Roll No. [ ] Total No. of Pages : 03

Total No. of Questions : 07

M.Sc (Mathematics) (2017 Batch) (Sem.–3)

MATHEMATICAL STATISTICS-I

Subject Code : MSM-303

Paper ID : [75383]

Time : 3 Hrs. **Max. Marks : 80**

INSTRUCTION TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.
2. SECTION - B & C. have THREE questions in each section carrying SIXTEEN marks each.
3. Select atleast TWO questions from SECTION - B & C EACH.

SECTION-A

1. Answer all the questions briefly :

a) If the moment generating function of X is given by $M_x(t) = \dfrac{1}{1-t^2}$ where $|t| < 1$.

Find the moment generating function of $Y = \dfrac{X-4}{4}$.

b) State the axioms of probability.

c) Is weak law of large numbers holds for the sequence of independent and identical distribution random variables? Explain.

d) Show that the characteristic function of the sum of the two independent random variables is equal to the product of their characteristic functions.

e) A sample space consists of four mutually exclusive events *A*, *B*, *C* and *D*. Given that $P(A) = 0.2$, $P(B) = 0.3$, $P(C) = 0.4$. Find the probability of $(A' \cap B' \cap C') \cup D$.

f) A continuous random variable x follows the probability law $f(x) = Ax^2$, $0 \le x \le 1$. Determine A

g) State Jensen and Chebychev's inequalities.

h) Let X be the random variable such that $P(X = -2) = P(X = -1)$, $P(X = 2) = P(X=1)$ and $P(X>0) = P(X<0) = P(X = 0)$. Obtain the probability mass function of X and its distribution functions.

SECTION-B

2. (a) Two fair dice are thrown independently. Three events *A*, *B* and *C* is defined as follows:

   A: Even face with first dice.

   B: Even face with second dice.

   C: Sum of the points on the two dice is odd.

   Discuss the independence of events *A*, *B* and *C*.

   (b) A continuous random variable *X* takes the values from [2, 8] and has a density function of the form *ax*+*b*, *a* and *b* are constants. The expected value of *X* is $\frac{43}{8}$. Calculate (i) *a* and *b* ; (ii) $P(|X-5|<0.5)$.

3. (a) Let *A* and *B* be two events such that $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$, show that $P(A \cup B) \geq \frac{3}{4}$ and $\frac{3}{8} \leq P(A \cap B) \leq \frac{5}{8}$.

   (b) In a competitive examination, an examine either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is 0.35 and the probability that he copies the answer is 0.20. The probability that the answer is correct, given that he copied it is 0.15. Find the probability that he (i) guesses, (ii) copies, (iii) knows, the answer to the question, given that he correctly answered it.

4. (a) Let $f(x,y) = \binom{y}{x} p^x (1-p)^{y-x} \frac{e^{-\lambda}\lambda^y}{y!}$;x = 0,1,2,…; y = 0,1,2,…; with y≥x and *f(x,y)=0 elsewhere.* Find the marginal density function of X and the marginal density function of Y. Also determine whether the random variables X and Y are independent.

   (b) Two dimensional random variables (X,Y) have the joint density $f(x,y) = \begin{cases} \frac{8}{9}xy, 1 \leq x \leq y \leq 2 \\ 0 \quad , \quad otherwise \end{cases}$. Find the marginal density functions of X and Y. Also find the[U , otherwise conditional distribution of Y given X = x and conditional density function of X given Y = y.

SECTION-C

5. (a) Define convergence in probability. Let $X_1, X_2, \ldots, X_n$ be identical and independent distributed variables with mean μ and variance σ^2 and as $n \to \infty$, $(X_1^2 + X_2^2 + \ldots + X_n^2)/n$ converges in probability to some $c(0 \leq c < \infty)$. Find c.

(b) A random variable X has a density function $f(x) = \begin{cases} \lambda e^{-\lambda x}; & x > 0 \\ 0 & ; eleswhere \end{cases}$ Find the moment generating function and the mean and variance..

6. (a) Use Chebychev's inequality to show that for $n > 36$, the probability that, in n throws of a fair die, the number of sixes lie between $\frac{1}{6}n - \sqrt{n}$ and $\frac{1}{6}n + \sqrt{n}$ is atleast $\frac{31}{36}$.

(b) If X has the probability density function $f(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty.$ Find the characterstic function of X and show that variance of X is 2.

7. (a) Show that for the symmetrical density function $f(x) = \frac{2a}{\pi}\left[\frac{1}{a^2 + x^2}\right], \ -a < x < a,$ the odd central moments are zero and the second and fourth central moments are given by $\mu_2 = \frac{a^2(4-\pi)}{\pi}, \ \mu_4 = a^4\left[1 - \frac{8}{3\pi}\right].$

(b) A man draws 3 balls from an urn containing 5 white and 7 lack balls. He gets Rs. 10 for each white ball and Rs. 5 for each black ball. Find his expectation.