Roll No. | | | | | | | | | | | | Total No. of Pages : 2

Total No. of Questions : 7

M.Sc. (Mathematics) (2018 Batch) (Sem.–1)

REAL ANALYSIS-I

Subject Code : MSM-102-18

Paper ID : [75130]

Time : 3 Hrs. Max. Marks : 70

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.
2. SECTION - B & C. have THREE questions each.
3. Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.
4. Select atleast TWO questions from SECTION - B & C each.

SECTION-A

1. a) Show that set of rational numbers are countable.

b) Show that if the series $\sum_{n=1}^{\infty} a_n$ is divergent, then the series $\sum_{n=1}^{\infty} |a_n|$ also diverges.

c) Let f be monotonic on (a, b). Then show that the set of points of (a, b) at which f is discontinuous is atmost countable.

d) Construct sequences $\{f_n\}$ and $\{g_n\}$ of functions which converge uniformly on some set E, but $\{f_n g_n\}$ does not converge uniformly on E.

e) Give an example of a bounded real function f on $[a, b]$ which is not Riemann integrable but f^2 is Riemann integrable. [2×5=10]

SECTION-B

2. a) Prove that every subset of a compact metric space is closed if and only if it is compact. [8]

b) Let $\sum_{n=1}^{\infty} a_n$ be a convergent series of non-negative terms, then what can be said about the convergence/ divergence of the series $\sum_{n=1}^{\infty} \frac{a_n}{1+na_n}$. Justify your answer. [7]

3. a) Let f be a continuous mapping of a metric space X into a metric space Y and let E be a connected subset of X, then prove that $f(E)$ is also connected. [7]

b) Prove that a mapping f of a metric space X into a metric space Y is continuous if and only if inverse image of every open set is open. [8]

4. a) Prove that the cauchy sequences of two absolute convergent series converges absolutely. [7]

b) For $x, y \in \mathbb{R}$ define, $d(x, y) = \begin{Bmatrix} 1 & x \neq y \\ 0 & x = y \end{Bmatrix}$. Show that d is metric on X. Also, find all open and closed subsets of this metric space. [8]

SECTION-C

5. a) Let $f \in \Re$ on $[a, b]$, $m \leq f \leq M$, ϕ is continuous on $[m, M]$ and $h(x) = \phi(f(x))$ on $[a, b]$. Then show that $h \in \Re(\alpha)$. [7]

b) Let f be Riemann integrable on $[a, b]$, define $F(x) = \int_a^x f(t)dt$. Then show that F is continuous on $[a, b]$. Also, if f is continuous at a point x_0, of $[a, b]$, then prove that F is differentiable at x_0 and $F'(x_0) = f(x_0)$. [8]

6. a) Prove that there exists a real continuous function on the real line which is nowhere differential. [7]

b) If f maps $[a, b]$ into $\mathbb{R}^k$ and if $f \in \Re(\alpha)$ for some monotonically increasing function α on $[a, b]$, then prove that $|f| \in \Re(\alpha)$. [8]

7. a) If $\{f_n\}$ is a pointwise bounded sequence of complex functions on countable set E, then prove that $\{f_n\}$ has subsequence $\{f_{nk}\}$ converges for every $x \in E$. [7]

b) State and prove Stone Weierstrass Theorem. [8]