Roll No. Total No. of Pages : 02

Total No. of Questions : 07

M.Sc (Mathematics) (2018 Batch) (Sem.–1)

COMPLEX ANALYSIS

Subject Code : MSM-103-18

Paper ID : [75131]

Time : 3 Hrs. Max. Marks : 70

INSTRUCTION TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.
2. SECTION - B & C have THREE questions each.
3. Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.
4. Select atleast TWO questions from SECTION - B & C each.

SECTION-A

1. **Write short answers of the following :**

   a) Show that the function $u(x, y) = e^x \cos y + y$ is harmonic.

   b) Define radius of convergence for a power series.

   c) Define essential singularity, and discuss it for $f(z) = \sin(1/z)$.

   d) Define Bilinear transformation, and find its transformation which maps the points $z_1 = \infty, z_2 = i, z_3 = 0$ onto the points $w_1 = 0, w_2 = i, w_3 = \infty$.

   e) Evaluate : $\oint_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz,$ where C is the circle $|z| = 3$,

SECTION-B

2. a) Define Branch point of a function. Discuss the branch point of logZ .

   b) Derive the Cauchy-Riemann equations for Cartesian coordinates.

   c) If $u = (x - 1)^3 - 3xy^2 + 3y^2$, determine v so that $u + iv$ is an analytic function of $x + iy$.

3. a) State and prove Cauchy-Goursat Theorem.

b) Compute $\int_{|z|=1} \frac{dz}{|z-2|^2}$.

4. a) State and Prove Schwarz lemma.

b) Show that the function $f(z) = u + iv$, where $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2} & ; z \neq 0 \\ 0 & ; z = 0 \end{cases}$

is continuous and that the Cauchy-Riemann equations are satisfied at the origin, yet $f'(0)$ does not exist.

SECTION-C

5. a) Find the nature and location of singularities :

i) $(z+1)\sin\frac{1}{z-2}$ ii) $\frac{e^{az}}{1-e^{-z}}$

b) Find the Laurent series expansion of the function (in powers of z) in different domains :

$$f(z) = \frac{z}{(z+a)(z+b)}; |a| < |b|$$

Also mention the region of validity in each case.

6. a) Using Rouche's theorem, prove that any polynomial :

$$p(z)=a_0+a_1z+a_2z^2+\ldots\ldots+ a_{n-1} z^{n-1}+a_nz^n,(a_n \neq 0),$$

where $n \geq 1$, has precisely n zeros, counting multiplicities.

b) Find the region of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n z^{2n-1}}{(2n-1)!}$.

7. a) Find what regions of the w - plane correspond by the transformation $w = \frac{z-i}{z+i}$ to

i) Interior of a circle of centre $z = -i$,

ii) The region $y > 0, x > 0, |z+i| < 2$.

b) Find the fixed points and the normal form of the bilinear transformations $w = \frac{3iz+1}{z+i}$.