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Total No. of Questions : 07

B.Sc. (Computer Science) (2013 & Onwards) (Sem.–6)

REAL ANALYSIS

Subject Code : BCS-601

Paper ID : [72781]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains SIX questions carrying TEN marks each and students has to attempt any FOUR questions.

SECTION–A

1. Write briefly :

(a) Determine the radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{\lfloor\underline{n}}{n^{n}}x^{n}$.

(b) State Fourier series for odd and even functions.

(c) State a test of uniform convergence of sequence of functions.

(d) Define Rotation and Inversion.

(e) Differentiate between the pointwise convergence and uniform convergence of sequence of functions.

(f) What are Cauchy-Riemann equations?

(g) What is a harmonic function? Is the function $x^3 - 3xy^2$ harmonic?

(h) Show that the function $e^x(\cos y + i \sin y)$ is holomorphic and find its derivative.

(i) Define the limit of a function of a complex variable.

(j) State Abel's theorem on power series.

SECTION–B

2. Define an analytic function of a complex variable. If $f(z)$ is an analytic function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2.$$

3. (a) State and prove the test for uniform convergence of sequence of functions.

   (b) Check for uniform convergence the sequence $\{f_n(x)\}$ where $f_n(x) = x^{n-1}(1-x)$ in $[0, 1]$.

4. Define the Euler's formula for a Fourier Series. Expand $f(x) = x \sin x$; $0 < x < 2\pi$ as a Fourier series.

5. (a) Show that $u=\frac{1}{2}\log(x^2+y^2)$ is harmonic and find its harmonic conjugate.

   (b) Show that the function $f(z) = xy + iy$ is everywhere continuous but not analytic.

6. (a) Define a Mobius transformation. Find the Mobius transformation which maps $1, -i, 2$ into $0, 2, -i$ respectively.

   (b) State and prove Abel's theorem on Power Series.

7. Let $\{f_n\}$ be a sequence of real valued functions on a metric space X which converges uniformly to f on X. If each f_n is continuous on X, then f is also continuous on X.