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Total No. of Pages : 02

Total No. of Questions : 07

M.Sc. Mathematics (2017 Batch) (Sem.-1)

**COMPLEX ANALYSIS**

Subject Code : MSM-103

Paper ID : [74722]

Time : 3 Hrs.

Max. Marks : 80

**INSTRUCTION TO CANDIDATES :**

1. SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.
2. SECTION - B & C. have THREE questions in each section carrying SIXTEEN marks each.
3. Select atleast TWO questions from SECTION - B & C EACH.

**SECTION-A**

1) Answer briefly :

- a) Evaluate  $(1-i)^4$ .
- b) Prove that  $\operatorname{Re}(iz) = -\operatorname{Im} Z$ .
- c) Check whether the function  $f(z) = \operatorname{Im}(z)$  has a derivative at any point.
- d) Find the singular points of  $\frac{z^3 + i}{z^2 - 3z + 2}$ .
- e) Evaluate  $\operatorname{Log}(-ei)$ .
- f) Write MitagLefler's inequality of complex integration and explain it.
- g) State Rouché's theorem.
- h) Discuss the types of singularities.

**SECTION-B**

- 2) a) Find all roots of  $(2i)^{1/2}$  and exhibit them geometrically.  
b) Show that the function  $f(z) = \bar{z}$  is nowhere differentiable.
- 3) a) Show that the function  $f(z) = \frac{\bar{z}^2}{z}$  if  $z \neq 0$  and 0 if  $z = 0$  is not differentiable at  $z = 0$  but the CR equations are satisfied.  
b) Use the polar form to show that  $-(1+i)^7 = -8(1+i)$
- 4) a) Determine accumulation points of  $z_n = \left(\frac{1}{n}\right)i^n$  ( $n = 1, 2, \dots$ ).  
b) Establish Cauchy riemann equations.

**SECTION-C**

- 5) a) Show that  $u(x,y) = \sinh x \sin y$  is harmonic in some domain and find its harmonic conjugate of it.  
b) Show that when  $n = 0, \pm 1, \pm 2, \dots$
- $$(1+i)^i = \exp\left(-\frac{\pi}{4} + 2n\pi\right) \exp\left[\frac{i}{2} \log 2\right]$$
- 6) a) State and prove mitag lefler's inequality.  
b) Determine the domain of analyticity of the function  $f$  and apply Cauchy Goursat theorem to show that  $\int \frac{z^2}{z-3} dz = 0, |z|=1$
- 7) a) State and prove maximum modulus principle.  
b) Establish Schwarz lemma.