents Gamma function. (10)