Roll No. Total No. of Pages : 2

Total No. of Questions : 07

M.Sc. Mathematics (2017 Batch) (Sem.–1)

ALGEBRA-I

Subject Code : MSM-101

Paper ID : [74720]

Time : 3 Hrs. **Max. Marks : 80**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.**
2. **SECTION - B & C. have THREE questions in each section carrying SIXTEEN marks each.**
3. **Select atleast TWO questions from SECTION - B & C EACH.**

SECTION-A

1. Answer briefly :

a) Write all abelian groups of order 108.

b) State Sylow's second theorem.

c) Let R be a ring and $a \in R$. Prove that $I = \{x \in R / ax = 0\}$ is a right ideal of R.

d) Prove that any p-Sylow subgroup of a group G of order 33 is a normal subgroup of G.

e) Show that elements in the same class of a group must have the same order.

f) Write the composition series for a cyclic group of order 50.

g) Write the composition series for the group A_4.

h) Prove that group of order 55 is not simple.

SECTION-B

2 a) Prove that A_n, n>4, is the only nontrivial normal subgroup of S_n.

b) Let G be a group of order 2m where m is odd. Prove that G contains a normal subgroup of order m.

3 a) Prove that if an abelian group has a composition series, then G is a finite group.

b) State and prove Cayley's theorem.

4 a) Prove that any two finite sub normal series for a group G have isomorphic refinements.

b) Write the composition series for the symmetric group S_4.

SECTION-C

5 a) Prove that a divison ring is a simple ring.

b) Let G be a finite group such that x^2= e for all x ε G. Prove that G is the direct product of a finite number of cyclic groups of order 2.

6 a) State and prove sylow's third theorem.

b) Prove that there are only two non-abelian groups of order 8.

7 a) Prove that in an integral domain every prime element is an irreducible element. The converse may not be true.

b) Let R be a Boolean ring. Then each prime ideal P ≠ R is maximal.