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Total No. of Questions : 07

M.Sc Mathematics (2018 Batch) (Sem.–1)

MATHEMATICAL METHODS

Subject Code : MSM-105-18

Paper ID : [75133]

Time : 3 Hrs. Max. Marks : 70

INSTRUCTION TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.
2. SECTION - B & C. have THREE questions each.
3. Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.
4. Select atleast TWO questions from SECTION - B & C each.

SECTION-A

1. (a) Find Laplace Transformation of $F(t)$, where

$$F(t)=\begin{cases}4, & 0<t<1\\ 3, & t>1.\end{cases}$$

(b) Write a short note on Volterra integral equations.

(c) Apply change of scale property to find Laplace Transformation of sin (at).

(d) Find Fourier Transformation of $f(x)$, where

$$f(x)=\begin{cases}\dfrac{\sqrt{2\pi}}{2\in} & |x|\le\in\\ 0, & |x|>\in\end{cases}$$

(e) Write a short note on various applications of Laplace Transforms.

SECTION-B

2. Solve with the help of Laplace Transformation $\frac{\partial y}{\partial t} = 3\frac{\partial^2 y}{\partial x^2}$,

where $y\left(\frac{\pi}{2}, t\right) = 0, \left(\frac{\partial y}{\partial x}\right)_{x=0} = 0$ and $y(x, 0) = 30 \cos(5x)$.

3. Show that $L^{-1}\left(\frac{8}{(p^2+1)^3}\right) = (3 - t^2) \sin t - 3t \cos t$,

where L^{-1} stands for inverse Laplace Transformation.

4. Find Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$, and hence find Fourier sine transform of

$F(x) = \frac{x}{1+x^2}$.

SECTION-C

5. Reduce the initial value problem

$$\phi''(x) - 3\phi'(x) + 2\phi(x) = 4 \sin x$$

with conditions $\phi(0) = 1, \phi'(0) = -2$ to a non-homogeneous Volterra's integral equation of second kind. Conversely, derive the original differential equation with initial conditions from the integral equation obtained.

6. Solve the following integral equation :

$$\phi(x) = 1 + \lambda \int_0^{\pi} \sin(x + \xi)\, \phi(\xi)\, d\xi.$$

7. Determine the eigen values and eigen function for the following homogeneous integral equations with degenerate kernels:

$$\phi(x) = \lambda \int_{-1}^{1} (5x\xi^3 + 4x^2\xi + 3x\xi)\, \phi(\xi)\, d\xi.$$