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Total No. of Questions : 07

M.Sc (Mathematics) PIT (Sem.–4)
MATHEMATICAL METHODS
Subject Code : MMS-402
Paper ID : [72894]

Time : 3 Hrs.   Max. Marks : 100

INSTRUCTIONS TO CANDIDATES :
1. Attempt any FIVE Questions out of EIGHT questions.
2. All questions carry equal marks.

1. a. Find the inverse laplace transform of $\dfrac{s^2 + 2s + 3}{(s^2 + 2s + 2)(s^2 + 2s + 5)}$.

   b. Solve $y''' - 3y'' + 3y' - y = t^2 e^t; \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = -2,$ using Laplace transform.

2. a. Find the Fourier transform of $e^{-at^2}, a > 0.$ Write the inverse transform also.

   b. Using Fourier sine transform, solve the BVP $y'' - a^2 y = 0$ on $x \geq 0.$ $y'(0) = b, y(\infty) = 0.$

3. Find the eigenvalues and the eigenfunctions of the Fredholm integral equation

$$y(x) = \lambda \int_1^2 \left(st + \frac{1}{xt} \right) y(t)dt.$$

4. Solve the Volterra integral equation $y(x) = x + \int_0^x (t - x)y(t)dt.$

5. a. Find the extremals of the functional $J[y(x)] = \int_0^{\pi/2} y(2x - y)dx, \ y(0) = 0, \ y\left(\dfrac{\pi}{2}\right) = \dfrac{\pi}{2}.$

   b. Determine the plane curve down which a particle will slide without friction from the point $A(x_1, y_1)$ to $B(x_2, y_2)$ in the shortest time.

6. Find an extremal in the isometric problem of the functional

$$J\left[y(x), z(x)\right] = \int_{0}^{1} (y'^2 + z'^2 - 4xz' - 4z)dx$$

$y(0) = 0, z(0) = 0, \; y(1) = 1, \; z(1) = 1$ subject to the condition $\int_{0}^{1} \left(y'^2 - xy' - z'^2\right)dx = 2$.

7. Write the Sturm-liouville equation associated with the quadratic functional

$$J\left[y(x)\right] = \int_{a}^{b} (c_1 y'^2 + cy^2)dx$$

Where c amd $c_1 > 0$ are constants, subkect to the comditions $y(a) = 0, \; y(b) = 0$.

8. Find the eigenvalues and the eigenfunctions of the functional $J\left[y(x)\right] = \int_{0}^{1} (y^2 + y'^2)dx$

subject to the condition $y(0) = 0, y(1) = 0, \; \int_{0}^{1} y^2(x)dx = 1$.