Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐☐   **Total No. of Pages : 02**

Total No. of Questions : 08

M.Tech. (CSE Engg.) (2018 Batch) (Sem.–1)
MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE
Subject Code : MTCS-101-18
Paper ID : [75153]

Time : 3 Hrs.   **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :
1. Attempt any FIVE questions out of EIGHT questions.
2. Each question carries TWELVE marks.

1. Define the following concepts from Graph theory, with an example for each :

   a. Spanning subgraph of a graph.

   b. Eccentricity of a vertex of a graph.

   c. A full binary tree.

   d. A Hamiltonian Path in a graph.

2. a. The adjacency matrix of a graph G is :

$$A = \begin{matrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{matrix}$$

   Find $A + A^2 + A^3 + A^4$. Is G connected? Check your answer by drawing the graph G.

   b. Draw a graph with vertex set $V = \{v_1, v_2, v_3, v_4, v_5\}$ and such that $d(v_1) = 2$, $d(v_2) = 2$. $d(v_3) = 4$, $d(v_4) = 1$, and $d(v_5) = 1$. Is there a tree with given vertex set v and given degrees? Justify your answer.

3. a. Show that if every component of a graph is bipartite, then the graph is bipartite.

    b. Prove that if u is a vertex of odd degree in a graph, then there exists a path from u to another vertex v of the graph where v also has odd degree.

4. Two fair dice are thrown. If the scores are unequal, the larger of the two scores is recorded. If the scores are equal then that score is recorded. Let X denote the number recorded.

    a. Show that $P(X = 2) = 1/12$ and draw up a table showing the probability distribution of X.

    b. Find the mean and variance of this distribution.

5. a. Suppose you draw 21 samples a population with mean of 78 and a standard deviation of 8.8. What is the probability of obtaining a mean of 78.2 or more?

    b. Why does a small sample size cause problems? When is the Central Limit Theorem needed? How big does the sample have to be for the Central Limit Theorem?

6. a. Suppose that we wanted to estimate the true average number of eggs a queen bee lays with 95% confidence. The margin of error we are willing to accept is 0.5. Suppose we also know that s is about 10. What sample size should we use?

    b. Suppose 20 donors come to a blood drive. Assume that the blood donors are not related in any way, so that we can consider them independent. The probability that the donor has type-O blood is 0.06, which is constant from donor to donor. Let X = the number of donors that have type-O blood.

       For a sample of 100 donors, what is the sampling distribution of the sample proportion?

7. What are the recent trends in various distributions functions in mathematical field of computer science in Computer Vision?

8. Write short notes on :

    a. Mathematical applications in Network Protocol.

    b. Number theory applications in Cryptography.