Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐   **Total No. of Pages : 03**

Total No. of Questions : 08

Ph.D Examination
LINEAR ALGEBRA (Mathematics)
M.Code : 78005

Time : 3 Hrs.   **Max. Marks : 100**

INSTRUCTIONS TO CANDIDATES :
1. **Attempt any FIVE questions out of EIGHT questions.**
2. **Each question carries TWENTY marks.**

1. a) Solve the following system of equations using gauss Jordan method :

    $-x + y + 2z = 2$

    $3x - y + z = 6$

    $-x + 3y + 4z = 4$

    b) Give examples of two different basis of $V_2(R)$ and $V_3(R)$.

2. a) State and prove rank nullity theorem.

    b) Solve the following using determinants :

    $x + y + z = 1$

    $ax + by + cz = k$

    $a^2x + b^2y + c^2z = k^2$

3. a) Find the fourier transform of $f(x) = k$ if $0 < x < a$, and $f(x) = 0$

    b) Fit by a straight line

    $(0,3)$, $(2,1)$ $(3,-1)$, $(5,-2)$

4. a) Transform the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ to tridiagonal form by Given's method. Find the

    eigen vector corresponding to largest eigen value.

b) Find the linear transformation T: $R^3 \rightarrow R^4$ whose range is spanned by (1,2,0,-4) and (2,0,-1,-3)

5. a) Test the definiteness of the following function :

$$x_1^2 + x_2^2 + x_3^2 - 2x_1x_2$$

b) Prove that a linear system of equations Ax = b whose condition number is small is well conditioned.

6. a) Use Big M method to Maximize $Z = 6x_1 + 4x_2$

Subject to : $2x_1 + 3x_2 \leq 30$

$3x_1 + 2x_2 \leq 24$

$x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$

b) Use Dual Simplex Method to solve the LPP :

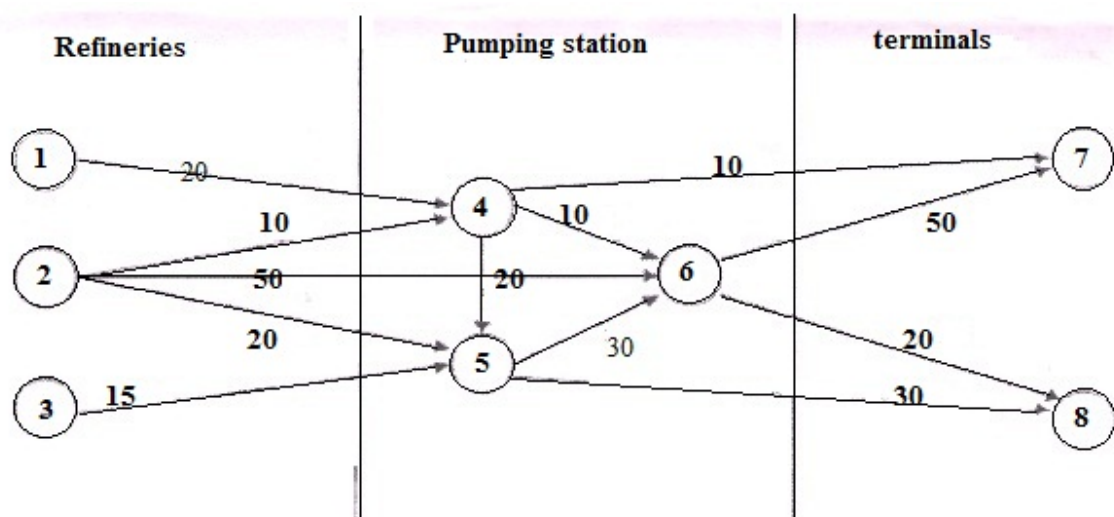Minimize $Z = 3x_1 + x_2$

Subject to : $x_1 + x_2 \geq 1$

$2x_1 + 3x_2 \geq 2$

$x_1, x_2 \geq 0$

7. Solve the following game by linear programming using simplex method and determine the best strategies for both the players.

		Player B		
		3	-4	2
PlayerA		1	-3	-7
		-2	4	7

8. Three refineries send LPG to two distribution terminals through a pipeline network. Any demand that cannot be satisfied through the network is acquired from other source. The pipeline network is served by 3 pumping stations as shown in the figure. The LPG flows in the network in the direction of arrows. The capacity of each pipeline is mentioned on the arcs in [million bbl per day]. To match the maximum capacity of network, determine the following :

a) Daily production at each refinery

b) The daily demand at each terminal

c) The daily capacity of each pump.