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Total No. of Questions : 07

M.Sc. (Mathematics) (2017 Batch) (Sem.–1)

REAL ANALYSIS – I

Subject Code : MSM-102

Paper ID : [74721]

Time : 3 Hrs. **Max. Marks : 80**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.**
2. **SECTION - B & C. have THREE questions in each section carrying SIXTEEN marks each.**
3. **Select atleast TWO questions from SECTION - B & C EACH.**

SECTION-A

1. a) Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of m and n.

b) Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{x^{n-1}}{n}$.

c) State Dirichlet's test for uniform convergence.

d) Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{2^n}{n^2} x^n$.

e) If f $\in$R(α) and g $\in$R(α) on [a, b] then prove that fg $\in$ R(α).

f) Prove that $\sum a_n n^{-x}$ is uniformly convergent on [0, 1} if $\sum a_n$ converges uniformly in [0,1].

g) Define a closed curve and rectifiable curve.

h) Show that the sequence {fn}, where $f_n(x) = nx\ e^{-nx^2}$, $x \geq 0$ is not uniformly convergent on [0, k], k > 0.

SECTION-B

2. a) State and prove Dirichlet's theorem on power series.

   b) State and prove Heine-Borel theorem.

3. a) Prove that the continuous image of a compact set is compact.

   b) State and prove Cauchy's General Principle of uniform convergence.

4. a) Prove that each closed and bounded set in R^n is compact.

   b) Prove that the set of real numbers in [0, 1] is uncountable.

SECTION-C

5. a) If f is continuous on [0, 1] and if $\int_0^1 f(x)x^n dx = 0,$ n = 1, 2, 3,... Prove that f(x) = 0 on [0, 1].

   b) Show that the sequence $\{f_n\}$ where $f_n : R \to R$ defined by $f_n(x) = x/n \ \forall\ x \in R, n \in N$ is convergent point wise but not uniformly.

6. a) Let α be monotonically increasing function on [a, b] and $f_n \in R(\alpha)$ on [a, b], for n = 1, 2, 3,..., such that $f_n \to f$ uniformly on [a, b]. Then $f \in R(\alpha)$ on [a, b] and $\int_a^b f\, d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha.$

   b) Prove that $\sum_{n=1}^{\infty} a_n \sin nx$ and $\sum_{n=1}^{\infty} a_n \cos nx$ are uniformly convergent on R if $\sum_{n=1}^{\infty} |a_n|$ converges.

7. State and prove Stone Weierstrass theorem.