Roll No. Total No. of Pages : 02

Total No. of Questions : 07

M.Sc Mathematics (2017 Batch) (Sem.-3)

TOPOLOGY

Subject Code : MSM-301

Paper ID : [75381]

Time : 3 Hrs. Max. Marks : 80

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.
2. SECTION - B & C. have THREE questions in each section carrying SIXTEEN marks each.
3. Select atleast TWO questions from SECTION - B & C EACH.

SECTION-A

1. Answer briefly :

i) Let (X, τ), where $X = \{1,2,3,4,5\}$, $\tau = \{\theta, X, \{2,3\}, \{1,4,5\}\}$ be a topological space. Find the derived set and exterior set of the set $A = \{1,3,4\}$.

ii) Show that in a topological space (X, τ), a point $x \in X$ is a boundary point of a set A iff it is a limit point of both A and $X - A$.

iii) In a topological space, prove that every closed subspace of a compact space is compact.

iv) Show that the function $f : (|R, u) \to (|R, u)$ defined by $f(x) = x^2$ is open, where $(|R, u)$ denotes the usual topological space.

v) Define a separable topological space. Give an example.

vi) Consider a topological space (X, τ), where $X = \{1,3,5\}$, $\tau = \{\theta, X, \{1\}, \{3\}, \{1,3\}\}$. Check whether (X, τ) is T_1 or not? Justify your claim.

vii) State the Urysohn's Lemma.

viii) Can every topological space be generated by a metric space? Justify your claim.

SECTION-B

2. State the properties of Kuratowski closure operator. Show that a topology can be defined in terms of Kuratowski closure operator.

3. Let $f: (X, \tau) \rightarrow (Y, \mu)$ be any map from a topological space (X, τ) to another topological space $\{Y, \mu)$. Show that the following statements are equivalent:

   i) The function f is continuous.

   ii) The inverse image of each closed set is closed.

   iii) The inverse image of each member of a subbase for the topology for Y is open.

   iv) For each $x \in X$, the inverse image of each neighborhood of $f(x)$ is a neighborhood of x.

   v) For each $x \in X$ and each neighborhood U of $f(x)$, there is a neighborhood V of x such that $f(V) \subset U$.

4. Show that every regular topological space with a countable basis is normal.

SECTION-C

5. i) Show that in a topological space, interior of any set A is the largest open subset of A. (6)

   ii) Prove that a subfamily β of a topology τ on X is a basis for τ iff each member of τ can be written as a union of members of β. (10)

6. Prove that a second countable topological space is Lindelof. Is the converse true? Justify your claim. (16)

7. i) Show that a Hausdorff space is both T_0 and T_1 but converse is not true. Give examples. (8)

   ii) Show that the image of a connected space under a continuous map is connected. (8)