Roll No. Total No. of Pages : 02

Total No. of Questions : 07

M.Sc(Mathematics) (2018 Batch) (Sem.-1)

ALGEBRA-I

Subject Code : MSM-101-18

Paper ID : [75129]

Time : 3 Hrs. Max. Marks : 70

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.
2. SECTION - B & C. have THREE questions each.
3. Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.
4. Select atleast TWO questions from SECTION - B & C each.

SECTION-A

1. **Write short answers :**

a) Prove that a subgroup of index 2 is normal.

b) Prove that no group of order 15 is simple.

c) Find all the non-isomorphic Abelian group of order 12.

d) Show that a group of order 6 is solvable.

e) Let H and K be normal subgroups of a group G of coprime orders. Prove that $H \cap K = \{e\}$.

SECTION-B

2. a) Let H and K be subgroups of a group G such that $K \Delta G$. Prove that $\frac{HK}{K} \cong \frac{H}{(H \cap K)}$.

b) Prove that the intersection of two subgroups of finite indices is of finite index.

3. a) Define a composition series of a group G. Find all the composition series for $\frac{Z}{<30>}$ and verify that they are equivalent.

b) Define the derived series of a group G. Prove that G is solvable if and only if G has a normal series with Abelian factors.

4. a) Show that the alternative group A_n is simple if $n \geq 5$. Deduce that S_n is not solvable for $n \geq 5$.

b) Prove that if an abelian group G has a composition series then G is finite.

SECTION-C

5. a) State and Prove Cauchy theorem of finite abelian group.

b) If R is a non-zero ring with unity 1 and A any ideal of R such that $A \neq R$, then there exists a maximal ideal M of R such that $A \leq M$.

6. a) Let $o(G) = pq$ where p , q are distinct prime, $p < q$ and p does not divide (q -1) then prove that G is cyclic.

b) Let P is a Sylow p – subgroup of G ; (G is a finite group). Prove that if K is normal in G, then $P \cap$ K is a p – subgroup of K and $\frac{PK}{K}$ is a Sylow p-subgroup of $\frac{G}{K}$.

7. a) Let R be a commutative ring and P a prime ideal. Then $S = R - P$ is a multiplicative set and R_s is a local ring with unique maximal ideal $P_S = \left\{\frac{a}{s} : a \in P, s \notin P\right\}$.

b) Show that the centre of a ring R is a subring of R.