

[illegible]

- h) Change the order of integration and evaluate $\int_0^1 \int_{x^2}^{2-x} xy dx dy$
- i) Find $\text{grad}\Phi$ when $\phi = 3x^2y - y^3z^2$ at the point $(1, -2, -1)$.
- j) Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$.

SECTION-B

2. Find the values of a and b for which the equations

$$x + ay + z = 3$$

$$x + 2y + 2z = b$$

$x + 5y + 3z = 9$ are consistent. When will these equations have unique solution?

3. Prove that $\tan^{-1}\left(\frac{63}{16}\right) = \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$.
4. If $\theta = t^n e^{\frac{-r^2}{4t}}$, what value of n will make $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \theta}{\partial r} \right) = \frac{\partial \theta}{\partial t}$?
5. Show that area between parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$.
6. Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken round the rectangle bounded by the lines $x = \pm a$, $y = 0$, $y = b$.

SECTION-C

7. Find the matrix P which transforms the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to diagonal form. Hence

calculate A^4

8. State Euler theorem. Using euler theorem prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$

$$\text{And } x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u) \sin 2u \text{ when } \tan u = \frac{x^3 + y^3}{x - y}.$$

9. Verify Divergence theorem for $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ taken over the rectangular parallelepiped $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq c$.