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[**Note:** Distribution of Questions between Problems and Theory of this paper must be 60: 40 i:e, Problem Questions: 60 % & Theory Questions : 40 %]

References

Kanishka Bedi, PRODUCTION & OPERATIONS MANAGEMENT, *Oxford, NewDelhi, 2007*

Panneerselvam, R, OPERATIONS RESEARCH, *Prentice-Hall of India, New Delhi, 2002.*

G.Srinivasan, OPERATIONS RESEARCH, *PHI Learning, NewDelhi,2010*

Tulsian & Pandey, QUANTITATIVE TECHNIQUES, *Pearson, NewDelhi, 2002*

Vohra, QUANTATIVE TECHNIQUES IN MANAGEMENT, *Tata McGrawHill, NewDelhi, 2010*

UNIT-I

Lesson.1 - Introduction to Operations Management

Lesson Objectives

- To Introduce The Evolution Of The Field Of Om
- To Brief The Role Of Operations Management In An Organization
- To Introduce The Role Of A Operations Managers
- To Discuss The Role And Importance Of Process Planning
- To Discuss The Importance In Deciding Plant Location
- To Give An Introduction About Various Types Of Layouts
- To Learn The Importance Of Production Planning

Chapter Structure

This Chapter is organized in the following order

1.1 Introduction to Operations Management

1.1.1 Historical Evolution of Operations Management
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1.1 Introduction to Operations Management

Operations Management is a field of management science that deals with the design and management of products, processes, services and supply chains. It deals with acquisition, development, and utilization of various resources that any firms need to deliver the goods and services to their clients want.

The subject coverage in Operations Management ranges from strategic to tactical and to operational levels. For example, it deals with strategic issues such as determining the location for a manufacturing company, type of manufacturing process and size for the factory, expansion strategy for plant location, other manufacturing locations, deciding the structure of service or telecommunications networks, and designing technology supply chains etc.

Also, various tactical issues like, plant layout and structure, project management methods, and equipment selection and replacement, the application of Operations Management is evident. Operational issues include production scheduling and control, inventory management, quality control and inspection, traffic and materials handling, and equipment maintenance policies.

Production and Operations Management ("POM") is about the transformation of production and operational inputs into "outputs" that, when distributed, meet the needs of customers. The process is often referred to as the **"Conversion Process".**

There are several different methods of handling the conversion or production process - **Job, Batch, Flow and Group.** POM incorporates many tasks that are interdependent, but which can be grouped under five main headings, which is briefly discussed in the following pages.

Product

Marketers in any business concerns about selling products that meet customer needs and wants. In fulfilling this objective, the role of Production and Operations play a major role; it has to ensure that the business actually makes the required products in accordance with the expectations of market and consumers and translated as a plan. The role of PRODUCT in POM therefore concerns areas such as:

- Performance
- Aesthetics
- Quality
- Reliability
- Quantity
- Production costs
- Delivery dates

Plant

To make the needed product, the 'PLANT' of some kind is needed for any business house. This will comprise the bulk of the fixed assets and many short term assets, set of creditors, who supply the requirement materials and many others to the business. In determining which PLANT to use, management must consider areas such as

- Future demand (volume, timing)
- Design and layout of factory, equipment, offices
- Productivity and reliability of equipment
- Need for (and costs of) maintenance
- Health and safety (particularly the operation of equipment)
- Environmental issues (e.g. creation of waste products)

Processes

There are many different ways of producing a product. Management must choose the best process, or series of processes. They will consider

- Available capacity
- Available skills
- Type of production
- Layout of plant and equipment
- Safety
- Production costs
- Maintenance requirements

Programmes

In the production management terminology, Programme concerns the dates and times of the products that are to be produced and supplied to customers. The decisions made about programme will be influenced by factors such as

- Purchasing patterns (e.g. lead time)
- Cash flow
- Need for / availability of storage
- Transportation

People

ộ *Production depends on PEOPLE, whose skills, experience and motivation vary. Key people-related decisions will consider the following areas*

- Wages and salaries
- Safety and training
- Work conditions
- Leadership and motivation
- Unionisation
- Communication

1.1.1 Historical Evolution of Operations Management

The subject Operations management has its own connection with the age old Industrial Revolution, which has started during the late 17th century in England and later spread to the rest of Europe and to the United States during the 19th century. Prior to that time, goods were manufactured in small quantities in smaller shops / factories by the local craftsmen and their apprentices, who were mostly their family members. Under that system, it was common for one person to be responsible for making a product, such as a horse-drawn wagon or a piece of furniture, from start to finish. Only simple tools were available; the machines that we use today had not been invented.

Later, in the 18th century, many scientific inventions came into existence and changed the face of production / operations by substituting huge machines, which are operated by steam power and electric power. Perhaps the most significant of these inventions, was the steam engine; it had the ability to provide power to operate huge machineries in the factories. For example, the spinning jenny and the power looms revolutionized the textile industry. Ample supplies of coal and iron ore provided materials for generating power and making machinery. The new machines, made of iron, were much stronger and more durable than the simple wooden machines they replaced.

From the late 17th century (1770) to the early years of the 18th century, series of events took place in England which together is called the Industrial Revolution.

Industrial Revolution resulted in two major developments: widespread substitution of machine power for human power and establishment of the organized production system known as factory system.

The events that took place from 1770 to the 1800s are characterized by great inventions. The great inventions were eight in number ,with six of them having been conceived in England, one in France and one in the United States .The eight inventions are—Hargreaves Spinning Jenny, Arkwright's Water Frame, Crompton's Mule, Cartwright's Power Loom, Watt's steam engine, Berthollet's Chlorine Bleaching Discovery, Mandslay's Screw-Cutting Lathe and Eli Whitney's Interchangeable Manufacture.

As observed from eight inventions, most of them have to do with the spinning of yarn and weaving of cloth. This is logical from the point of view that cloth was the principal export commodity of England at that time and was in short supply owing to the considerable expansion of England's colonial empire and its commercial trade.

The availability of machine power greatly facilitated the gathering of workers in factories that housed the machines. The large number of workers congregated in the factories, created the need for organizing them in logical ways to produce goods.

The publication of Adam Smith's The Wealth of Nations in 1776 advocated the benefits of the division of labor or specialization of labor, which broke production of goods into small specialized tasks that were assigned to workers on production lines. Thus, the factories of late 1700s not only had developed production machinery, but also ways of planning and controlling the output of workers.

The impact of the Industrial Revolution was first felt in England. From here, it spread to other European countries and to the United States. The Industrial Revolution advanced further with the development of the gasoline engine and electricity in the 1800s. Other industries emerged and along with them new factories came into being. By the middle of 18th century, the old cottage system of production had been replaced by the large scale factory system. As days went by, production capacities expanded, demand for capital grew and labor became highly dependent on jobs and urbanized. At the commencement of the 20th century, the one element that was missing was a management –the ability to develop and use the existing facilities to produce on a large scale to meet massive markets of today.

Later, the Scientific Management Era has brought widespread changes to the practices and management of factories. The movement was spearheaded by the efficiency engineer and inventor ***Frederick Winslow Taylor,*** who is often referred to as the ***Father of Scientific Management.*** Taylor believed in a "science of management" based on observation, measurement, analysis and improvement of work methods, and economic incentives. He studied work methods in great detail to identify the best method for doing each job. Taylor also believed that management should be responsible for planning, carefully selecting and training workers, finding the best way to perform each job, achieving cooperation between management and workers, and separating management activities from work activities.

1.1.2 Operations performance objectives

An important point to be noted at this section is that operations management deals with set of objectives, which are very broad. In general, we can classify operations management impact on the five broad categories of stakeholders; customers, suppliers, shareholders, employees and society.

Stakeholders is a broad term but is generally used to mean anybody who could have an interest in, or is affected by, the operation.

- **Customers** – These are the most obvious people who will be affected by any business.
- **Suppliers** – Operations can have a major impact on suppliers, both on how they prosper themselves, and on how effective they are at supplying the operation.
- **Shareholders** – Clearly, the better operation is at producing goods and services, the more likely the whole business is to prosper and shareholders will be one of the major beneficiaries of this.
- **Employees** – Similarly, employees will be generally better off if the company is prosperous; if only because they are more likely to be employed in the future. However operations responsibilities to employees go far beyond this. It includes the general working conditions which are determined by the way the operation has been designed.
- **Society** – Although often having no direct economic connection with the company, individuals and groups in society at large can be impacted by the way its operations managers behave. The most

obvious example is in the environmental responsibility exhibited by operations managers.

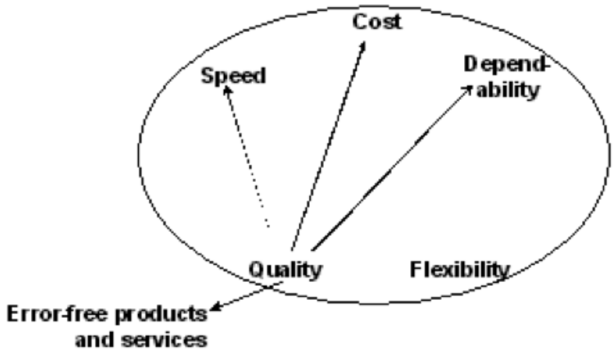
We will discuss briefly the five performance objectives, namely, quality, speed, dependability, flexibility, and cost in the following paragraph.

Quality

Quality is placed first in our list of performance objectives because many authorities believe it to be the most important. Certainly more has been written about it than almost any other operations performance objective over the last twenty years. As far as this introduction to the topic is concerned, quality is discussed largely in terms of it meaning 'conformance'. That is, the most basic definition of quality is that a product or service is as it is supposed to be. In other words, it conforms to its specifications.

There are two important points to remember when reading the section on quality as a performance objective.

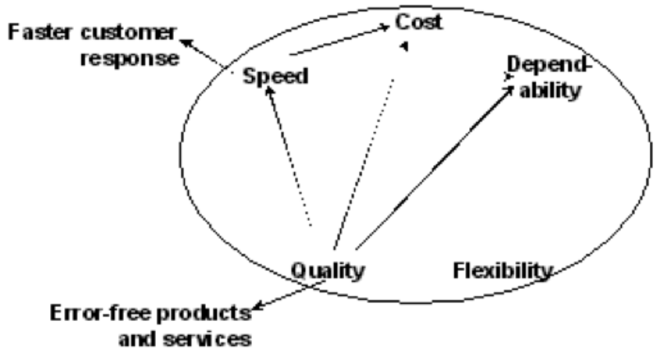
- The external affect of good quality within in operations is that the customers who 'consume' the operations products and services will have less (or nothing) to complain about. And if they have nothing to complain about they will (presumably) be happy with their products and services and are more likely to consume them again. This brings in more revenue for the company (or clients satisfaction in a not-for-profit organisation).
- Inside the operation quality has a different affect. If conformance quality is high in all the operations processes and activities very few mistakes will be being made. This generally means that cost is saved, dependability increases and (although it is not mentioned explicitly in the chapter) speed of response increases. This is because, if an operation is continually correcting mistakes, it finds it difficult to respond quickly to customers requests. See the figure below.



Speed

Speed is a shorthand way of saying 'Speed of response'. It means the time between an external or internal customer requesting a product or service, and them getting it. Again, there are internal and external affects.

- Externally speed is important because it helps to respond quickly to customers. Again, this is usually viewed positively by customers who will be more likely to return with more business. Sometimes also it is possible to charge higher prices when service is fast. The postal service in most countries and most transportation and delivery services charge more for faster delivery, for example.
- The internal affects of speed have much to do with cost reduction. Two areas where speed reduces cost is reducing inventories and reducing risks. Usually, faster throughput of information (or customers) will mean reduced costs. So, for example, processing passengers quickly through the terminal gate at an airport can reduce the turn round time of the aircraft, thereby increasing its utilization. This is best thought of the other way round, 'how is it possible to be on time when the speed of internal throughput within an operation is slow?' When materials, or information, or customers 'hangs around' in a system for long periods (slow throughput speed) there is more chance of them getting lost or damaged with a knock-on effect on dependability.

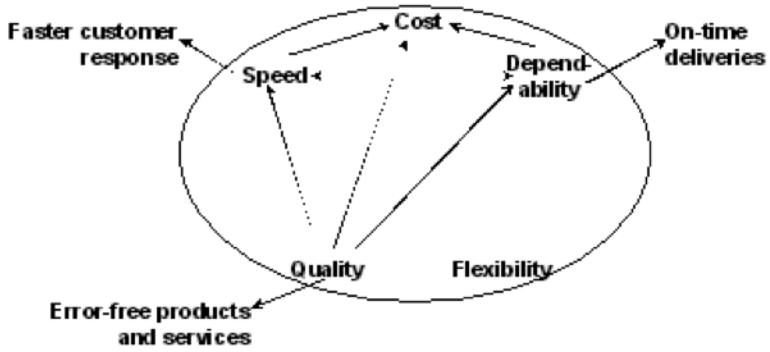


Dependability

Dependability means 'being on time'. In other words, customers receive their products or services on time. In practice, although this definition sounds simple, it can be difficult to measure. What exactly is on time? Is it when the customer needed delivery of the product or service? Is it when they expected delivery? Is it when they were promised delivery? Is it when they were promised delivery the second time after it failed to be delivered the first time? Again, it has external and internal affects.

- Externally (no matter how it is defined) dependability is generally regarded by customers as a good thing. Certainly being late with delivery of goods and services can be a considerable irritation to customers. Especially with business customers, dependability is a particularly important criterion used to determine whether suppliers have their contracts renewed. So, again, the external affects of this performance objective are to increase the chances of customers returning with more business.

- Internally dependability has an effect on cost. Three ways in which costs are affected – by saving time (and therefore money), by saving money directly, and by giving an organisation the stability which allows it to improve its efficiencies.



Flexibility

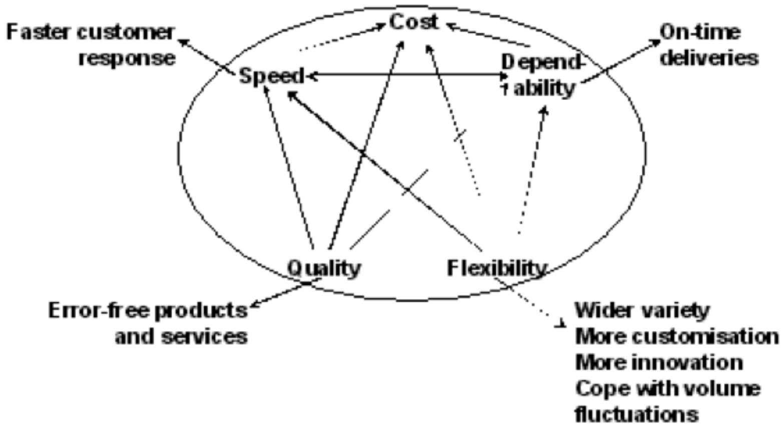
This is a more complex objective because we use the word 'flexibility' to mean so many different things. The important point to remember is that flexibility always means 'being able to change the operation in some way'. Some of the different types of flexibility include product/service flexibility, mix flexibility, volume flexibility, and delivery flexibility. It is important to understand the difference between these different types of flexibility, but it is more important to understand the affect flexibility can have on the operation.

Externally the different types of flexibility allow an operation to fit its products and services to its customers in some way. Mix flexibility allows an operation to produce a wide variety of products and services for its customers to choose from.

Product/service flexibility allows it develops new products and services incorporating new ideas which customers may find attractive.

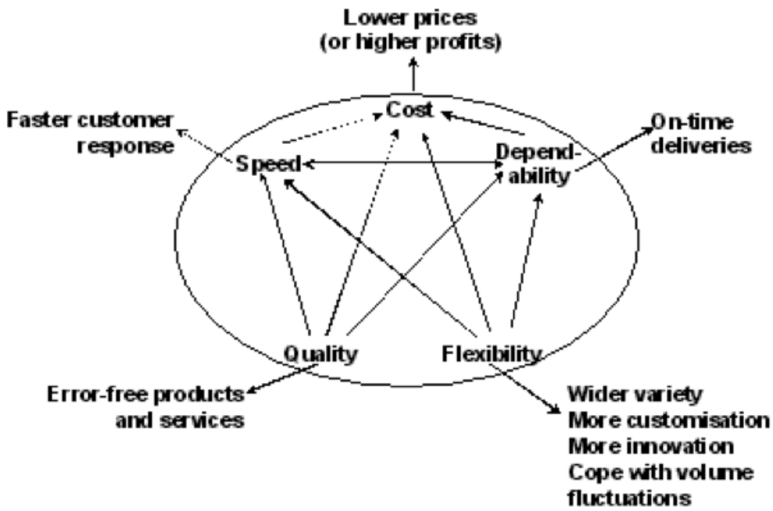
Volume and delivery flexibility allow the operation to adjust its output levels and its delivery procedures in order to cope with unexpected changes in how many products and services customers want, or when they want them, or where they want them.

- Once again, there are several internal affects associated with this performance objective. Among them, three most important factors are flexibility speeds up response, flexibility saves time (and therefore money), and flexibility helps maintain dependability.



Cost

The first important point on cost is that the cost structure of different organisations can vary greatly. Second, and most importantly, the other four performance objectives all contribute, internally, to reducing cost. This has been one of the major revelations within operations management over the last twenty years.



"If managed properly, high quality, high speed, high dependability and high flexibility can not only bring their own external rewards, they can also save the operation cost."

1.1.3 Role of Operations Management

Even if you want to specialize in the domains like, finance or marketing, still you have to study a course on Operations Management. There are number of reasons to quote; but the most important among them is that 50 percent or more of all jobs are in operations management or related fields. Moreover, recall the image of a business organization as a car, with operations as its engine, in order for that car to function properly, all of the parts must work together. So, too, all of the parts of a business organization must work together in order for the organization to function successfully.

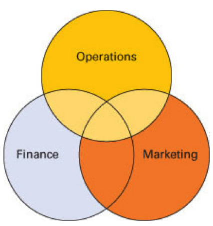
For the successful functioning of the organisation, members of various functional domains shall work together; thus, it is very much essential for all members of the organization to understand not only their own role in their functional specialization, but, they also understand the roles of others.

This is precisely why all business students, regardless of their majors, are required to take a common core of courses that will enable them to learn about all aspects of business. Because operations management is central to the functioning of all business organizations, it is included in the core of courses business students are required to take.

And even though individual courses have a narrow focus (e.g., accounting, marketing), in practice, there is significant interfacing and collaboration among the various functional areas, involving exchange of information and cooperative decision making. For example, although the three primary functions in business organizations perform different activities, many of their decisions impact the other areas of the organization. Consequently, these functions have numerous interactions, as depicted by the overlapping circles shown in diagram.

figure 1

The three major functions of business organizations overlap



Finance and operations management personnel cooperate by exchanging information and expertise in various activities the following are the illustrative but not an exhaustive list.

- ***Budgeting:*** Budgets must be periodically prepared to plan financial requirements. Budgets must sometimes be adjusted, and performance relative to a budget must be evaluated.
- ***Economic analysis of investment proposal:*** Evaluation of alternative investments in plant and equipment requires inputs from both operations and finance people.
- ***Provision of funds:*** The necessary funding of operations and the amount and timing of funding can be important and even critical when funds are tight. Careful planning can help avoid cash-flow problems.

Marketing's focus is on selling and/or promoting the goods or services of an organization. Often, the marketing department share invaluable information to the operations managers and their team.

- ***Demand Estimation:*** Marketing, which is also responsible for assessing customer wants and needs, communicating those to operations people (short term) and to design people (long term); that is, operations needs information about demand over the short to intermediate term so that it can plan accordingly (e.g., purchase materials or schedule work), while design people need information that relates to improving current products and services and designing new ones.
- Marketing, design, and production must work closely together to successfully implement design changes and to develop and produce new products. Marketing can provide valuable insight on what competitors are doing. Marketing also can supply information on consumer preferences so that design will know the kinds of products and features needed; operations can supply information about capacities and judge the manufacturability of designs.

- Operations will also have advance warning if new equipment or skills will be needed for new products or services. Finance people should be included in these exchanges in order to provide information on what funds might be available (short term) and to learn what funds might be needed for new products or services (intermediate to long term). One important piece of information marketing needs from operations is the manufacturing or service lead time in order to give customers realistic estimates of how long it will take to fill their orders.

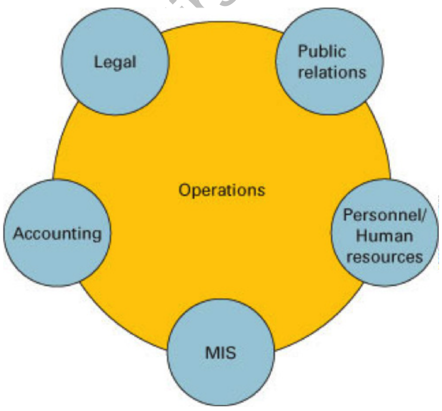
Thus, marketing, operations, and finance must interface on product and process design, forecasting, setting realistic schedules, quality and quantity decisions, and keeping each other informed on the other's strengths and weaknesses.

People in every area of business need to appreciate the importance of managing and coordinating operations decisions that affect the supply chain and the matching of supply and demand, and how those decisions impact other functions in an organization.

Operations also interacts with other functional areas of the organization, including legal, management information systems (MIS), accounting, personnel/human resources, and public relations, as depicted in the following diagram.

Figure 2

Operations interfaces with a number of supporting functions



The legal department must be consulted on contracts with employees, customers, suppliers, and transporters, as well as on liability and environmental issues. Accounting supplies information to management on costs of labor, materials, and overhead, and may provide reports on items such as scrap, downtime, and inventories.

Management information systems *(MIS)* is concerned with providing management with the information it needs to effectively manage. This occurs mainly through designing systems to capture relevant information and designing reports. MIS is also important for managing the control and decision-making tools used in operations management.

The personnel or human resources department is concerned with recruitment and training of personnel, labor relations, contract negotiations, wage and salary administration, assisting in manpower projections, and ensuring the health and safety of employees.

Public relations department has responsibility for building and maintaining a positive public image of the organization. Good public relations provide many potential benefits to the organisation. An obvious one is in the marketplace. Other potential benefits include public awareness of the organization as a good place to work (labor supply), improved chances of approval of zoning change requests, community acceptance of expansion plans, and instilling a positive attitude among employees.

1.1.4 Roles and Responsibility of an Operations Manager

Some people, particularly, those professionally involved in operations management, argue that operations management involves everything an organisation does. In this sense, every manager is an operations manager, since all managers are responsible for contributing to the activities required to create and deliver an organization's goods or services. However, others argue that this definition is too wide, and that the operations function is about producing the right amount of a good or service, at the right time, of the right quality and at the right cost to meet customer requirements.

A stereotypical example of an operations manager would be a plant manager in charge of a factory, such as an automobile assembly plant. But other managers who work in the factory in departments like quality assurance, production and inventory control and line supervisions can also be considered to be working in operations management. In service industries, managers in hotels, restaurants, banks, airline operations, hospital and stores are operations managers. In the not-for-profit sector, the manager of a nursing home or day centre for older people is an operations manager, as they are the managers of a local government tax-collection office and the manager of a charity shop staffed entirely by volunteers.

Operations managers are responsible for managing activities that are part of the production of goods and services. Their direct responsibilities include managing the operations process, embracing design, planning, control, performance improvement, and operations strategy. Their indirect responsibilities include interacting with those managers in other functional areas within the organisation whose roles have an impact on operations. Such areas include marketing, finance, accounting, personnel and engineering.

Operations managers' responsibilities include:

- ***Human resource management*** – the people employed by an organisation either work directly to create a good or service or provide support to those who do. People and the way they are managed are a key resource of all organisations.
- ***Asset management*** – an organization's buildings, facilities, equipment and stock are directly involved in or support the operations function.
- ***Cost management*** – most of the costs of producing goods or services are directly related to the costs of acquiring resources, transforming them or delivering them to customers. For many organisations in the private sector, driving down costs through efficient operations management gives them a critical competitive edge. For organisations in the not-for-profit sector, the ability to manage costs is no less important.

The chief role of an operations manager is planning and decision making. As an operations manager in an organisation, he exerts considerable influence over the degree to which the goals and objectives of the organization are realized.

Most decisions involve many possible alternatives that can have quite different impacts on costs or profits. Consequently, it is important to make informed decisions.

Decision making is a central role of all operations managers. Decisions need to be made in:

- Designing the operations system
- Managing the operations system
- Improving the operations system

Operations management professionals make a number of key decisions that affect the entire organization. These include the following:

1. The processes by which goods and services are produced
2. The quality of goods or services
3. The quantity of goods or services (the capacity of operations)
4. The stock of materials (inventory) needed to produce goods or services
5. The management of human resources

You can put them under the following questions

What: What resources will be needed, and in what amounts?

When: When will each resource be needed? When should the work be scheduled? When should materials and other supplies be ordered? When is corrective action needed?

Where: Where will the work be done?

How: How will the product or service be designed? How will the work be done (organization, methods, equipment)?

How will resources be allocated?

Who: Who will do the work?

The operations function consists of all activities *directly* related to producing goods or providing services. Hence, it exists both in manufacturing and assembly operations, which are *goods-oriented,* and in areas such as health care, transportation, food handling, and retailing, which are primarily *service-oriented.*

The following table provides examples of the diversity of operations management settings.

Table 1 Examples of types of operations

Type of Operations	Examples
Goods producing	Farming, mining, construction, manufacturing, power generating
Storage/transportation	Warehousing, trucking, mail service, moving, taxis, buses, hotels, airlines
Exchange	Retailing, wholesaling, financial advising, renting or leasing, library loans, stock exchange
Entertainment	Films, radio and television, plays, concerts, recording
Communication	Newspapers, radio and TV newscasts, telephone, satellites, the Internet

A primary function of an operations manager is to guide the system by decision making. Certain decisions affect the design of the system, and others affect the operation of the system.

System design involves decisions that relate to system capacity, the geographic location of facilities, arrangement of departments and placement of equipment within physical structures, product and service planning, and acquisition of equipment. These decisions usually, but not always, require long-term commitments. Moreover, they are typically *strategic* decisions.

System operation involves management of personnel, inventory planning and control, scheduling, project management, and quality assurance. These are generally *tactical* and *operational* decisions.

Feedback on these decisions involves *measurement* and *control.* In many instances, the operations manager is more involved in day-to-day operating decisions than with decisions relating to system design. However, the operations manager has a vital stake in system design because *system design essentially determines many of the parameters of system operation.* For example, costs, space, capacities, and quality are directly affected by design decisions. Even though the operations manager is not responsible for making all design decisions, he or she can provide those decision makers with a wide range of information that will have a bearing on their decisions.

Purchasing has responsibility for procurement of materials, supplies, and equipment. Close contact with operations is necessary to ensure correct quantities and timing of purchases. The purchasing department is often called on to evaluate vendors for quality, reliability, service, price, and ability to adjust to changing demand. Purchasing is also involved in receiving and inspecting the purchased goods.

Industrial engineering is often concerned with scheduling, performance standards, work methods, quality control, and material handling.

Distribution involves the shipping of goods to warehouses, retail outlets, or final customers.

Maintenance is responsible for general upkeep and repair of equipment, buildings and grounds, heating and air-conditioning; removing toxic wastes; parking; and perhaps security. The operations manager is the key figure in the system: He or she has the ultimate responsibility for the creation of goods or provision of services.

The kinds of jobs that operations managers oversee vary tremendously from organization to organization largely because of the different products or services involved. Thus, managing a banking operation obviously requires a different kind of expertise than managing a steelmaking operation. However, in a very important respect, the jobs are the same: They are both essentially *managerial*. The same thing can be said for the job of any operations manager regardless of the kinds of goods or services being created.

The importance of operations management, both for organizations and for society, should be fairly obvious: The consumption of goods and services is an integral part of our society. Operations management is responsible for creating those goods and services. Organizations exist primarily to provide services or create goods. Hence, operations management is the *core function* of an organization. Without this core, there would be no need for any of the other functions—the organization would have no purpose. Given the central nature of its function, it is not surprising that more than half of all employed people in this country have jobs in operations. Furthermore, the operations function is responsible for a major portion of the assets in most business organizations.

The service sector and the manufacturing sector are both important to the economy. The service sector now accounts for more than 70 percent of jobs in the country, and it is growing in other countries as well. Moreover, the number of people working in services is increasing, while the number of people working in manufacturing is not. The reason for the decline in manufacturing jobs is two fold:

- As the operations function in manufacturing companies finds more productive ways of producing goods, the companies are able to maintain or even increase their output using fewer workers.
- Furthermore, some manufacturing work has been outsourced to more productive companies, many in other countries that are able to produce goods at lower costs.

1.1.5 Productions/Operations Management Problems

POM is a functional field of business with clear line management responsibilities. Problems of management in the production/operations function basically concerns two types of decision:

Those relating to the design or establishment of the production/operations system.

i. Those relating to the operation, performance and running of the production/operations system.

Problems in the design of production/operations system are as follows:

i. Design/specification of goods/service,
ii. Location of facilities,
iii. Layout of facilities/resources and materials handling,
iv. Determination of capacity/capability,

v. Design of works or jobs,

vi. Involvement in determination of remuneration system and work standards.

Problems in the operation of system are:

i. Planning and scheduling of activities,

ii. Inventory (Stock) control,

iii. Quality control,

iv. Maintenance and replacement,

v. Involvement in performance measurement.

Every business organization will embrace these problems areas to a greater or lesser extent. The relative emphasis will differ between companies and industries, and also over a period of time. Problems in the first section are of long term nature and will assume considerable importance at only infrequent intervals. Problems in the second section will be of a resurring nature, i.e. they are of short term nature.

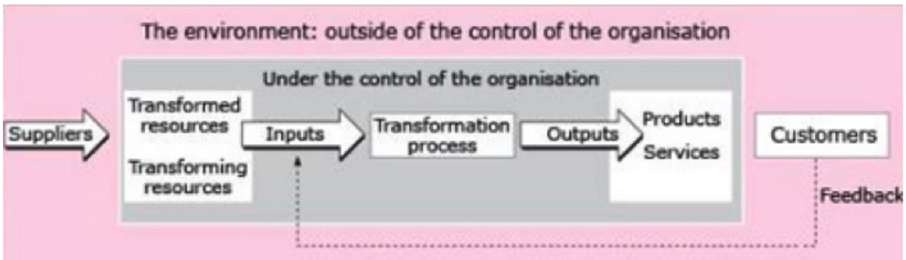
1.1.6 The boundary of the operations system

The simple transformation model given in the following diagram provides significant understanding and powerful tool for looking at operations in many different contexts.

It helps the decision maker to analyse and design operations in many types of organisation at many levels.

This model can be developed by identifying the boundaries of the operations system through which an organization's goods or services are provided to its customers or clients. The diagram shows this boundary and added three components that are located outside it:

- Suppliers
- Customers
- The environment



In any business, the set of suppliers provides inputs to the operations system. They may supply raw materials (for example sugarcane manufacturers provide sugarcane to Sugar Manufacturing units such as EID Parry / Sakthi Sugar etc; TVS is providing various nuts and bolts to automobile manufacturers / other equipment manufacturers), components (Prical provides speed measuring device to two wheeler manufacturers such as Hero Honda, Yamaha), finished products (for example a pharmaceutical company providing drugs to a hospital, or an office supplies company providing it with stationery) or services (as in the case of a law firm providing legal advice).

The customers (or clients) are the users of the outputs of the transformation process. The boundary drawn in the above diagram, represents the transforming process can be thought of as the boundary of the organisation, so that the whole organisation is viewed as an operations system, with its customers external to it. This may be an appropriate way of viewing a small organisation, whose outputs go directly to its external customers.

However, many macro operations are made up of a number of micro operations, or sub-systems. Only the outputs of the final micro operation go directly to a customer or client who is not part of the organisation that is carrying out the macro operation. The final user or client of the good or service is the organisation's external customer, and the users or clients of the outputs of the other micro operations internal customers. Most of the operations in a large organisation serve internal, rather than external customers. For example, if you are the manager of a human resources department, a printing unit or a building maintenance section within a large organisation, your customers are internal: they are other sub-systems within the larger organisation that are external to your operations system but internal to the organisation as a whole.

All operating systems are influenced by the organization's environment. This environment includes both other functional areas within the organisation, each with its own policies, resources, forecasts, goals, assumptions and constraints, and the wider world outside the organisation – the legal, political, social and economic conditions within which it is operating.

Changes in either the internal or the external environment may affect the operations function. Traditionally, organisations have kept the operations function separate from both its customers and its suppliers, in order to protect it from environmental disturbances (Thompson, 1967). This can lead to a 'closed system' mentality, in which the operations function loses contact with external customers and suppliers, and focuses only on the transformation process that it controls. A closed system tends to limit flexibility and result in a loss of competitiveness. An 'open system' mentality, in which communication with customers and suppliers is encouraged, seeks to reduce the barriers between the operations function and its environment, in order to enhance the organization's competitiveness.

An added complication is that, as organisations become more complex, it becomes increasingly difficult to draw neat boundaries around the operations function. Operations management must therefore focus its attention on key interfaces within the organisation, as well as on interfaces between the organisation and its external customers and suppliers. Most operations systems are part of a supply chain that involves materials, information and customers, and the distribution of finished goods or services to customers or clients. It is therefore the responsibility of the operations function to co-ordinate the flow of information that links these activities through the supply chain. Thus, while some operations managers are concerned only with the transformation process within a single organisational unit, such as a factory or service outlet, many are involved in managing operations across several organisational units or even across separate organisations.

1.2 Process Planning

Any business, the success predominantally depenps upon the effective production/operations process. There are numerous types of production processess and there are also many ways of classifying or grouping them for descriptive purposes. Classifying production/ operations processes by their characteristics can provide valuable insights into how they should be managed.

In general, the processes by which goods and services are produced can be categorised in two traditional ways.

1. Firstly, we can identify continuous, repetitive, intermittent and job shop production process.
2. Second and similar classification divides production processes into Process production, Mass production, Batch production and jobbing production.

We will breifly introduce these methods in the following paragraphs.t

Job shop

A wide variety of customized products are made by a highly skilled workforce using general-purpose equipment. These processes are referred to as jumbled-flow processes because there are many possible routings through the process.

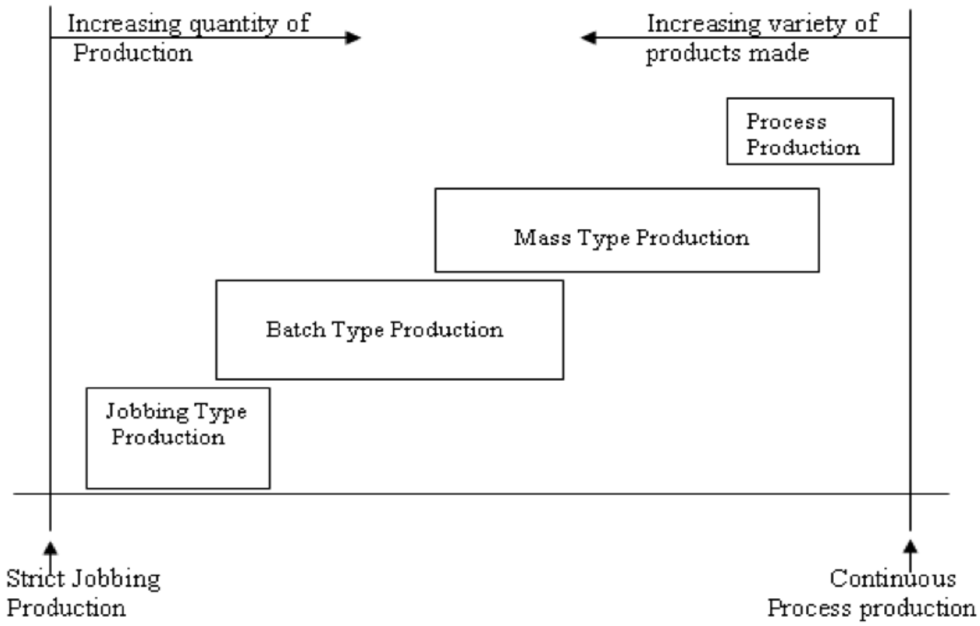
Examples: Home renovating firm, stereo repair shop, restaurants.

Intermittent (batch) flow

A mixture of general-purpose and special-purpose equipment is used to produce small to large batches of products.

Examples: Clothing and book manufacturers, winery, caterer.

Figure 3 Types Of Production Process



Repetitive flow (mass production)

The product or products are processed in lots, each item of production passing through the same sequence of operations, i.e. several standardized products follow a predetermined flow through sequentially dependent work centers. Workers typically are assigned to a narrow range of tasks and work with highly specialised equipment.

Examples: Automobile and computer assembly lines, insurance home office.

Continuous flow (flow shop)

Commodity like products flow continuously through a linear process. This type of process will theoretically run for 24 hrs/day, 7 days/ week and 52 weeks/year and, whilst this is often the objective, it is rarely achieved.

Examples: chemical, oil, and sugar refineries, power and light utilities.

These four categories represent points on continuum of process organisations. Processes that fall within a particular category share many characteristics that fundamentally influence how a process should be managed.

The second and similar classification divides production processes into:

Process Production

Processes that operate continually to produce a very high volume of a standard product are termed "Processes". This type of process involves the continuous production of a commodity in bulk, often by chemical rather than mechanical means, such as oil and gas. Extra examples of a continuous processes oil refinery, electricity production and steel making.

Mass Production

It is conceptually similar to process production, except that discrete items such as motorcars and domestic appliances are usually involved. A single or a very small range of similar items is produced in very large numbers. In other words, processes that produce high-volume and low-variety products are termed line or mass processes. Because of the high volumes of product it is cost-effective to use specialised labour and equipment.

Batch Production

Processes that produce products of medium variety and medium volume are termed "batch processes". It occurs where the number of discrete items to be manufactured in a period is insufficient to enable mass production to be used. Similar items are, where possible, manufactured together in batches. In other words, batch processes cover a relatively wide range of volume and variety combination. Products are grouped into batches whose batch size can range from two to hundreds.

Jobbing Production (Project Type Production)

Processes that produce high-variety and low-volume products are known as "jobbing". Although strictly consisting of the manufacture of different products in unit quantities (in practice corresponds to the intermittent process mentioned above). This type of production assumes a one-of-a-kind production output, such as a new building or developing a new software application. The equipment is typically designed for flexibility and often general purpose, meaning it can be used for many different production requirements.

Often, it is a practice that a firm has more than one type of operating process in its production system to manage the resources optimally. Sometime, the labour may not be available; on other occasion, the raw material may be short; market may slow down or go up exponentially. For instance, a firm may use a repetitive-flow process to produce high-volume parts but use an intermittent-flow process for lower-volume parts.

A link often exists between a firm's product line and its operating processes. Job shop organisations are commonly utilised when a product or family of products is first introduced. As sales volumes increase and the product's design stabilises, the process tends to move along the continuum toward a continuous-flow shop. Thus, as products evolve, the nature of the operating processes used to produce them evolves as well.

1.2.1 Efficiency of the production process

The creation of goods and services requires changing resources into goods and services. Productivity is used to indicate how good an operation is at converting inputs to outputs efficiently. The more efficiently we make this change the more productive we are. The production/operations manager's job is to enhance (improve) this ratio of outputs to inputs.

Productivity

It is the ratio of outputs (goods and service) divided by one or more inputs (such as labour, capital or management)

Productivity is a measure of operational performance. Thus improving productivity means improving efficiency. This improvement can be achieved in two ways:

1. *Reduction in inputs while output remains constant,* ***or***
2. *Increase in output while inputs remain constant.*

Both represent an improvement in productivity. Production is the total goods and services produced. High Production may imply only that more people are working and that employment levels are high (low unemployment), but it does not imply high productivity.

Productivity measures can be based on a single input (Single-Factor Productivity or Partial Productivity) or on more than one input (Multi-Factor Productivity) or on all inputs. The choice depends on the purpose of the measurement.

Single-factor Productivity

It indicates the ratio of one resource (input) to the goods and services produced (outputs).

For example, for labour productivity, the single input to the operation would be employee hours.

Productivity = {Output of a specific Product}/ {Input of a specific Resource}

Multi-factor Productivity

Indicates the ratio of many or all resources (inputs) to the goods and services produced (outputs). When calculating multi-factor productivity, all inputs must be converted into a common unit of measure, typically cost.

$$Productivity = \frac{Output}{Labour + Material + Energy + Capital + Miscellaneaus}$$

1.3 Plant Location

Every business is facing the issue of selecting the suitable location for their factory plant. Units concerning both manufacturing as well as the assembling of the products are on a very large scale affected by the decisions involving the location of the plant. Location of the plant itself becomes a very important factor concerning service facilities, as the plant location decisions are strategic and long-term in nature. ***Plant location*** *refers to the choice of region and the selection of a particular site for setting up a business or factory.*

An ideal location is on where the cost of the product is kept to minimum, with a large market share, the least risk and the maximum social gain. It is the place of maximum net advantage or which gives lowest unit cost of production and distribution. For achieving this objective, small-scale business can make use of location analysis for this purpose.

1.3.1Need for the plant location analysis

The strategic nature of the decision on Plant location, require very detailed analysis due to several reasons. But the choice is made only after considering various costs associated and comparing the benefits of different alternative sites. It is a strategic decision that cannot be changed once taken. If changed, it can happen at the cost of huge cash outflow as well as considerable deployment of various firm's resources. Each individual plant is a case in itself. The major reasons are,

1. Wrong plant location generally affects cost parameters i.e. poor location can act as a continuous stimulus of higher cost. Marketing, transportation, quality, customer satisfaction are some of the other factors which are greatly influenced by the plant location decisions – hence these decisions require in-depth analysis.

2. Once a plant is set up at a location which is not much suitable, it is a very disturbing as well as very expensive process to shift works of a company to some other place, as it would largely affect the cycle of production.

3. The investments involved in the in setting up of the plant premises .buying of the land etc are very large and especially in the case of big multinational companies, the investments can go into millions of rupees, so economic factors of the location should be very minutely and carefully checked and discussed in order to achieve good returns on the money which has been invested.

1.3.2 Plant location analysis

Location analysis is a dynamic process where the business analyses and compares the appropriateness or otherwise of alternative sites with the aim of selecting the best site for a given firm. It consists of the following:

Demographic Analysis

It involves study of population in the area in terms of total population, age of the population group, per capita income of the state, country and at times, the district, adjacent district per capita incomes, educational level, occupational structure etc. This will give an insight about the market, availability of manpower and composition of trained manpower.

Trade Area Analysis

It is an analysis of the geographic area that provides continued clientele to the firm. The business would also see the feasibility of accessing the trade area from alternative sites. It involves the transportation cost, mode of transportation, availability of infrastructure such as road, railway lines and sea and air ports and facilities such as storages, climate condition, which may also influence the firm's decision.

Competitive Analysis

It helps to judge the nature, location, size and quality of competition in a given trade area.

Traffic analysis

To have a rough idea about the number of potential customers passing by the proposed site during the working hours of the shop, the traffic analysis aims at judging the alternative sites in terms of pedestrian and vehicular traffic passing a site. This will give an idea about how other business units evaluate the site.

Site economics

Alternative sites are evaluated in terms of establishment costs and operational costs under this. Costs of establishment is basically cost incurred for permanent physical facilities but operational costs are incurred for running business on day to day basis, they are also call d as running costs.

1.3.3 Factors influencing Manufacturing Plant Location

Plant location decisions are needed when a new plant is to be set up or when the operations involved in the company at the present location need to be expanded but expansion becomes difficult because of the poor selection of the site for such operations. These decisions are sometimes taken because of the social or the political conditions engulfing the working of a company.

The way the works of a company have to be performed, largely depends upon the industrial policies issued by the government. Any change that creeps in the industrial policy of the government which favors decentralization and hence does not permit any change or any expansion of the existing plant – requires strictly evaluated location decisions. We will broadly put the factors into four heads;

Operational Factors

Operational factors that play a key role in factory location or relocation are diverse, touching on everything from cost consciousness and labor management to strategic direction and regulatory compliance. Other elements in the plant-opening equation include government stimulus programs -- such as fiscal incentives -- and geographical convenience—availability of land / power and other related infrastructure.

A company's top brass may take various steps to analyze plant location issues and remedy problems with factory site selection. Senior executives may develop an objective understanding of the best locations to pick, why some sites are inappropriate, how to avert logistical nightmares with respect to worker commutes and how the site-search team can collaborate effectively with corporate manufacturing personnel to make the search a success.

1. Availability of qualified employees
2. Stable climate
3. Secure area due to good policing
4. Socially acceptable in the surrounding region

Materials Management

Materials management deals with the mixture of processes and tools a company relies on to determine how much merchandise it has at a given point, to instill in warehouse personnel the need to prevent product decay, to arrange for shipping companies to quickly access storage areas and to expand factory capacity while heeding the importance of profit management and sales growth. Simply put, materials administration helps the business produce items it can sell, minimize waste and make more money. Materials falling under the items management function include finished goods, work-in-process merchandise and raw materials.

1. Raw material availability and the transport of these resources to the plant at minimal cost
2. Forecast of present and future demand and supply of the product being produced.
3. Availability of waste disposal sites: the manufacturing plant must be as environmentally as possible
4. Availability of governmental support, tax benefits, and other incentives

Connection

Plant location considerations connect with the material management work stream in corporate processes, especially in businesses with a large manufacturing base or those relying heavily on a continued stream of supplies to make money. Examples include large grocery stores and multi-channel food distribution centers.

The operational symbiosis between the two concepts often helps corporate management do away with the primary dilemma of modern business management: how to produce goods quickly and not too far from distribution centers so customers can have them when needed.

1. Availability of the market and potential for future growth
2. The cost of transporting goods and services to people must be minimal
3. Competition analysis in the region using relevant market intelligence.

Deal Economics

Before locating -- or relocating -- a factory or production process, company principals sit department heads and business consultants at a table, asking them to ponder costs associated with the move. Senior executives focus on clarity of thought and idea generation and do not let the group trundle off with a hazy idea of what relocation expenses will be. In this context, deal economics includes things like land cost, factory construction, labor expense, fiscal implications and production capacity.

1. Land availability in terms of future expansion of the plant and the ability of the soil to support a factory
2. Labor and raw material availability and the transport of these resources to the plant at minimal cost
3. Availability of transportation and communication facilities like airports, railway, telephone, etc
4. Availability of infrastructure: running water, electricity, schools, hospitals, libraries, etc.

Furthermore, political, technical and economic considerations must also be taken into account before setting up a new manufacturing plant.

1.4 Plant Layout

The efficiency of any production system depends on well-organized factors such as various machines, production facilities and employee's amenities located in a plant. Properly laid out plant can ensure the smooth and rapid movement of material, from the raw material stage to the end product stage. Plant layout deals with new layout as well as improvement in the existing layout.

Plant layout can be defined as the arrangement of physical facilities such as machinery, equipment, furniture etc. within the factory building in such a manner so as to have quickest flow of material at the lowest cost and with the least amount of handling in processing the product from the receipt of material to the shipment of the finished product.

Overall objective of plant layout is to design a physical arrangement that most economically meets the required output – quantity and quality. Plant layout ideally involves allocation of space and arrangement of equipment in such a manner that overall operating costs are minimized.

The problems related to plant layout are generally observed because of the various developments that occur. These developments generally include adoption of the new standards of safety, changes in the design of the product, decision to set up a new plant, introducing a new product, withdrawing the various obsolete facilities etc.

1.4.1 Objectives of a good plant layout

1. Proper and efficient utilization of available floor space
2. Giving good and improved working conditions
3. To ensure that work proceeds from one point to another point without any delay
4. Provide enough production capacity
5. Minimizing delays in production
6. Reduce material handling costs
7. Reduce hazards to personnel
8. Utilize labour efficiently
9. Increase employee morale
10. Reduce accidents
11. Provide for volume and product flexibility
12. Provide ease of supervision and control
13. Provide for employee safety and health
14. Allow ease of maintenance
15. Allow high machine or equipment utilization
16. Improve productivity

Sometime, providing comfort to the workers and catering to worker's taste and liking, better control over the production cycle by having greater flexibility for changes in the design of the product may also be objective behind designing the layout.

1.4.2 Principles of a good plant layout

1. A good plant layout is the one which is able to integrate its workmen, materials, machines in the best possible way.

2. A good plant layout is the one which sees very little or minimum possible movement of the materials during the operations.
3. A good layout is the one that is able to make effective and proper use of the space that is available for use.
4. A good layout is the one which involves unidirectional flow of the materials during operations without involving any back tracking.
5. A good plant layout is the one which ensures proper security with maximum flexibility.
6. Maximum visibility, minimum handling and maximum accessibility, all form other important features of a good plant layout.

1.4.3 Types of layout

There are mainly four types of plant layout:

(a) Product or line layout

(b) Process or functional layout

(c) Fixed position or location layout

(d) Combined or group layout

(a) Product or Line layout

In an industrial set up, sometime, the machines and equipments are arranged in one line depending upon the sequence of operations required for the product. The raw materials and semi-finished materials move from one workstation to another sequentially without any backtracking or deviation.

Under this, machines are grouped in one sequence. Therefore materials are fed into the first machine and finished goods travel automatically from machine to machine, the output of one machine becoming input of the next, e.g. in a paper mill, bamboos are fed into the machine at one end and paper comes out at the other end.

The raw material moves very fast from one workstation to other stations with a minimum work in progress storage and material handling. The grouping of machines is done on following general principles.

- All the machine tools or other items of equipments must be placed at the point demanded by the sequence of operations
- There should no points where one line crossed another line.
- Materials may be fed where they are required for assembly but not necessarily at one point.
- All the operations including assembly, testing packing must be included in the line

Advantages of Product layout

- Low cost of material handling, due to straight and short route and absence of backtracking
- Smooth and continuous operations
- Continuous flow of work
- Lesser inventory and work in progress
- Optimum use of floor space
- Simple and effective inspection of work and simplified

production control

- Lower manufacturing cost per unit

Disadvantages of Product layout

- Higher initial capital investment in special purpose machine (SPM)
- High overhead charges
- Breakdown of one machine will disturb the production process.
- Lesser flexibility of physical resources

Thus, these types of layouts are able to make better utilization of the equipment that is available, with greater flexibility in allocation of work to the equipment and also to the workers one should be very cautious about any imbalance caused in one section is not allowed to affect the working of the other sections.

(b) Process or functional layout

In this type of layout machines of a similar type are arranged together at one place.

`For example, machines performing drilling operations are arranged in the drilling department, machines performing casting operations be grouped in the casting department. Therefore the machines are installed in the plants, according to various processes in the factory layout.

Hence, such layouts typically have drilling department, milling department, welding department, heating department and painting department etc. The process or functional layout is followed from historical period. It evolved from the handicraft method of production. The work has to be allocated to each department in such a way that no machines are chosen to do as many different job as possible i.e. the emphasis is on general purpose machine.

The work, which has to be done, is allocated to the machines according to loading schedules with the object of ensuring that each machine is fully loaded.

Advantages of Process layout

- Lower initial capital investment is required
- There is high degree of machine utilization, as a machine is not blocked for a single product
- The overhead costs are relatively low
- Breakdown of one machine does not disturb the production process
- Supervision can be more effective and specialized.
- Greater flexibility of resources

Disadvantages of Process layout

- Material handling costs are high due to backtracking
- More skilled labour is required resulting in higher cost
- Work in progress inventory is high needing greater storage space
- More frequent inspection is needed which results in costly supervision

Thus, the process layout or functional layout is suitable for factories / businesses which have job order production;

that is involving non-repetitive processes and customer specifications and non-standardized products, e.g. tailoring, light and heavy engineering products, made to order furniture industries, jewelry etc.

(c)Fixed position or location layout

Fixed position layout involves the movement of manpower and machines to the product which remains stationary. The movement of men and machines is advisable as the cost of moving them would be lesser. This type of layout is preferred where the size of the job is bulky and heavy. Example of such type of layout is locomotives, ships, boilers, generators, wagon building, aircraft manufacturing, etc.

Advantages of Fixed position layout

- The investment on layout is very small.
- The layout is flexible as change in job design and operation sequence can be easily incorporated.
- Adjustments can be made to meet shortage of materials or absence of workers by changing the sequence of operations.

Disadvantages of Fixed position layout

- As the production period being very long so the capital investment is very high.
- Very large space is required for storage of material and equipment near the product.
- As several operations are often carried out simultaneously so there is possibility of confusion and conflicts among different workgroups.

(d) Combined or group layout

Certain manufacturing units may require all three processes namely intermittent process (job shops), the continuous process (mass production shops) and the representative process combined process [i.e. miscellaneous shops]. In most of industries, only a product layout or a process layout or a fixed location layout does not exist. Thus, in manufacturing concerns where several products are produced in repeated numbers with no likelihood of continuous production, combined layout is followed.

Generally, a combination of the product and process layout or other combination are found, in practice, e.g. for industries involving the fabrication of parts and assembly, fabrication tends to employ the process layout, while the assembly areas often employ the product layout.

In soap, manufacturing plant, the machinery manufacturing soap is arranged on the product line principle, but ancillary services such as heating, the manufacturing of glycerin, the power house, the water treatment plant etc. are arranged on a functional basis.

1.5 Introduction to Production Planning

In any product manufacturing company, considerable time is spent on planning the output to be produced. Production planning means to fix the production goals and to estimate the resources which are required to achieve these goals. It prepares a detailed plan for achieving the production goals economically, efficiently and in time.

It forecasts each step in the production process. It forecasts the problems, which may arise in the production process. It tries to provide remedial measures to resolve these issues. It also tries to remove the causes of wastage.

Thus, ***Production Planning may be defined as*** "Production Planning is concerned with the determination, acquisition and arrangement of all facilities necessary for future operations."

Production planning provides answers for two major questions, viz.,

- *What work should be done?*
- *How much time will be taken to perform the work?*

So, production planning decides the ways and means of production. It shows the direction. It is based on sales forecasting. It is a pre-requisite of production control.

1.5.1 Objectives of Production Planning

The need, main **functions** or objectives of production planning in any organisation could be:

- Effective utilization of all the resources in the organisation
- Steady flow of production process without any hurdles / bottlenecks
- Estimate the resources – men, machinery and material requirements for the future
- Ensures optimum inventory level, without blocking the organization's resources

- Co-ordinates activities of various departments
- Minimize wastage of raw materials
- Improves the labour productivity
- Helps to capture the market
- Provides a better work environment
- Facilitates quality improvement
- Results in consumer satisfaction
- Reduces the production costs
- Now let's discuss each objective of production planning one by one

We will give a brief introduction about these points in the following paragraphs.

1. Effective utilization of resources

Production planning results in effective utilization of resources, plant capacity and equipments. This results in low-cost and high returns for the organization. Thus, the operations manager in charge, need to have discussion with various departments – such as purchases, inventory, sales and human resources to arrive better utilization of all the resources.

2. Steady flow of production

Production planning ensures a regular and steady flow of production. Here, all the machines are put to maximum use. This results in a regular production, which helps to give a routine supply to customers. Moreover, to ensure the steady flow, the plan should include an element of human resource plan to maintenance of all the equipments.

3. Estimate the resources

Production planning helps to estimate the resources like men, materials, etc. The estimate is made based on sales forecast. So production is planned to meet sales requirements.

4. Ensures optimum inventory

Production planning ensures optimum inventory. It prevents over-stocking and under-stocking. Necessary stocks are maintained. Stock of raw material is maintained at a proper level in order to meet the production demands. Stock of finished goods is also maintained to meet regular demands from customers.

5. Co-ordinates activities of departments

Production planning helps to co-ordinate the activities of different departments. For instance, the department has to coordinate with marketing department to set the targets / goals for production department to sell the goods. This results in profit to the organization.

6. Minimize wastage of raw materials

Production planning minimizes wastage of raw materials. It ensures proper inventory of raw materials and materials handling. This helps to minimize wastages of raw material and ensures production of quality goods. This will result in minimum rejections; thus, proper production planning and control results in minimum wastage.

7. Improves the labour productivity

Production planning improves the labour productivity. Here, there is maximum utilization of manpower. Training is provided to the workers. The profits are shared with the workers in form of increased wages and other incentives. Workers are motivated to perform their best. This results in improved labour efficiency.

8. Helps to capture the market

Production planning helps to give delivery of goods to customers in time. This is because of regular flow of quality production. So the company can face competition effectively, and it can capture the market.

9. Provides a better work environment

Production planning provides a better work environment to the workers. Workers get improved working conditions, proper working hours, leave and holidays, increased wages and other incentives. This is because the company is working very efficiently.

10. Facilitates quality improvement

Production planning facilitates quality improvement because the production is checked regularly. Quality consciousness is developed among the employees through training, suggestion schemes, quality circles, etc.

11. Results in consumer satisfaction

Production planning helps to give a regular supply of goods and services to the consumers at far prices. It results in consumer satisfaction. If the product / brand are not available regularly in the market, it will create lot of chaos in the market and in the consumer mind. Also, there is a scope for the firm to lose the market share to the competitors.

12. Reduces the production costs

Production planning makes optimum utilization of resources, and it minimizes wastage. It also maintains optimum size of inventories. All this reduces the production costs. Thus, in the planning, elements of financial implications are also involved.

1.5.2 Characteristics of a good production plan

Any manufacturing or service company success and higher productivity highly depend upon an effective and efficient production plan. Effective planning is fundamental in any business; however, making a plan is not an easy task. It is a complex process that covers a wide range of diverse activities, which relate and link materials, equipment and human resources available in the organisation and complete the work. Production planning is like a roadmap to reach destination set by the top management. It helps you know where you are going and how long it will take you to get there.

Advantages of an effective production plan and scheduling:

1. Reduces labour by eliminating wasted time and improving process flow
2. Reduces inventory costs by reducing the need for safety stocks and excessive work-in-process inventories
3. Optimizes equipment usage and maximizes capacity

4. Utilizes human resources to their full potential
5. Improves on-time deliveries of products and services

1.5.3 Key factors of a production plan

Effective planning hinges on a sound understanding of key activities that entrepreneurs and business managers should apply to the planning process. Here are some examples:

Forecast Market Expectations

To plan effectively you will need to estimate potential sales with some reliability. Most businesses don't have firm sales or service figures. However, they can forecast sales based on historical information, market trends and/or established orders.

Inventory Control

Reliable inventory levels feeding the pipeline have to be established and a sound inventory system should be in place.

Availability of Equipment and Human Resources

Also known as open time, this is the period of time allowed between processes so that all orders flow within your production line or service. Production planning helps you manage open time, ensuring it is well-utilized, while being careful not to create delays. Planning should maximize your operational capacity but not exceed it. It's also wise not to plan for full capacity and leave room for the unexpected priorities and changes that may arise.

Standardized Steps and Time

Typically, the most efficient means to determine your production steps is to map processes in the order that they happen and then incorporate the average time it took to complete the work. Remember that all steps don't happen in sequence and that many may occur at the same time.

After completing a process map, you will understand how long it will take to complete the entire process. Where work is repeated or similar, it is best to standardize the work and time involved. Document similar activities for future use and use them as a base-line to establish future routings and times. This will speed up your planning process significantly.

During the process map stage, you may identify waste. You can use operational efficiency/lean manufacturing principles to eliminate waste, shorten the process and improve deliveries and costs. BDC Consulting can assist businesses in process mapping and other operational efficiency principles and tools.

Risk Factors

Evaluate these by collecting historical information on similar work experiences, detailing the actual time, materials and failures encountered. Where risks are significant, you should conduct a failure mode effect analysis method (FMEA) and ensure that controls are put in place to eliminate or minimize them. This method allows you to study and determine ways to diminish potential problems within your business operations. This type of analysis is more common in manufacturing and assembly businesses.

1.5.4 Planning activities

All other activities are initiated from the production plan and each area is dependent on the interaction of the activities. Typically, a plan addresses materials, equipment, human resources, training, capacity and the routing or methods to complete the work in a standard time. In order to do a good sales forecast, you should base it on a history of firm orders.

The production plan initially needs to address specific key elements well in advance of production in order to ensure an uninterrupted flow of work as it unfolds.

Material Ordering

Materials and services that require a long lead time or are at an extended shipping distance, also known as blanket orders, should be ordered in advance of production requirements. Suppliers should send you materials periodically to ensure an uninterrupted pipeline

Equipment Procurement

Procuring specialized tools and equipment to initiate the production process may require a longer lead time. Keep in mind that the equipment may have to be custom made or simply difficult to set up. This type of equipment may also require special training

Bottlenecks

These are constraints or restrictions in the process flow and should be assessed in advance so you can plan around them or eliminate them before you begin production. When you assess possible bottlenecks, be aware that they may shift to another area of the process. Dealing with bottlenecks is a continual challenge for any business

Human Resources Acquisitions and Training

Key or specialized positions may demand extensive training on specialized equipment, technical processes or regulatory requirements. These employees should be interviewed thoroughly about their skills. When hiring them, allow sufficient time for training and be sure that they are competent in their work before the job begins. This will ensure that your process or service flows smoothly

The production plan provides a foundation to schedule the actual work and plan the details of day-to-day activities. As sales orders come in, you will need to address them individually based on their priority. The importance of the sales order will determine the work flow and when it should be scheduled. After this, you should evaluate whether or not you are ready for production or to offer the service. You will need to determine:

1. If the inventory is available at the point where work is to start? If not, then the work needs to be rescheduled when supplies become available. There is no point in scheduling work that you will not be able to complete
2. Are your resources available? Do you have the necessary staff to complete the task? Are the machines being used?
3. Does the standard time fit within the open time allowed? If not, then the work should be rescheduled
4. You should be careful to minimize risk factors; allowing too many what-ifs can delay delivery and be counter productive

1.5.5 Communicate the plan

After you have determined that you have met the criteria to start production, you will need to communicate the plan to the employees who will implement it. You can plan the production on spreadsheets, databases or software which usually speeds the process up. However, a visual representation is preferred as a means to communicate operation schedules to floor employees. Some businesses post work orders on boards or use computer monitors to display the floor schedule.

The schedule also needs to be available to employees ahead of time and kept up to date.

Consider change

One of the many challenges of production planning and scheduling is following up with changes to orders. Changes happen every day; you may lack materials; delivery time is moved up or work parameters have to be adapted. You will need to adjust your plan in line with these changes and advise the plant. Dealing with change is not always easy and may take as much effort as creating the original production plan. You will need to follow up with the various departments involved in order to rectify any problems. As well, computer software can be helpful in tracking changes, inventory, employees and equipment.

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UNIT-II

Lesson-2 Introduction to Operations Research

Lesson Objectives

- To Introduce The Evolution Of The Field Of Or & Various Features
- To Brief The Various Phases In Or & Limitations Of Or
- To Introduce Method For Mathematical Formulations / Modeling
- To Illustrate The Method Of Solving Lpp By Graphical Solution Procedure
- To Introduce The Simplex Algorithm To Solve Lp Problems
- To Brief The Big-M Method & Concept Of Duality

Chapter Structure

This Chapter is organized in the following order

2.1 Introduction to the field of Operations Research

2.2 Linear Programming – Problem formulation [Mathematical Modeling]

2.3 Linear Programming – Problem Solving [GRAPHICAL METHOD]

2.4 Some Special Cases of Graphical Solution Methods of LP Problems

2.5 Linear Programming – Problem Solving [SIMPLEX METHOD]

2.6 BIG-M Method [INTRODUCTION]

2.7 Duality – Introduction

2.1 Operations Research – An Introduction

2.1.1 Genesis of Operations Research

Different authors define Operations Research in different ways. The subject is emerged from the War Field; OR was first used during the **Second World** War by **England** to solve their complex war related issues and problems, such as moving troops to the war field, allocating appropriating resources, managing the inventory and so on. England made OR teams. Since the problems in the war field are multi-disciplinary in nature, the teams included experts from various branches of science such as mathematicians, statisticians, scientists, engineers, etc.

These OR teams were very successful in solving England's war problems. Therefore, United States of America (**USA**) also started using OR to solve their war problems. It is research designed to determine most efficient way to do something new. OR is the use of mathematical models, statistics and algorithm to aid in decision-making. It is most often used to analyze complex real life problems typically with the goal of improving or optimizing performance.

After the war, soon industries and businesses were also started using OR to solve their complex management problems. Scientists and technocrats formed team to study the problem arising out of difficult situations and at the later stage solutions to these problems. With the growth in computing facilities, Operations Research (OR) not only applies scientific method to the management of organized systems in business and industry, but even in government departments and non-governmental enterprises.

OR is very aggressively applied in domains such as,

1. Supply Chain Management [SCM]
2. Marketing and revenue management systems
3. Manufacturing plants
4. Financial Engineering
5. Telecommunication networks

6. Healthcare management
7. Transportation networks
8. Energy and the Environment Management
9. Service systems
10. Web commerce
11. Military /Defense applications

Typically, applications of OR in these and other areas deal with decisions involved in planning and efficiently allocating various resources – which may be material / skilled / semi-skilled labors, machines, money and time, through which the achieve stated goals and objectives under conditions of uncertainty and over a span of time. Efficient allocation of resources may entail establishing policies, designing processes, or relocating assets. OR analysts solve such management decision problems with an array of mathematical methodologies.

2.1.2 Definition of Operations Research

It is very difficult task to define OR, since it is evolving as a domain of knowledge and its boundaries and content are not yet fixed. You can think that OR uses mathematical and quantitative techniques to substantiate the decision being taken. Further, it is multidisciplinary which takes various theories, techniques and tools from subjects like mathematics, statistics, engineering, economics, psychology etc. and uses them to evaluate the consequences of possible alternative actions with the decision makers.

"Operations research is a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control".

"Operations research is the application of scientific methods to arrive at the optimal solutions to the problems".

2.1.3 Features of OR

There are various significant features available for the domain Operations Research.

Decision-Making

Many business houses, in the recent times, come across issues, which are multifaceted problems and these problems require identifying best possible solutions. OR as a body of knowledge, aims to provide the business executives and decision makers an optimal solution through the use of OR techniques. It also helps the decision maker to improve his creative and judicious capabilities, analyse and understand the problem situation leading to better control, better co-ordination, better systems and finally better decisions.

Scientific Approach

OR applies scientific methods, techniques and tools for the purpose of analysis and solution of the complex problems. It completely eliminates heuristics and personal bias of the decision maker.

Inter-disciplinary Team Approach

Basically the industrial problems are of complex nature and require a team effort to handle it. More often, the team comprises of scientist/mathematician and technocrats joint effort to obtain a feasible and optimum solution for the enterprise. The tries to analyse the cause and effect relationship between various parameters of the problem and evaluates the outcome of various alternative strategies.

System Approach

The main aim of the system approach is to trace for each proposal all significant and indirect effects on all sub-system on a system and to evaluate each action in terms of effects for the system as a whole. The interrelationship and interaction of each sub-system can be handled with the help of mathematical/analytical models of OR to obtain acceptable solution.

Use of Computers

With the advances in computing sciences, various models in OR efficiently use the advances in computing / computers. With the use of computers, it is possible to handle problems with high degree of complexity, which require large amount of calculations. The objective of the operations research models is to attempt and to locate best or optimal solution under the specified conditions. For the above purpose, it is necessary that a measure of effectiveness has to be defined which must be based on the goals of the organisation. These measures can be used to compare the alternative courses of action taken during the analysis.

2.1.4 Phases of OR

Operations Research is a logical and systematic approach to provide a rational basis for decision-making. You may think of the following steps which is required for the analysis of a problem under OR

Step I. Observe the Problem Environment

The first step of OR study is the observation of the environment in which the problem exists. The activities that constitute this step are visits, conferences, observations, research of past data etc.

With the help of such activities, the OR analyst / team gets sufficient insight and information to proceed. Also, this gives the team better preparedness level to formulate the problem.

Step II. Analyse and Define the Problem

In this step not only the problem is defined, but also uses objectives and limitations of the study that are stressed in the light of the problem. The end results of this step are clear grasp of need for a solution and understanding of its nature.

Step III. Develop a Model

The next step is to develop model, which is representation of same real or abstract situation. OR models are basically mathematical models representing systems, process or environment in form of equations, relationships or formulae. The activities in this step is to defining interrelationships among variables, formulating equations, using known OR models or searching suitable alternate models. The proposed model may be field tested and modified in order to work under stated environmental constraints. A model may also be modified if the management is not satisfied with the answer that it gives.

Step IV. Selection of Data Input

It is an established fact that without authentic and appropriate data the results of the OR models cannot be trusted. Hence, taping right kind of data is a vital step in OR process.

Important activities in this step are analysing internal-external data and facts, collecting opinions and using computer data banks. The purpose of this step is to have sufficient input to operate and test the model.

Step V. Solution and Testing

In this step the solution of the problems is obtained with the help of model and data input. Such a solution is not implemented immediately and this solution is used to test the model and to find its limitations if any. If the solution is not reasonable or if the model is not behaving properly, updating and modification of the model is considered at this stage. The end result of this step is solution that is desirable and supports current organisational objectives.

Step VI. Implementation of the Solution

This is the last phase of the OR study. In OR the decision-making is scientific but implementation of decision involves many behavioural issues. Therefore, implementation authority has to resolve the behavioural issues, involving the workers and supervisors to avoid further conflicts.

The gap between management and OR scientist may offer some resistance but must be eliminated before solution is accepted in totality. Both the parties should play positive role, since the implementation will help the organisation as a whole. A properly implemented solution obtained through OR techniques results in improved working conditions and wins management support.

2.1.5 Limitations of Operations Research

OR has some limitations however, these are related to the problem of model building and the time and money factors involved in application rather than its practical utility. Some of them are as follows:

(i) Magnitude of Computation

Operations research models try to find out optimal solution taking into account all the factors. These factors are enormous and expressing them in quantity and establishing relationships among these require voluminous calculations which can be handled by computers.

(ii) Non-Quantifiable Factors

OR provides solution only when all elements related to a problem can be quantified. All relevant variables do not lend themselves to quantification. Factors which cannot be quantified find no place in OR study. Models in OR do not take into account qualitative factors or emotional factors which may be quite important.

(iii) Distance between User and Analyst

OR being specialist's job requires a mathematician or statistician, who might not be aware of the business problems. Similarly, a manager fails to understand the complex working of OR. Thus there is a gap between the two. Management itself may offer a lot of resistance due to conventional thinking.

(iv)Time and Money Costs

When basic data are subjected to frequent changes, incorporating them into the OR models is a costly proposition. Moreover, a fairly good solution at present may be more desirable than a perfect OR solution available after sometime. The computational time increases depending upon the size of the problem and accuracy of results desired.

(v) Implementation

Implementation of any decision is a delicate task. It must take into account the complexities of human relations and behaviour. Sometimes, resistance is offered due to psychological factors which may not have any bearing on the problem as well as its solution.

2.2 Linear Programming – Problem formulation [Mathematical Modeling]

We have briefly discussed the meaning of models, various types of models; we are particularly more interested in the mathematical models. Let us consider the following situation pertaining to a furniture manufacturing firm.

A furniture company manufactures desks and chairs. There are four departments namely carpentry, upholstery, painting and varnishing with capacities as given below:

Departments	Man - Hours per week	Number of man-hours required for one	
		Desk	Chair
Carpentry	120	4	2
Upholstery	240	0	3
Painting	90	2	2
Varnishing	100	1	2

Assuming that raw materials are available in adequate quantities and the manufacturer wishes to know how many desks and chairs he should produce. He enjoys a good market share. The contribution from a desk is Rs.40/- and that from a chair is Rs.25/-

Let us develop the mathematical model of the given situation as follows:

Let us say, we want to produce X1 units of Desks and X2 Units of Chairs.

By producing one unit of desk, the manufacturer gets Rs. 40/- as profit and for chairs, he gets Rs. 25/-

Therefore, the profits from desks = 40 * X1

Profits from chairs = 25 * X2

Therefore, the total profits = 40X1 + 25X2

Obviously, the objective of the firm, therefore, is Maximize the total profits, which is expressed as a mathematical expression (function) as

Max Z = 40X1 + 25X2

However, in achieving the objective, the firm faces several constraints. The first constraint is from the carpentry section; to produce, one unit of desk, it needs 4 hours from this section and for producing chairs, and this section needs to spend 2 hours. However, this section, in total, available, only for 120 hours in the week.

Thus, the first constraint is formulated as follows:

4X1 + 2X2 <= 120 …………………. (1)

Upholstery section's service is needed only for producing chairs and to produce one unit of chair, this section has to spend 3 hours and its maximum availability is 240 hours per week. Thus, the second constraint is derived as follows:

$3X2 <= 240$ .... (2)

Similarly, the painting section's availability per week is limited to 90 hours. To produce a desk or chair, this section has to spend 2 hours for each unit. Hence, the constraint for painting section is,

$2X1 + 2X2 <= 90$ .... (3)

Finally, the varnishing section's services are needed for both the products, and it is limited to 100 hours per week. To produce a desk, it needs to spend 1 hour and to produce a chair; it has to spend 2 hours. Thus, the constraint for varnishing section is,

$X1 + 2X2 <= 100$ .... (4)

Obviously, the production quantities like number of desks (X1) and number of chairs (X2) cannot be negative; we add two more constraints to this situation. They are

$X1 \geq 0$ and $X2 \geq 0$

Therefore, the mathematical formulation of the given situation is

Max $Z = 40X1 + 25X2$

Subject to

$$4X1 + 2X2 <= 120$$
$$3X2 <= 240$$
$$2X1 + 2X2 <= 90$$

$X1 + 2X2 <= 100$

$\mathbf{X1 \geq 0 \text{ and } X2 \geq 0}$

Therefore by reading,

Max Z = 40X1 + 25X2

Subject to

$$4X1 + 2X2 <= 120$$
$$3X2 <= 240$$
$$2X1 + 2X2 <= 90$$
$$X1 + 2X2 <= 100$$
$$\mathbf{X1 \geq 0 \text{ and } X2 \geq 0}$$

The reader can understood that the firm has to decide the quantities to be produced in desks and chairs, so as to make the maximize profit / contribution and achieving this objective is subject to 4 constrains – availability of raw materials from its 4 departments.

Thus, the mathematical model summarizes the information provided in the context / situations in terms of mathematical symbols and notations. We will take few more examples.

Example1: Distillers Production Schedule

M/S. RK Distillers Ltd **(RKDL)** has two bottling plants, one located at Pondicherry and the other at Chennai. Each plant produces three brands of liquor products, Challenge, Royal and Salute under the job order contract to the leading liquor baron MB Distillers and Bottlers Ltd (**MBDL). **The number of cases produced per day is as follows:

Brand of Liquor	Plant at	
	Pondicherry	Chennai
Challenge	1500	1500
Royal	3000	1000
Salute	2000	5000

As per the sales forecast given by the marketing team, MBDL expects a minimum of 30000 cases of Challenge, 40000 cases of Royal and 44000 cases of Salute from RKDL for the next fortnight. The operating costs per day for plants at Chennai and Pondicherry are 600 and 450 thousands per day. How many days each plant should run to fulfill the orders for the next fortnight? [You can assume that the factory runs all the 7 days in a week-since there is a shift system to take care of weekly off]

Solution

Let X1 be the number of days Chennai plant to be operated and X2 be the number of days Pondicherry plant should run.

The firm aims to reduce the overall operating cost arising out operations subject to fulfilling the market demand.

Therefore, the objective is to minimize the total operating cost; to operate the Chennai plant for X1 days, the firm has to incur 600X1+ 450X2

Min Z = 600X1 + 450X2

Subject to

$$1500X1 + 1500X2 >= 30,000$$

3000X1 + 1000X2 >= 40,000

2000X1 + 5000X2 >= 44000

Where, X1, X2 >= 0

Example2: Product Mix Decision

A company producing a standard line and a deluxe line of dish washers has the following time requirements (in minutes) in departments where either model can be processed.

	Standard	Deluxe
Stamping	3	6
Motor installation	10	10
Wiring	10	15

The standard models contribute Rs. 20 each and the deluxe Rs. 30 each to profits. Because the company produces other items that share resources used to make the dishwashers, the stamping machine is available only 30 minutes per hour, on average. The motor installation production line has 60 minutes available each hour. There are two lines for wiring, so the time availability is 90 minutes per hour. Write the formulation for this linear program

Solution

Let X = number of standard dishwashers produced per hour

Y = number of deluxe dishwashers produced per hour

Therefore, the objective is to maximize the total contribution from these two products.

Thus, it is written as

$$\text{Max } Z = 20X + 30Y$$

This contribution realization is subject to the following constraints;

$3X + 6Y \leq 30$ -------------- (Stamping Machine constraint)

$10X + 10Y \leq 60$ -------------- (Motor installation constraint)

$10X + 15Y \leq 90$ -------------- (Wiring machine constraint)

And obviously, X & Y cannot be negative quantities, hence, X>=0 & Y>=0

Example3: Product Mix Decision @ Whoppy Soft Drinks

The production manager for the Whoppy soft drink company is considering the production of 2 kinds of soft drinks: regular (R) and diet (D). The company operates one "8 hour" shift per day. Therefore, the production time is 480 minutes per day. During the production process, one of the main ingredients, syrup is limited to maximum production capacity of 675 gallons per day. Production of a regular case requires 2 minutes and 5 gallons of syrup, while production of a diet case needs 4 minutes and 3 gallons of syrup. Profits for regular soft drink are Rs.3.00 per case and profits for diet soft drink are Rs.2.00 per case. Write the formulation for this linear program.

Solution

Let R = number of regular drinks produced per days

D = number of diet drinks produced per days

Therefore, the objective is to maximize the total contribution from these two products.

Thus, it is written as

$$\text{Max } Z = 3R + 2D$$

This contribution realization is subject to the following constraints;

$2R + 4D \leq 480$ -------------- (Production time constraint)

$5R + 3D \leq 675$ -------------- (Syrup availability constraint)

And obviously, R & D cannot be negative quantities, hence, R>=0 & D>=0

Example4: Sales Mix Decision for computer retail sales

A computer retail store sells two types of flat screen monitors: 17 inches and 19 inches, with a profit contribution of Rs. 300 and Rs. 250, respectively. The monitors are ordered each week from an outside supplier. As an added feature, the retail store installs on each monitor a privacy filter that narrows the viewing angle so that only persons sitting directly in front of the monitor are able to see on-screen data. Each 19" monitor consumes about 30 minutes of installation time, while each 17" monitor requires about 10 minutes of installation time. The retail store has approximately 40 hours of labor time available each week. The total combined demand for both monitors is at least 40 monitors each week. How many units of each monitor should the retail store order each week to maximize its weekly profits and meet its weekly demand?

Solution

Let

$X1$ = number of 17 inches monitor to be ordered per week

$X2$ = number of 19 inches monitor to be ordered per week

Therefore, the objective of the retail service firm is to maximize the total contribution from these computer monitor sales. The retail firm gets Rs. 300 and Rs. 250 per monitor for 17 and 19 inches respectively, therefore, the objective function arrived as follows:

$$\text{Max } Z = 300\,X1 + 250\,X2$$

This contribution realization is subject to the following constraints;

$10X1 + 30X2 \leq 2400$ -------------- (Labor time constraint)

$X1 + X2 \geq 40$ -------------- (Market demand for the computer monitor per week constraint)

And obviously, X1 & X2 cannot be negative quantities, hence, $X1>=0$ & $X2>=0$

Exercises [Try on your own]

1. A furniture store produces beds and desks for college students. The production process requires assembly and painting. Each bed requires 6 hours of assembly and 4 hours of painting. Each desk requires 4 hours of assembly and 8 hours of painting. There are 40 hours of assembly time and 45 hours of painting time available each week. Each bed generates $35 of profit and each desk generates $45 of profit. As a result of a labor strike, the furniture store is

limited to producing at most 8 beds each week. Formulate the situation as a linear programming problem, which can determine number beds and desks should be produced each week to maximize weekly profits.

2. A bank is attempting to determine where its assets should be allocated in order to maximize its annual return. At present, $750,000 is available for investment in three types of mutual funds: A, B, and C. The annual rate of return on each type of fund is as follows: fund A, 15%; fund B, 12%; fund C; 13%. The bank's manager has placed the following restrictions on the bank's portfolio:

- No more than 20% of the total amount invested may be in fund A
- The amount invested in fund B cannot exceed the amount invested in fund C

Determine the optimal allocation that maximizes the bank's annual return.

2.3 Linear Programming – Problem Solving [GRAPHICAL METHOD]

2.3.1 Introduction

An optimal as well as feasible solution to an LP problem is obtained by choosing among several values of decision variables X1,X2,…….Xn, the one set of values that satisfy the given set of constraints simultaneously and also provide the optimal (maximum or minimum) value to the given objective function.

For LP problems that have only two variables it is possible that the entire set of feasible solutions can be displayed graphically by plotting linear constraints to locate a best (optimal) solution. The technique used to identify the optimal solution is called the Graphical Solution Technique for an LP problem with two variables.

The two graphical solution techniques are

1. Extreme point enumeration approach, and
2. Iso-profit (cost) function approach.

We will list out various definitions that are associated with the graphical solution method.

2.3.2 DEFINITIONS

Solution

Solution values of decision variables X1, X2, X3… (i=1, 2…n) which satisfies the constraints of a general LP model, is called the solution to that LP model.

Feasible Solution

Solution values of decision variables X1, X2, X3… (i=1, 2…n), which satisfies the constraints of the given problem as well as the non-negativity conditions of a LP model, are called as the feasible solution to that LP model.

Basic Solution

We know that to solve 2 variables, we need a minimum of 2 simultaneous equations; to solve 3 variables, a minimum of 3 simultaneous equations and so on.

For a set of m equations in n variables (n>m), a solution is obtained by setting (n-m) variables values equal to zero and solving for remaining m equations in m variables is called a Basic solution to the problem.

The (n-m) values, whose value did not appear in this solution, are called non-basic variables and the remaining m variables are called basic variables.

Basic Feasible Solution

A feasible solution to an LP problem that is also the basic solution is called the basic feasible solution to the LPP. That is, all basic variables assume non-negative values. Basic feasible solutions, in general, can be classified into two types:

a. Degenerate: A basic feasible solution is called degenerate if at least one basic variable possesses zero value.

b. Non-degenerate: A basic feasible solution is called non-degenerate if all m basic variables are non-zero and positive.

Optimum Basic Feasible Solution

A basic feasible solution which optimizes (maximizes or minimizes) the objective function of the given LP model is called an optimum feasible solution to the given LPP.

Unbounded Solution

Some occasions, a solution which can be increased or decreased the value of objective function of LP problem indefinitely, is known as an unbounded solution to the given problem.

2.3.3 GRAPHICAL SOLUTION PROCEDURE OF LP PROBLEMS

While obtaining the optimal solution to the LP problem by the graphical method, the statement of the following theorems of linear programming is used.

1. The collection of all feasible solutions to an LP problem constitutes a convex set whose extreme points correspond to the basic feasible solutions.
2. There are a finite number of basic feasible solutions within the feasible solution space.
3. If the convex set of the feasible solutions of the system Ax=b, x>=0, is a convex polyhedron, then at least one of the extreme points gives an optimal solution.
4. If the optimal solution occurs at more than one extreme point, then the value of the objective function will be the same for all convex combinations of these extreme points.

METHOD-1: Extreme Point Enumeration Approach

This solution method for an LP problem is divided into five steps.

Step 1

State the given problem in the mathematical form.

Step 2

Graph the constraints, by temporarily ignoring the inequality sign and decide about the area of feasible solutions according to the inequality sign of the constraints. Indicate the area of feasible solutions by a shaded area, which forms a convex polyhedron.

Step 3

Determine the coordinates of the extreme points of the feasible solution space.

Step 4

Evaluate the value of the objective function at each extreme point.

Step 5

Determine the extreme point to obtain the optimum (best) value of the objective function.

Example-1

Use the graphical method to solve the following LPP problem:

Maximize z = 15X1 + 10X2

Subject to constraints

$$4X1+6X2 <=360$$
$$3X1+0X2<=180$$
$$0X1+5X2 <=200$$
$$X1, X2>=0$$

Solution

Step 1

State the problem in mathematical form.

The problem we have considered here is already in mathematical form [given in terms of mathematical symbols and notations]

Step 2

Plot the constraints on the graph paper and find the feasible region.

We shall treat x1 as the horizontal axis and x2 as the vertical axis. Each constraint will be plotted on the graph by treating it as a linear equation and then appropriate inequality conditions will be used to mark the area of the feasible solutions.

Consider the first constraint 4X1+6X2<=360 .Treated it as the equation,

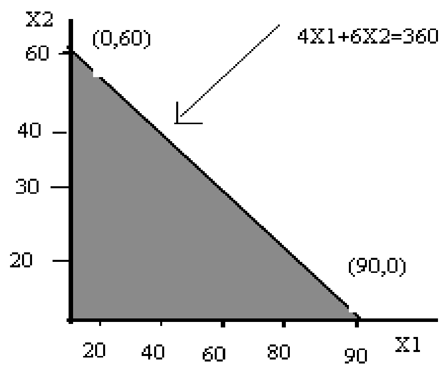
$$4X1+6X2 = 360.$$

Or

$$\frac{X1}{360/4} + \frac{X2}{360/6} = 1 \quad \text{Or} \quad \frac{X1}{90} + \frac{X2}{60} = 1$$

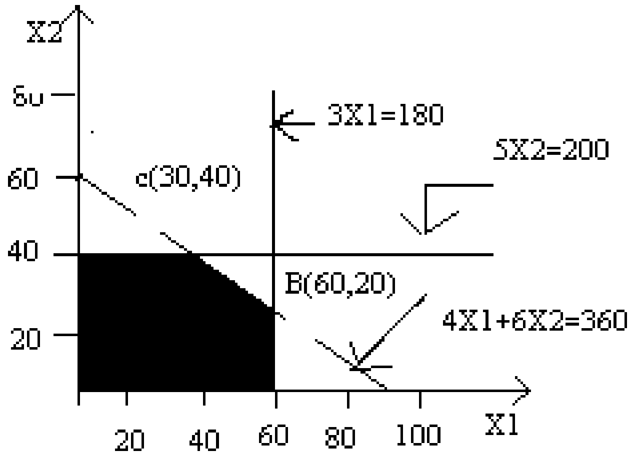
This equation indicates that when it is plotted, the graph cuts X1 axis [intercept] at 90 and X2 axis [x2 intercept] at 60. These two points are then connected by a straight line as shown in figure.

But the inequality and non-negativity condition can only be satisfied by the shaded area (feasible region) as shown in figure below:



GRAPHICAL SOLUTION OF LP PROBLEM

Similarly the constraints 3X1<=180 and 5X2<=200 are also plotted on the same graph, which is shown above and the required region is indicated by the shaded area, which is common to all the 3 constraint as shown in figure below:



Since all constraints have been graphed, the area which is bounded by all the constraints lines including all the boundary points is called the feasible region or solution space. The feasible region is shown in fig by the area OABCD.

Step 3

Determine the coordinates of extreme points
Since the optimal value of the objective function occurs at one of the extreme points of the feasible region, it is necessary to determine their coordinates. The coordinates of extreme points of the feasible region are:

O = (0, 0),
A= (60, 0)
B= (60, 20),
C= (30, 40),
D= (0, 40)

Step 4

Evaluate the value of the objective at extreme points

Now, the next step is to evaluate the objective function value at each extreme point of the feasible region, which is shown below:

Extreme point	Coordinates (X1, X2)	Objective function value Z= 15X1+10X2
O	(0,0)	15(0)+10(0) = 0
A	(60,0)	15(60)+10(0) = 900
B	(60,20)	15(60)+10(20) = 1100
C	(30,40)	15(30)+10(40) = 850
D	(0,40)	15(0)+10(40) = 400

Step 5

Determining the optimal value of the objective function

Since our desire it to maximize the value of the objective function, Z, therefore from step 4 we can conclude that maximum value of Z=1100 is achieved at the point B (60, 20). Hence the optimal solution to the given LP problem is:

X1=60,

X2=20 and

Value of the objective function, Z=1100

Example-2: Use the graphical method to solve the following LP problem

Minimize Z=20X1+10X2

Subject to constraints,

$X1+2X2 \leq 40$

$3X1+X2 \geq 30$

$4X1+3X2 \geq 60$

And $X1, X2 \geq 0$

Solution

Step 1

State the problem in mathematical form.
The problem we have considered here is already in mathematical form [given in terms of mathematical symbols and notations]

Step 2

Plot the constraints on the graph paper and find the feasible region.

We shall treat x1 as the horizontal axis and x2 as the vertical axis. Each constraint will be plotted on the graph by treating it as a linear equation and then appropriate inequality conditions will be used to mark the area of the feasible solutions.

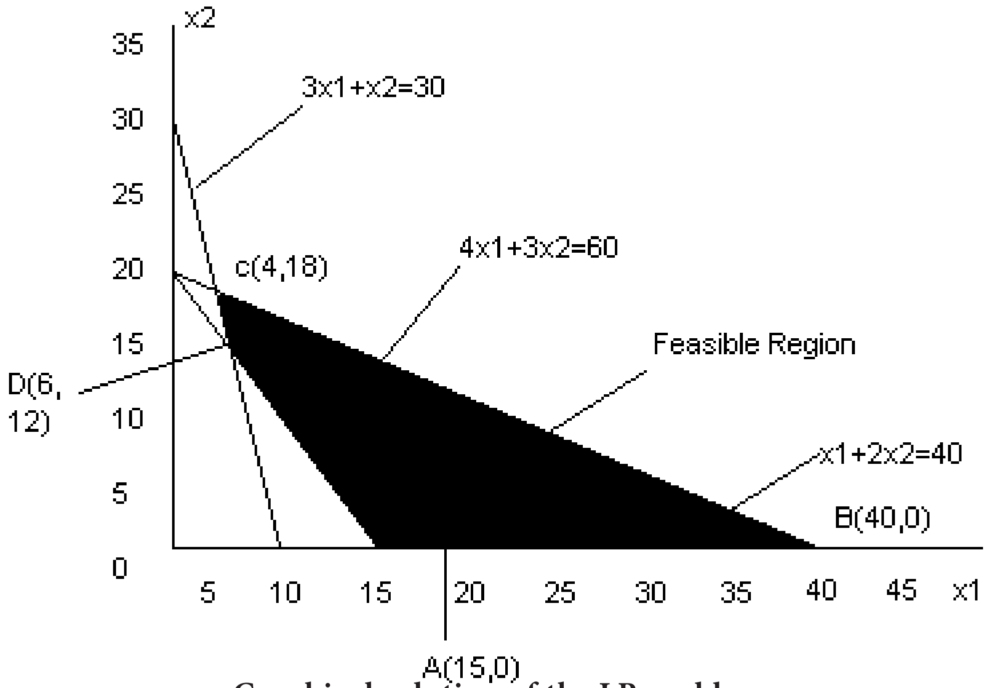
The constraints are written in the following form;

$X1/40 + X2/(40/2) \leq 1$ or $X1/40 + X2/20 \leq 1$-----(1)

$X1/(30/3) + X2/3 \geq 1$ or $X1/10 + X2/30 \leq 1$----(2)

$X1/(60/4) + X2/(60/3) \geq 1$ or $X1/15 + X2/20 \geq 1$----(3)

Graph each constraint by first treating it as a linear equation. Then use the inequality condition of each constraint to make the feasible region as shown in following figure.



Graphical solution of the LP problem

Step 3

Determine the coordinates of extreme points

Since the optimal value of the objective function occurs at one of the extreme points of the feasible region, it is necessary to determine their coordinates. The coordinates of extreme points of the feasible region are:

A= (15, 0), B= (40, 0),C= (4, 18) and D= (6, 12).

Step 4

Evaluate the value of the objective at extreme points

Now, the next step is to evaluate the objective function value at each extreme point of the feasible region, which is shown below:

Extreme point	Coordinates (X1, X2)	Objective function value Z= 15X1+10X2
A	(15,0)	20(15)+10(0) = 300
B	(40,0)	20(40)+10(0) = 800
C	(4,18)	20(4)+10(18) = 260
D	(6,12)	20(6)+10(12) = 240

Therefore, the minimum value of the objective function Z=240 occurs at the extreme point D (6, 12).

Hence, the optimal solution to the given LP problem is:

$$X1 = 6, X2 = 12$$

And the value of the objective function is

Z = 240.

2.4 Some Special Cases of Graphical Solution Methods of Lp Problems

Case-1: No Feasible Solution space

Consider the following LPP

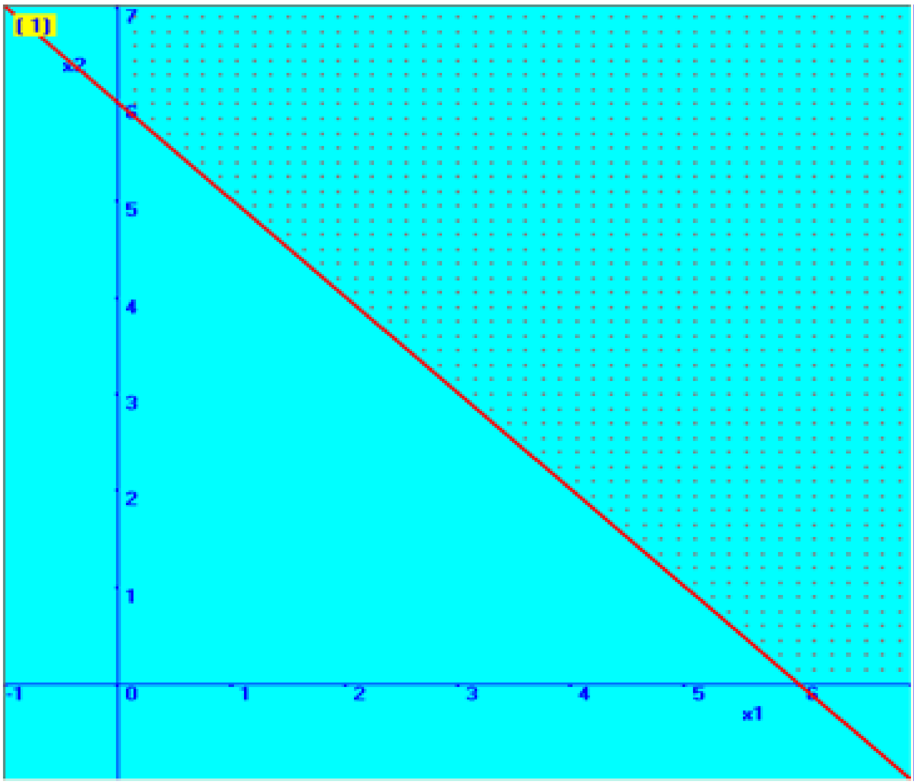
$$\text{Max } Z = 4X1 + 4\,X2$$

Subject to

$$2X1 + 2X2 >= 12$$
$$3X1 + 3X2 <= 9$$
$$X1, X2 >= 0$$

The solution is arrived through the graphical method; various steps are given below.

We plot the first constraint in the graphical plane



Consider 2X1 + 2X2 = 12

If X1 = 0, then X2 = 6

Therefore, one point on the line is (0, 6)

If X2 =0, then X1 = 6

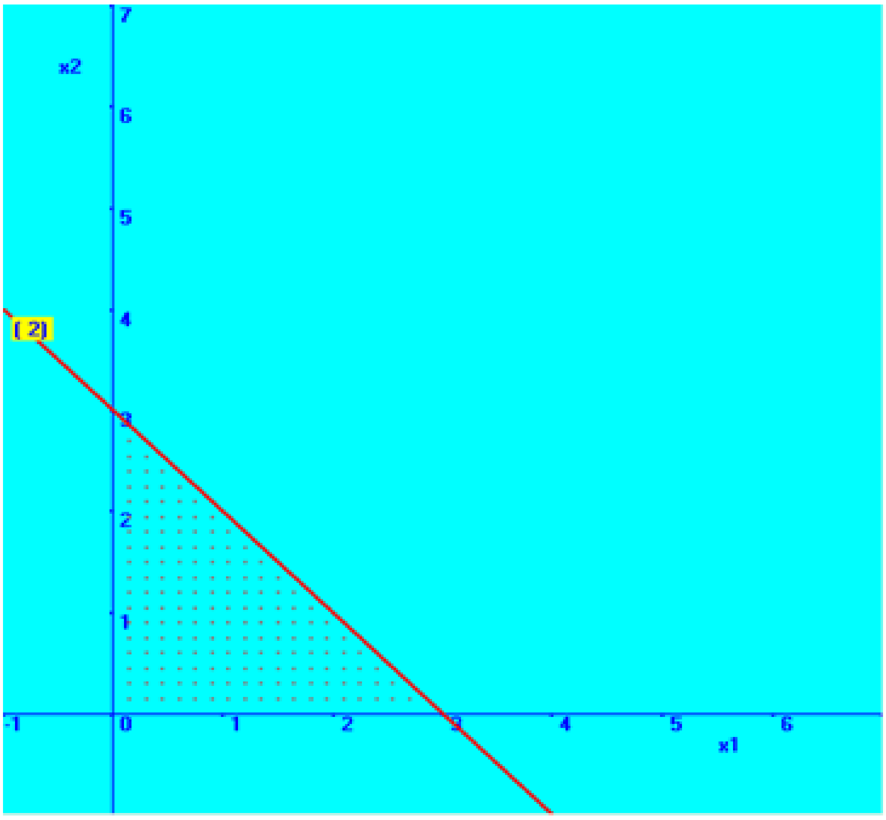
Therefore another point on the line is (6, 0)

We will fix with the help of these two points in the graphical plane, which is shown here. Then, based on the inequality sign, the required region is marked here.

In similar way, we fixed the second constraint

3X1 + 3X2 = 9

If X1 = 0, then X2 = 3



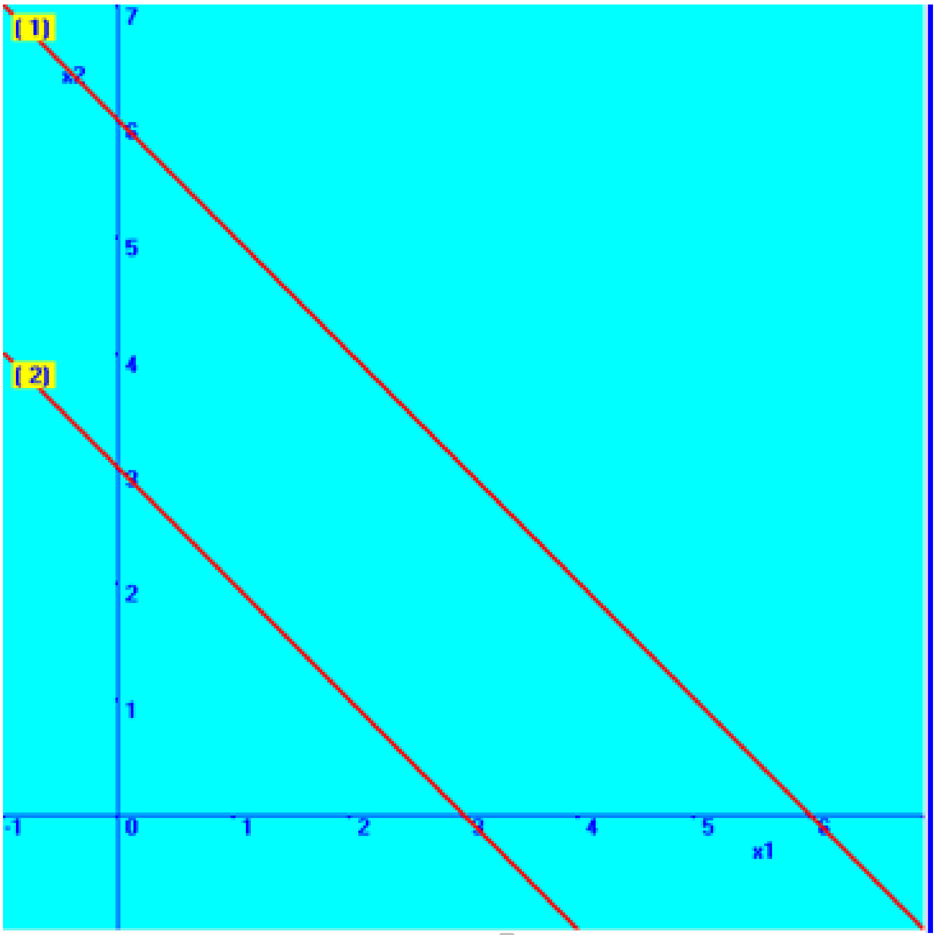
Therefore, one point on the line is (0, 3)

If X2 =0, then X1 = 3

Therefore another point on the line is (3, 0)

We will fix straight line with the help of these two points in the graphical plane, which is shown here. Then, based on the inequality sign, the required region is marked here.

Then the combined graphical solution space is arrived and shown here in the diagram.



You can note that there is no common solution area between these two constraints.

Hence this kind of problem is categorized as 'No Feasible solution'.

Case-2: Unbounded Solution space

Consider the following LPP

$$\text{Max } Z = 4X1 + 4\ X2$$

Subject to

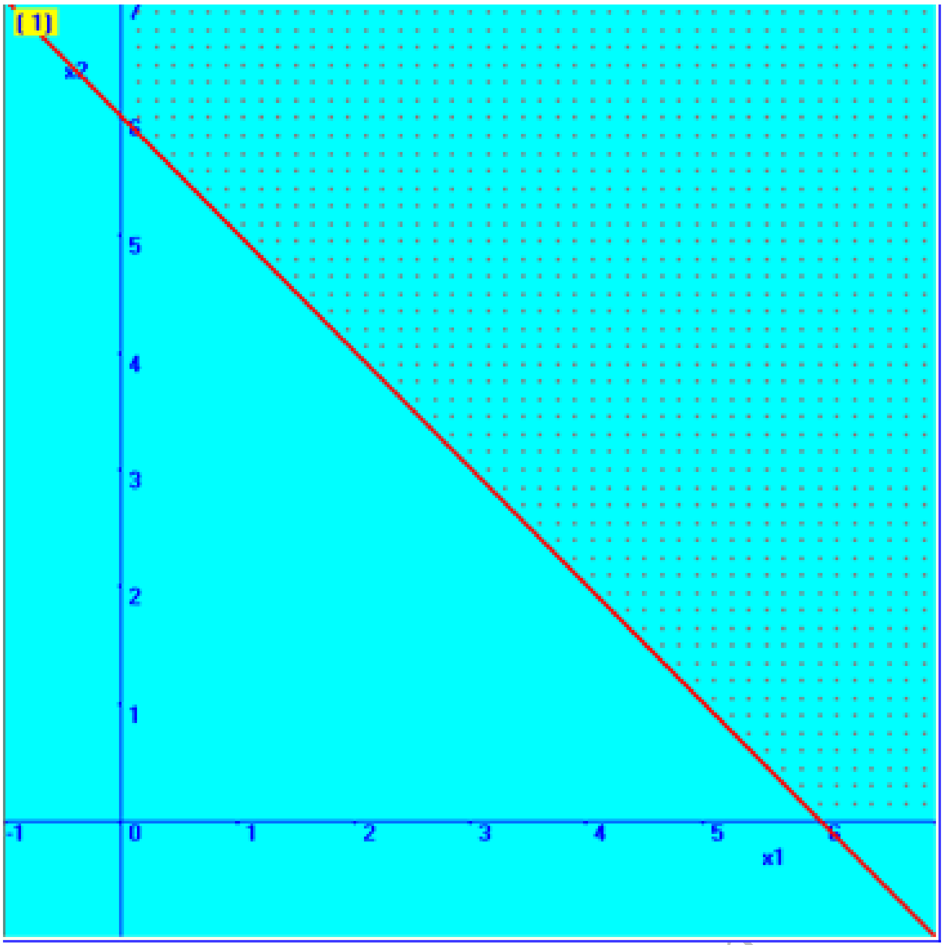
$$2X1 + 2X2 >= 12$$

$$3X2 <= 9$$

$$X1, X2 >= 0$$

The solution is arrived through the graphical method; various steps are given below.

We plot the first constraint in the graphical plane



Consider 2X1 + 2X2 = 12

If X1 = 0, then X2 = 6

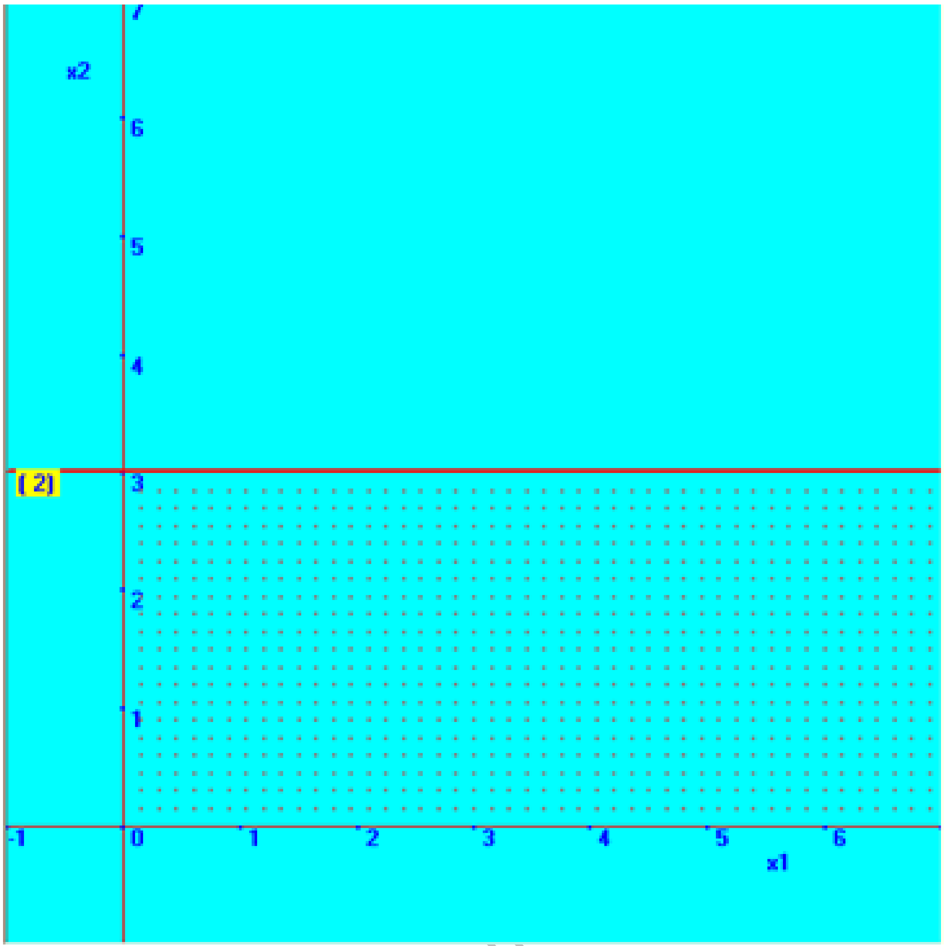
Therefore, one point on the line is (0, 6)

If X2 =0, then X1 = 6

Therefore another point on the line is (6, 0)

We will fix with the help of these two points in the graphical plane, which is shown here. Then, based on the inequality sign, the required region is marked here.

In similar way, we fixed the second constraint;



$$3X2 = 9$$

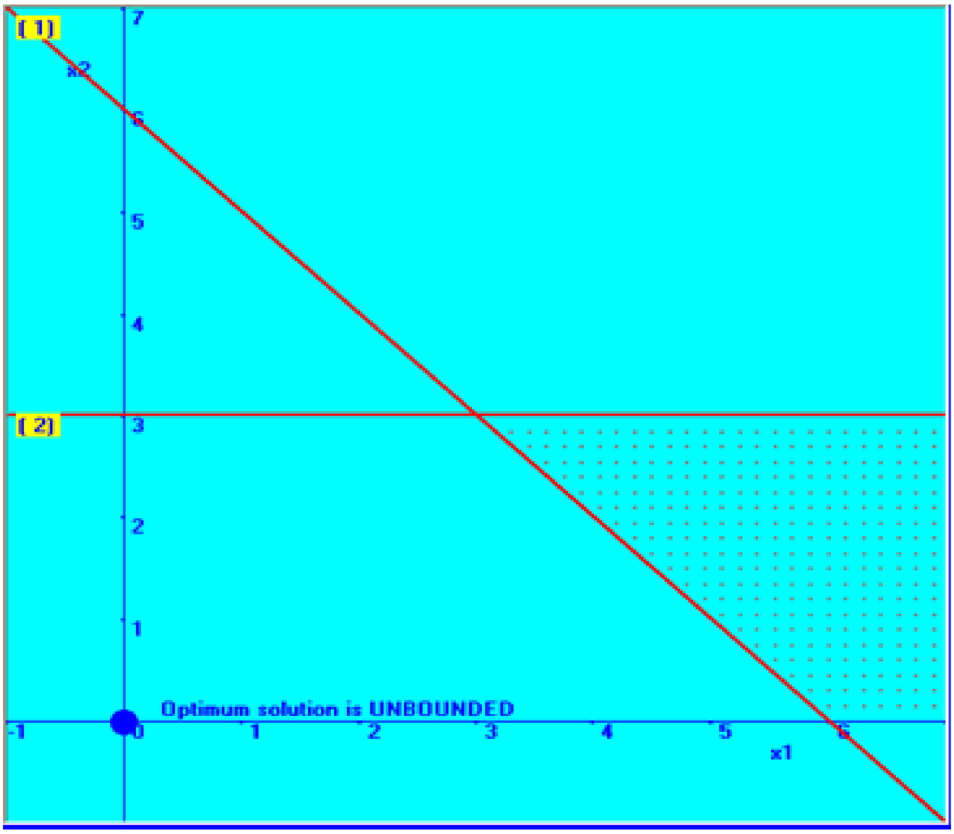
Since, $X1 = 0,$

then $X2 = 3$

Therefore, the straight line is parallel to X1 axis and passes through X2=3

Now, we fixed the straight line with the help of this point in the graphical plane, which is shown here. Then, based on the inequality sign, the required region is marked here.

Then the combined graphical solution space is arrived and shown below in the diagram.



You can note that the solution area between these two constraints is not bounded one; that is there is no finite solution space and this has implied that the solution can be infinitely increased with the limited resources. This is not obviously possible. Hence this kind of problem is categorized as 'unbounded feasible solution'.

Case-3: Unbounded Solution space-possibility of optimum solution

Consider the following LPP

Min Z= 4X1 + 4 X2

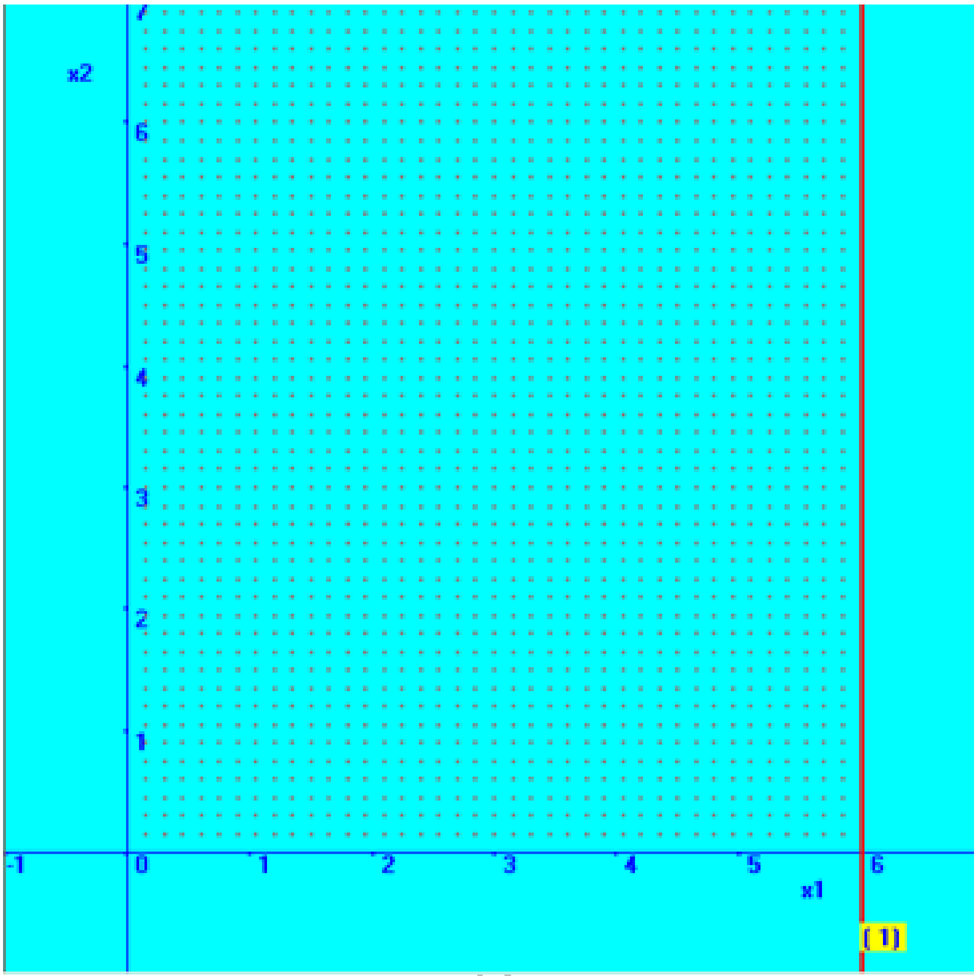
Subject to

2X1 + 0X2<= 12

0X1+ 3X2 <= 9

X1, X2 >= 0

The solution is arrived through the graphical method; various steps are given below.



Consider the first constraint-2X1 + <= 12

From this constraint, we consider the following straight line equation,

2X1 = 12

Since, X2 = 0,

then X1 = 6

Therefore, the straight line is parallel to X2 axis and passes through X1=6

Then the required region for the constraint, 2X1 + <= 12 is deducted by substituting an arbitrary point – say origin (0, 0) and check whether it is satisfied by the constraint. It is shown below in the diagram.

In a similar way, the second constraint is considered; 0X1+ 3X2 <= 9

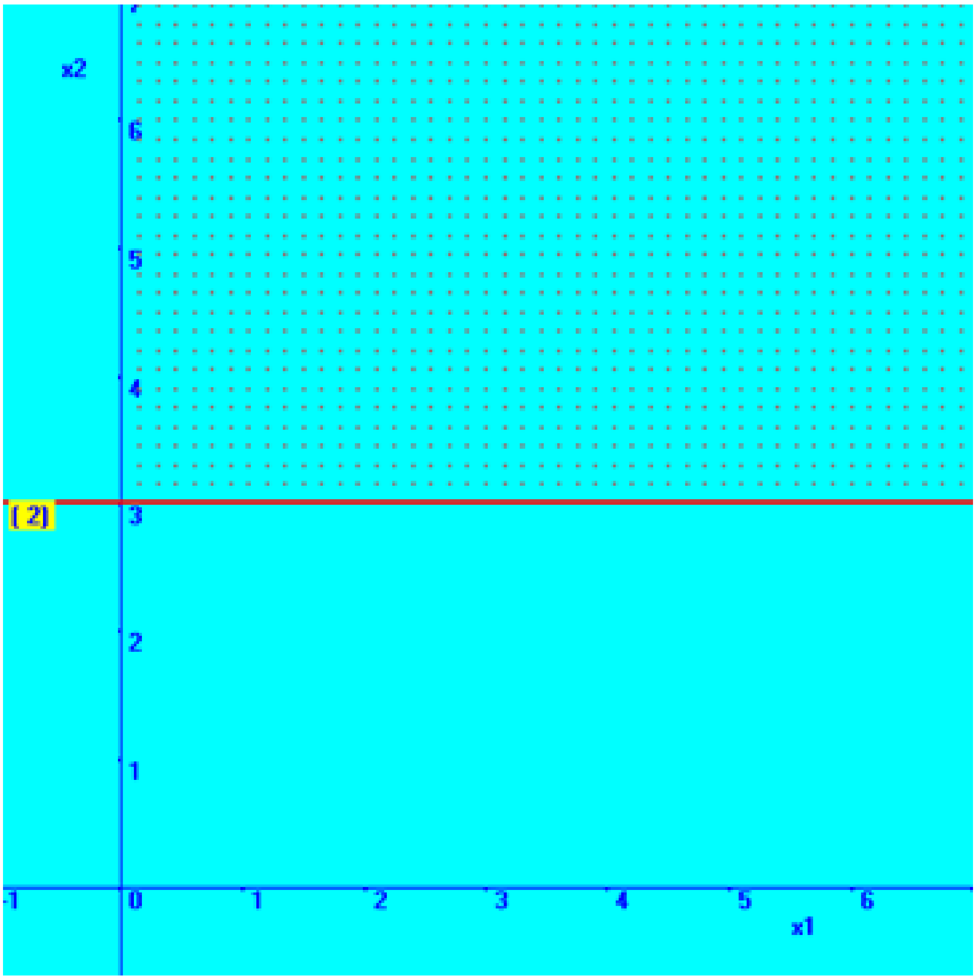
From this constraint, we consider the following straight line equation,

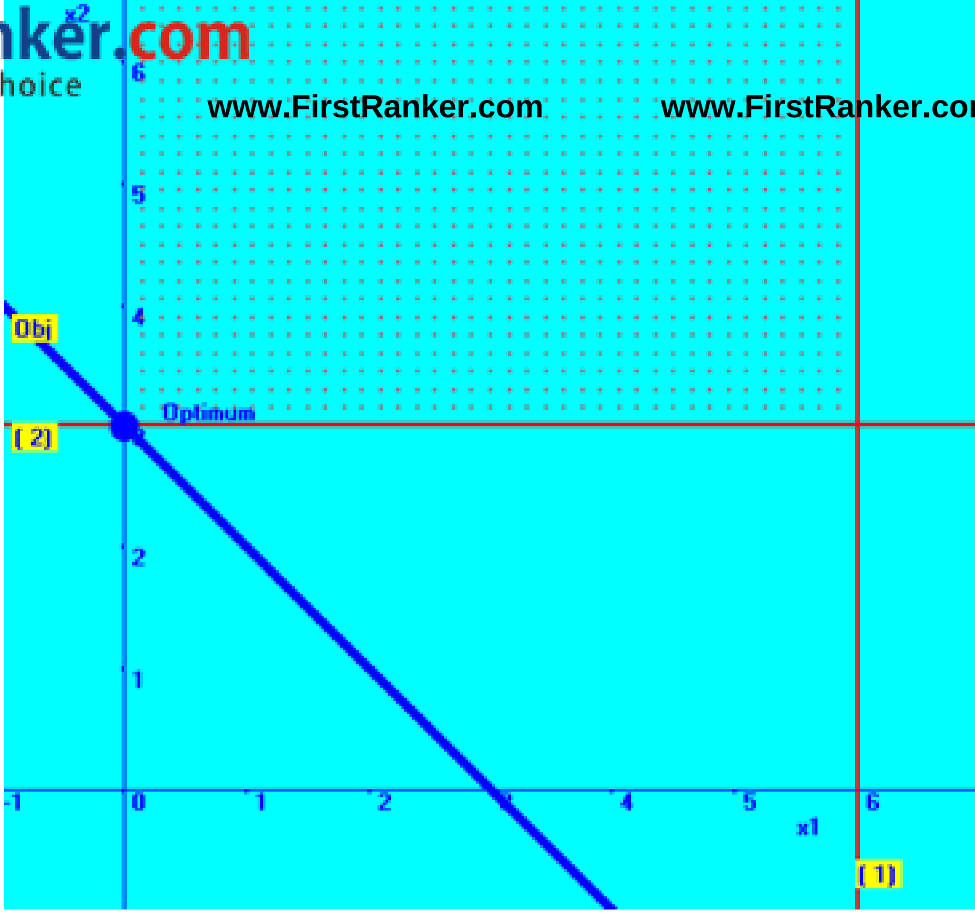
$$3X2 = 9$$

Since, $X1 = 0$,

then $X2 = 3$

Thus, you can sense that the straight line is parallel to X1 axis and passes through X2=3





Then the required region for the constraint, 3X2 <= 9 is deducted by substituting an arbitrary point – say origin (0, 0) and check whether it is satisfied by the constraint. It is shown above in the diagram.

In the diagram in the right hand side, the combined graphical solution space is shown.
You can notice that there exists an optimum solution, even though the solution space is not bounded.
At X1=0 & X2= 2, there is minimum value obtained for the problem.

Case-4: Multiple Optimum Solutions -possibility of more than one optimum solution
Consider the following LPP.

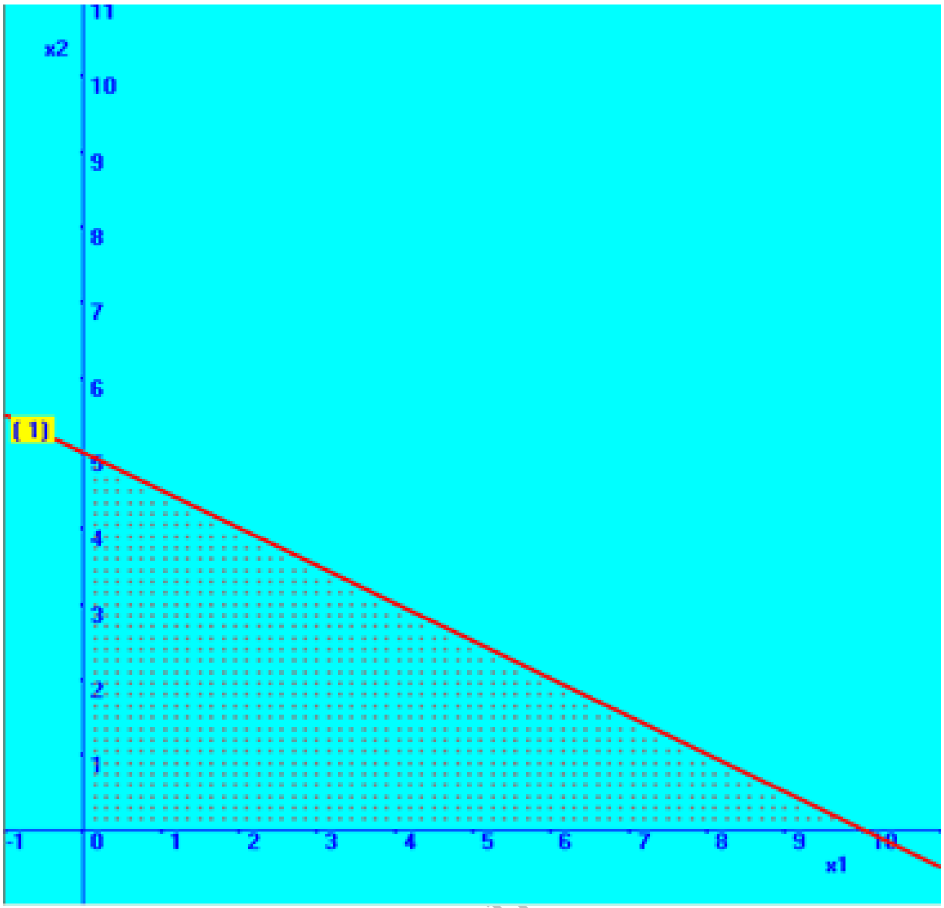
Max Z = X1 + 2X2
Subject to

$$X1 + 2X2 \leq 10$$
$$X1 + X2 \geq 1$$
$$X2 \leq 4$$
$$X1 \ \& \ X2 \geq 0$$

The solution is arrived through the graphical method; various steps are given below.

Consider the first constraint X1 + 2X2 ≤ 10

We plot the first constraint in the graphical plane;



Consider the straight line equation,

X1 + 2X2 = 10

If X1 = 0,

then X2 = 5

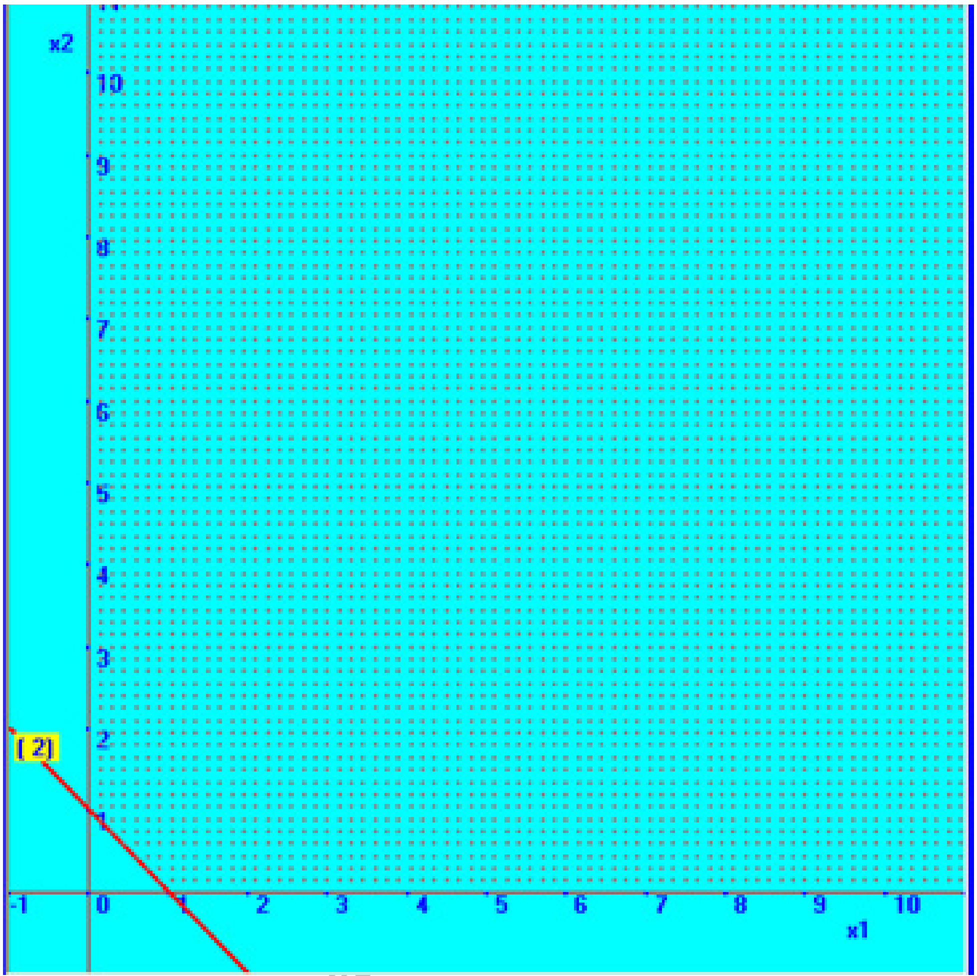
Therefore, one point on the line is (0, 5)

If X2 =0,

then X1 = 10

Therefore another point on the line is (10, 0)

We will fix with the help of these two points in the graphical plane, which is shown here. Then, based on the inequality sign, the required region is marked here.



Consider the second constraint, X1 + X2 ≥ 1

Now the respective straight line equation is considered for fixing the line.

X1 + X2 = 1

If X1 = 0,

then X2 = 1

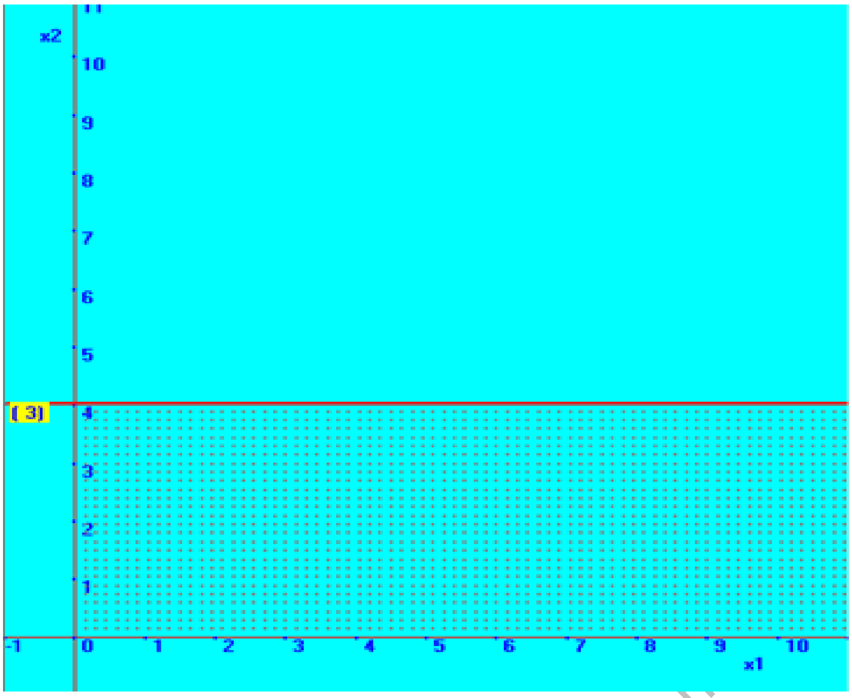
Therefore, one point on the line is (0, 1)

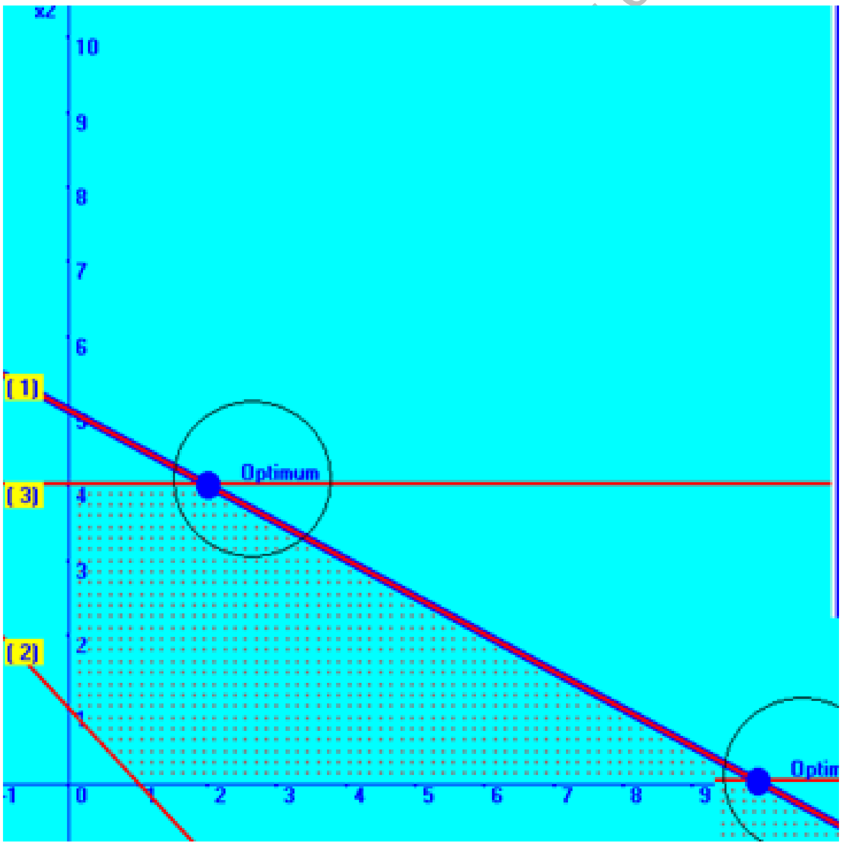
If X2 =0,

then X1 = 1

Therefore another point on the line is (1, 0).

We will fix with the help of these two points in the graphical plane, which is shown here. To fix the required region, we take arbitrary point, (0, 0) and substituting the values, in the second constraint, we find that it is not satisfying the inequality. Therefore, the region, above the straight line forms the required region.





In a similar way, the third constraint is also placed in the graphical plane and the combined picture of all the straight lines is also placed below with the solution space.

You can notice that the present problem, at two points, there is a possibility of getting maximum value for the objective function.

At X1= 2 & X2 = 4, the objective function value, Z = 10

And X1 = 10 & X2=0, again the objective function value,

Z = 10

These kinds of problems are called as linear programming problem with multiple optimum solutions.

2.5 Linear Programming – Problem Solving [SIMPLEX METHOD]

2.5.1 Introduction

The **Simplex Method** also called the '**Simplex Technique**' or the **Simplex Algorithm** is an iterative procedure for solving a linear programming problem in a finite number of steps. The method provides an algorithm which consists in moving from one vertex / corner point of the region of feasible solutions to another vertex in such a manner that the value of the objective function at the succeeding vertex is improved [lesser in case of minimization and more in case of maximization] than at the preceding vertex. This procedure of jumping from one vertex to another is then repeated. Since the number of vertices is finite, the method leads to an optimal vertex in a finite number of steps or indicates the existence of an unbounded solution.

We will introduce various concepts / definitions that are related to simplex method in the following paragraphs.

2.5.2 Definations

1. Objective Function

The function that is to be either minimized or maximized is called as objective function. For example, it may represent the cost that you are trying to minimize or total revenue that is to be maximized and so on.

2. Constraints

A set of equalities and inequalities that the feasible solution must satisfy is called as constraints of the problem.

3. Optimal Solution

A vector X, which is both feasible (satisfying all the constraints in the given problem) and optimal (obtaining the largest or smallest value for the objective function, depends upon the case) is known as optimal solution.

4. Feasible Solution

A solution vector, X, which satisfies all the constraints of the given problem is called feasible solution to the given LPP.

5. Basic Solution

X of (AX=b) is a basic solution if the n components of X can be partitioned into m "basic" and n-m "non-basic" variables in such a way that: the m columns of A corresponding to the basic variables form a nonsingular basis and the value of each "non-basic" variable is 0. The constraint matrix A has m rows (constraints) and n columns (variables).

6. Basis

The set of basic variables is called the basis for the given problem.

7. Basic Variables

Basic variables are set of variables, which are obtained by setting n-m variables values to zero, and are solving the resulting system.

8. Non-basic Variables

A variable not in the basic solution, not part of the solution is called non-basic variable.

9. Slack Variable

If we have a 'less or equal' to constraint, to convert that as an equation, a variable is added to the left hand side of the constraint; the new variable, which is added to the left hand side of the constraint is called as slack variable.

Ex: $2X1 + 5X2 \leq 10$

$$2X1 + 5X2 + SX3 = 10$$

The variable, SX3, is called as slack variable for the given constraint.

10. Surplus Variable

If we have a 'greater or equal' to constraint, to convert that as an equation, a variable is subtracted from the left hand side of the constraint; the new variable, which is subtracted, to the left hand side of the constraint is called as surplus variable.

Ex: $2X1 + 5X2 \geq 10$

$$2X1 + 5X2 - SX4 = 10$$

The variable, SX4, is called as surplus variable for the given constraint. Therefore, it is a variable added to the problem to eliminate greater-than constraints.

11. Artificial Variable

To get the initial basis in a 'greater than or' equal to constraint, additional variable is added in addition to the surplus variable. The additional variable added to a linear programming problem is called as 'artificial variable'.

12. Unbounded Solution

For some linear programs it is possible for the objective function to achieve infinitely high / low values, depends upon the objective. Such an LP is said to have an unbounded solution.

13. Standard form of LPP

Let the objective function be

$$\text{Max } Z = CX$$

And the set of constraints are represented as

$$AX \le b$$

Where, b- the vector obtained by collecting all the right hand side of the constraints.

If we add set of slack variables to all the constraints and if the constraints are equation, then that particular form is called as standard form of linear programming problem.

Therefore, $\text{Max } Z = CX$

And the set of constraints are represented as

$$AX = b$$

Example

Consider the following LPP

$$\text{Maximize } Z = 15X1 + 10X2$$

Subject to constraints

$$4X1+6X2 \le 360$$
$$3X1+0X2 \le 180$$
$$0X1+5X2 \le 200$$
$$X1, X2 \ge 0$$

The standard form of the given problem is obtained by adding slack variable X3 to the first constraint, X4 to the second and X5 to the third constraint.

$$4X1+6X2 + X3=360$$
$$3X1+0X2 +X4=180$$
$$0X1+5X2 +X5=200$$
$$X1, X2, X3, X4 \ \& \ X5 \ge 0$$

The modified objective function is

Maximize Z = 15X1 + 10X2 + 0X3 + 0X4 +OX5

14. Canonical form of LPP

Let the objective function be

Max Z = CX

And the set of constraints are represented as

AX<=b

Where, b- the vector obtained by collecting all the right hand side of the constraints.

This form, where all the constraints are '<=' type for a maximization problem and '>=' type for a minimization problem is known as canonical form of LPP.

2.5.3 Simplex Algorithm

Step 1

Check whether the objective function of the given L.P.P. is to be maximized or minimized.

If it is to be minimized then convert it into a problem of maximizing it by using the result

Minimum= -Maximum (-z)

Step 2

Check whether all 'b' values are non-negative. If any one of b is negative then multiply the corresponding inequality constraints by –1, so as to get all b values as non-negative.

Step 3

Convert all the in equations of the constraints into equations by introducing slack and/or surplus variables in the constraints. Put the costs of these variables equal to zero in the objective function, if the variables are slack variables. If surplus / artificial variables are added, then we need to use 'Big M' Method, which is a modified algorithm of the same simplex method.

Step 4

Obtain an initial basic feasible solution to the problem in the form Xb=B^-1 b and put it in the first column of the simplex table.

Step 5

Compute the net evaluations zj-cj (j=1,2…n) by using the relation,

Zj-Cj=CB yj-cj.

Examine the sign zj-cj.

i. If all values are >=0, then initial basic feasible solution is an optimum feasible solution.
ii. If at least one value < 0, go to next step.

Step 6

If there is more than one negative value, then choose most negative.

Step 7

Compute the ratio

{xb/yi, yi>0,I=1,2…. m} and choose the minimum of them.

The common element in the kth row and rth column is known as the leading element (pivotal element) of the table.

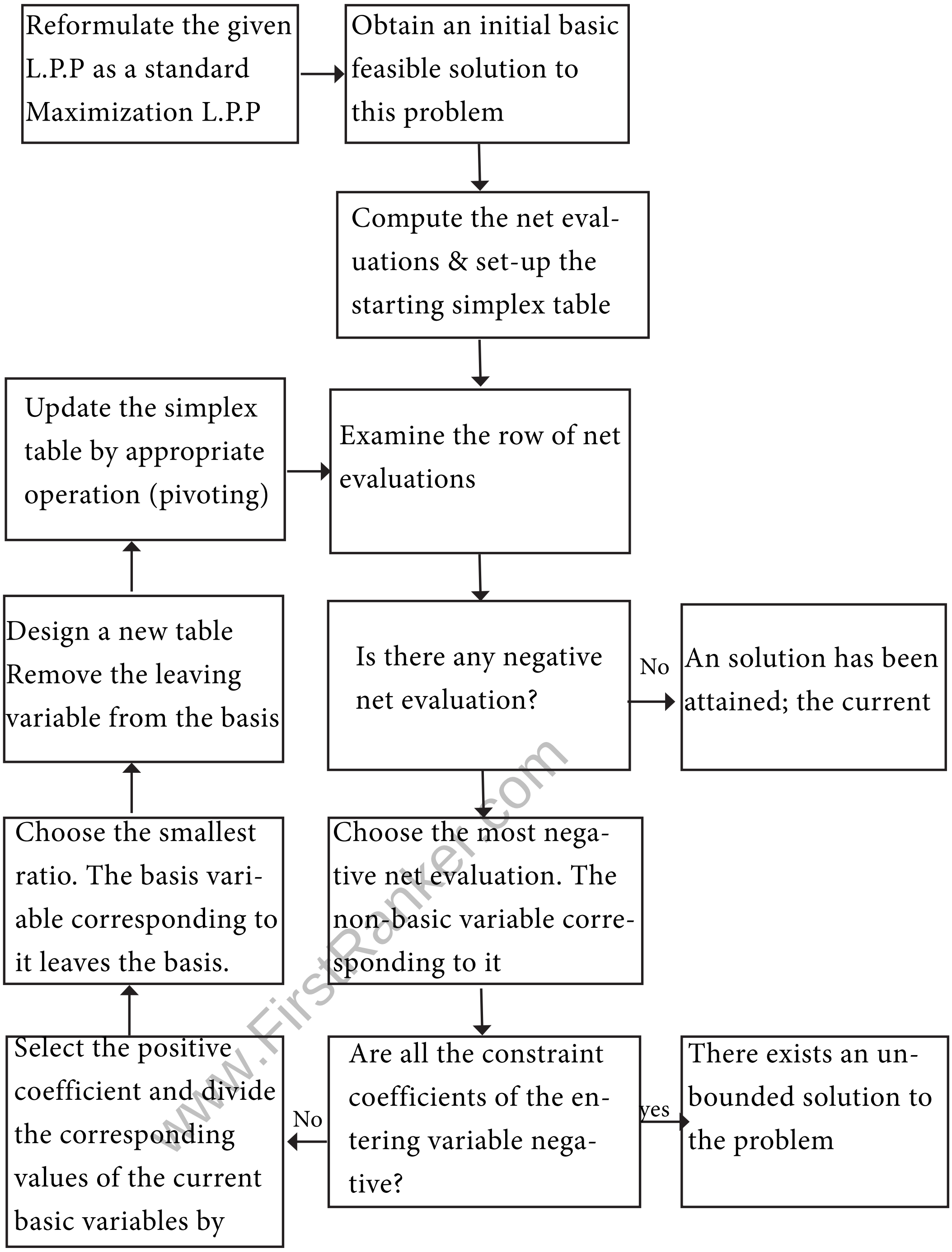
Step 8

Convert the leading element to unity by dividing its row by the leading element itself and all other elements in its column to zeros.

Step 9

Go to step 5 and repeat the process until either an optimum solution is obtained or there is an indication of unbounded solution.

Simplex Algorithm for Maximization L.P.P



2.5.4 Properties of The Simplex Method

1. The Simplex method for maximizing the objective function starts at a basic feasible solution for the equivalent model and moves to an adjacent basic feasible solution that does not decrease the value of the objective function. If such a solution does not exist, an optimal

solution for the equivalent model has been reached. That is, if all of the Coefficients of the non-basic variables in the objective function equation are greater than or equal to zero at some point, then an optimal solution for the equivalent model has been reached.

2. If an artificial variable is in an optimal solution of the equivalent model at a nonzero level, then no feasible solution for the original model exists. On the contrary, if the optimal solution of the equivalent model does not contain an artificial variable at a non-zero level, the solution is also optimal for the original model.

3. If all the slack, surplus, and artificial variables are zero when an optimal solution of the equivalent model is reached, then all of the constraints in the original model are strict "equalities" for the values of the variables that optimize the objective function.

4. If a non-basic variable has zero coefficients in the objective function equation when an optimal solution is reached, there are multiple optimal solutions. In fact, there is infinity of optimal solutions, the Simplex method finds only one optimal solution and stops.

5. Once an artificial variable leaves the set of basic variables (the basis), it will never enter the basis again, so all calculations for that variable can be ignored in future steps.

6. When selecting the variable to leave the current basis:

(a) If two or more ratios are smallest, choose one arbitrarily.

(b) If a positive ratio does not exist, the objective function in the original model is not bounded by the constraints. Thus a Finite optimal solution for The original model does not exist.

7. If a basis has a variable at zero level, it is called a degenerate basis.

8. Although cycling is possible, there have never been any practical problems for which the Simplex method failed to converge.

Example

Maximize $z = X_1+2X_2$

Subject to:

$$-X_1+2X_2<=8,$$
$$X_1+2X_2<=12,$$
$$X_1-X_2<=3;$$
$$X_1>=0 \text{ and } X_2>=0.$$

Solution

Step 1

Introducing the slack Variable $X_3>=0$, $X_4>=0$ and $X_5>=0$ to the first, second and third constraints respectively and convert the problem into standard form.

$$-X_1+2X_2 + X_3 = 8,$$
$$X_1+2X_2 \;\; +X_4 =12,$$
$$X_1- \;\; X_2 \;\; +X_5 = 3;$$

And the modified objective function is

$$Z=X_1+2X_2+0X_3+0X_4+0X_5$$
$$X_1, X_2, X_3, X_4 \ \& \ X_5 >0$$

The constraints the given L.P.P are converted into the system of equations:

$$\begin{pmatrix} -1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \\ 3 \end{pmatrix}$$

Step 2

An obvious initial basic feasible solution is given by

$$X_B=B^{-1}b.$$

Where $B=I_3$ and $X_B=[X_3\ X_4\ X_5]$, & I_3 stands for Identity matrix of order of 3 (that is a 3X3 matrix).

That is,

$$[X_3\ X_4\ X_5] = I_3\,[8 \quad 12 \quad 3] = [8 \quad 12 \quad 3]$$

Step 3

We compute yj and the net evaluations, zj-cj corresponding to the basic variables X3, X4 and X5:

$$y1=B\text{-}1a1 = I3\,[-1 \quad 1 \quad 1] = [1 \quad 1 \quad 1]$$

$$y2=B\text{-}1a1 = I3\,[2 \quad 2 \quad -1] = [2 \quad 2 \quad -1]$$

$$y3 = B\text{-}1e1 = e1, \quad y4 = B\text{-}1e2 = e2 \quad \text{and} \quad y5 = B\text{-}1e3 = e3.$$

$$Z1\text{-}C1 = cB\ y1\text{-}c1 = (0\ 0\ 0)\ [-1 \quad 1 \quad 1]\text{-}1 = -1.$$

$$Z2\text{-}C2 = cB\ y2\text{-}c2 = (0\ 0\ 0)\ [2 \quad 2 \quad -1]\text{-}2 = -2.$$

$$Z3\text{-}C3 = cB\ y3\text{-}c3 = (0\ 0\ 0)\ e1\text{-}0 = 0,$$

$$Z4\text{-}C4 = cB\ y4\text{-}c4 = (0\ 0\ 0)\ e2\text{-}0 = 0,$$

$$Z5\text{-}C5 = cB\ y5\text{-}c5 = (0\ 0\ 0)\ e3\text{-}0 = 0.$$

Step 4 - Deciding the entering variable

Making use of the above information, the starting simplex tableau is written as follows:

cB	yB	XB	y1	y2	y3	y4	y5
0	y3	8	-1	2	1	0	0
0	y4	12	1	2	0	1	0
0	y5	3	1	-1	0	0	1
	z	0	-1	-2	0	0	0

Simplex Table-1

From the starting tableau, it is apparent that there are two Zj-Cj values, which are having negative coefficients.

We choose the most negative of these, viz., -2. The corresponding column vector y2, therefore, enters the basis.

Step 5 - Deciding the leaving variable

Now, we will compute the ratios using the entering column elements and RHS of each constraint.

Each row of the table, the respective RHS coefficient of the constraint is divided by entering column, non-zero element and placed in the last column of the table. Then, the minimum among the value is chosen as leaving variable.

Min {XBi/Yi2, Yi2>0} = Min. {8/2, 12/2, no ratio for third row} = 4.

Since the minimum ratio occurs for the first row, basis vector Y3 leaves the basis. The common intersection element y12 (=2) become the leading element for updating. We indicate the leading element in bold type with a star *.

Step 6

Convert the leading element y12 to unity and all other elements in its column (i.e.y2) to zero by the following transformations:

Y11 =Y11/Y12 =1/2, Y10 = Y10/Y12 =8/2 or 4, so on,

Y20=y20-(y10/y11) y22=12-(8/2) (2) =4.

Y30=y30-(y10/y12) y32=3-(8/2) (-2) =11.

Y21=y21-(y11/y12) y22=1-(-1/2) (2) =2.

Y31=y31-(y11/y12) y32=1-(-1/2) (-2) =0. And so on………

Step 7

Using the above computations, the following iterated simplex tableau is obtained:

The above simplex tableau yields a new basic feasible solution with the increased value of z.

Now since z1-c1 < 0, y1 enters the basis.

Also, since Min. {X Bi / yi >0} = Min {4/2, 7/ (1/2)} = 2, y4 leaves the basis.

Thus the leading element will be y21 (=2).

Converting the leadwing element to unity and all other elements of yi to zero by usual row transformations the next iterated tableau is obtained.

Cb	YBa	Xba	Y1	Y2	Y3	Y4	Y5	Xbi/Yi1
2	Y2	X2=5	0	1	1/4	1/4	0	
1	Y1	X1=2	1	0	-1/2	1/2	0	
0	Y5	X5=6	0	0	3/4	-1/4	1	
Zj-Cj			0	0	0	1	0	

Since, all Zj-Cj>=0, we conclude that there is no more improvement possible and the problem is in its optimum stage.

Therefore, the optimal solution for the given problem is

X1= 2 X2= 5 Max Z = 12

Example-2

Solve the following problem by simplex method [the same problem is solved under graphical method already]

Maximize z = 15X1 + 10X2

Subject to constraints

4X1+6X2 <=360

3X1+0X2<=180

0X1+5X2 <=200

X1, X2>=0

Solution

The problem is converted into standard form by adding slack variables X3, X4 & X5 to the each of the constraint.

4X1+6X2 +X3=360

3X1+0X2+X4=180

0X1+5X2+X5=200

And the modified objective function is

Maximize z = 15X1 + 10X2 +0X3 + 0X4 + 0X5

Then the first simplex table is arrived.

Then we will compute the Zj-Cj (net evaluations) for each column as follows;

For the first column Y1, it is computed as follows;

$$Z1-C1= cB\ y1-c1$$

$$[(0*4) + (0*1) + (0* 0)]-15 = -15$$

In similar lines, the evaluations are calculated for all the columns and placed in the last row of the table.

Then we will compute the ratios, using RHS of each constraint and dividing it by the respective entering column non-zero, non-negative quantities; this is placed in the last column of the following table.

			15	10	0	0	0	
Cb	**YBa**	**Xba**	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	**Xbi/Yi1**
0	Y3	X3=360	4	6	1	0	0	360/4= 90
0	Y4	X4=180	3	0	0	1	0	180/3= 60
0	Y5	X5=200	0	5	0	0	1	-
Zj-Cj			-15	-10	0	0	0	

Then, the minimum among the value is chosen as leaving variable; therefore, X1 entering the basis and X4 leaving the basis.

Then, we need to do the simple 'row' operations or modified 'Gauss-Jordon' Elimination process. We have to have 1 at the row/column intersection [known as pivotal element] and the rest of the entries in the pivotal column as zero.

The first row is replaced by dividing all the entries by 4, which is the 'pivotal' element of the table [The number in the intersection of 'entering column' and 'leaving row'] and placed in the second iteration table in the corresponding position of the original place.

For example, consider the original first row

Y4	0	X3=180	3	0	0	1	0

Now this is replaced as follows [by dividing each entry by 3]

Y1	15	X1=60	1	0	0	1/3	0

Now the X3=360 is replaced by the following calculation

New element= Old element – [respective element in the pivotal row X respective element in the pivotal column/pivotal element]

New element for 360 = 360 – [(180 * 4) /3] = 360-240 = 120

Similarly, for the Y2 column, of the first row, for the '6', the new element is arrived in similar way

New element for 6 = 6 – [(4 * 0) /3] = 6-0 = 6

The new table-2 is arrived as follows with the following changes;

- X3 is replaced by X1
- Row operations are carried out

Table-2- which is showing the entering column vector and leaving variable

			15	10	0	0	0	
Cb	**YBa**	**Xba**	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	**Xbi/Yi1**
0	Y3	X3=120	0	6	1	-4/3	0	120/6= 20
15	Y1	X1=60	1	0	0	1/3	0	-
0	Y5	X5=200	0	5	0	0	1	200/5=40
		Zj-Cj	0	-10	0	5	0	

In the next step, X2 is entering the basis and X3 leaves the basis; the row operations are carried out and the new table-3 is given below;

Table-3

			15	10	0	0	0	
Cb	YBa	Xba	Y1	Y2	Y3	Y4	Y5	Xbi/Yi1
10	Y2	X2=20	0	1	1/6	-2/9	0	
15	Y1	X1=60	1	0	0	1/3	0	
0	Y5	X5=100	0	0	-5/6	10/9	1	
		Zj-Cj	0	0	10/6	25/3	0	

You can notice that in the end of second iteration, all the net evaluations (Zj-Cj) are non-negative; this implies that the current solution is in the optimal stage.

Therefore,

X1 = 60 X2= 20 Max Z=(15* 60) + (10 * 20) = 900+200=1100

You can recall that the same answer was obtained through the graphical method.

In the next sections, we solve the special cases, attempted by graphical method.

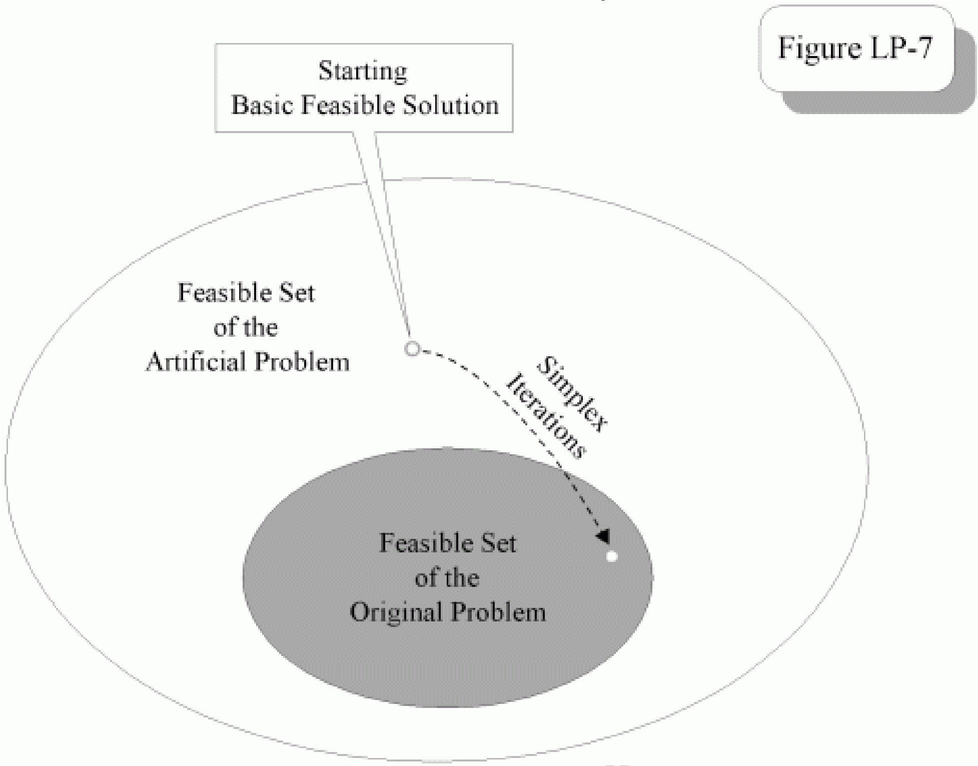
2.6 Big-M Method [Introduction]

In order to obtain an initial basic feasible solution, it is necessary to convert the given LPP into its standard form; in order to obtain the standard form; a non-negative variable is added to the left side of each of the equation that lacks the much needed starting basic variables. So added variable is called an artificial variable and it places the role in providing initial basic feasible solution.

The starting point of the big-M method is a basic solution that is feasible to the artificial problem. This solution allows us to start the

Simplex algorithm expeditiously, but it is not a feasible solution to the original problem. Our goal is to iterate toward solutions that are inside the original feasible set, assuming that it is not empty.

The artificial variables have no physical meaning from the view of the original problem; the method will be valid only if it is possible to force these variables at zero level when the optimum solution is attained.



To accomplish this goal, we designed an artificial objective function that is an aggregation of two functional parts, of which one is a copy of the original objective function and the other is a "penalty function" associated with the artificial variables.

The magnitude of the penalty, M, needs to be chosen to ensure that the contribution to this aggregate objective function from the second part, whenever it is positive, always outweighs that from the first part, for any solution.

Artificial Variable

If there is a constraint which is of > = or equality type then it is needed to write the constraint in standard form because it is not possible to get unit column vector. The additional variable, which is added in order to get unit column vector, is called an artificial variable.

Two methods are generally employed for the solution of linear programming problems having artificial variables:

1. Two-Phase Method.
2. Big-M Method (or) Method of Penalties.

We will have discussion only on Big-M Method here.

The Big M Method Procedure

If an LP has any > or = constraints, the Big M method or the two-phase simplex method may be used to solve the problem.

The Big M method is a version of the Simplex Algorithm that first finds a best feasible solution by adding “artificial” variables to the problem. The objective function of the original LP must, of course, be modified to ensure that the artificial variables are all equal to 0 at the conclusion of the simplex algorithm. The iterative procedure of the algorithm is given below.

Step-1

Modify the constraints so that the RHS of each constraint is non-negative (This requires that each constraint with a negative RHS be multiplied by -1. Remember that if any negative number multiplies an inequality, the direction of the inequality is reversed). After modification, identify each constraint as a <, >, or = constraint.

Step-2

Convert each inequality constraint to standard form (If a constraint is a ≤ constraint, then add a slack variable Xi; and if any constraint is a ≥ constraint, then subtract an excess variable Xi , known as surplus variable).

Step-3

Add an artificial variable a1 to the constraints identified as '≥' or with '=' constraints at the end of Step2. Also add the sign restriction ai ≥ 0.

Step-4

Let M denote a very large positive number. If the LP is a minimization problem, add (for each artificial variable) Mai to the objective function. If the LP is a maximization problem, add (for each artificial variable) -Mai to the objective function.

Step-5

Since each artificial variable will be in the starting basis, all artificial variables must be eliminated from row 0 before beginning the simplex.

Now we solve the transformed problem by the simplex (In choosing the entering variable, remember that M is a very large positive number). If all artificial variables are equal to zero in the optimal solution, we have found the optimal solution to the original problem. If any artificial variables are positive in the optimal solution, the original problem is infeasible!!!

- If at least one artificial variable is present in the basis with zero value, in such a case the current optimum basic feasible solution is a degenerate solution.
- If at least one artificial variable is present in the basis with a positive value, then in such a case, the given LPP does not have an optimum basic feasible solution. The given problem is said to have a pseudo-optimum basic feasible solution.

Example-1

Solve the following LPP by using Big -M Method

Maximize Z = 6X1+4X2

Subject to constraints:

2X1+3X2<=30

3X1+2X2<=24

X1+X2>=3

Solution

Introducing slack variables S1>=0, S2>=0 to the first and second equations in order to convert <= type to equality and add surplus variable to the third equation S3 >=0 to convert >= type to equality.

Then the standard form of LPP is

MAX Z=6X1+4X2+0S1+0S2+0S3

Subject to constraints

2X1+3X2+S1=30

3X1+2X2+S2=24

X1+X2-S3=3

Clearly there is no initial basic feasible solution. So an artificial variable A1>=0 is added in the third equation. Now the standard form will be

MAX Z=6X1+4X2+0S1+0S2+0S3+A1

Subject to constraints

2X1+3X2+S1=30

3X1+2X2+S2=24

X1+X2-S3+A1=3

Now the first simplex table is placed below with following additional information.

It clearly shows the net evaluations for each column variable; from these calculations, it is clear that X1 is the entering variable and A1, the artificial variable leaves the basis.

Introduce Y1 and drop Y6

			6	4	0	0	0	-M	Xbi/ Yi1
Cb	YBa	Xba	Y1	Y2	Y3	Y4	Y5	Y6	
0	Y3	X3=30	2	3	1	0	0	0	30/2= 15
0	Y4	X4= 24	3	2	0	1	0	0	24/3= 8
-M	Y6	A1=3	1	1	0	0	-1	1	3/1= 3
	Zj-Cj		-M-6	-M-4	0	0	-1	1	

Then, in the usual row operations, we modify this table and the new table is arrived.

Table-2

			6	4	0	0	0	-M	Xbi/Yi1
Cb	YBa	Xba	Y1	Y2	Y3	Y4	Y5	Y6	
0	Y3	X3=24	0	1	1	0	2	-2	24/2= 12
0	Y4	X4=15	0	-1	0	1	3	-3	15/3= 5
6	Y1	X1= 3	1	1	0	0	-1	1	-
	Zj-Cj		0	2	0	0	-6	6	

Introduce Y5 and drop Y4 from the basis; the new modified **table-3** is given below;

			6	4	0	0	0	-M	Xbi/i1
Cb	YBa	Xba	Y1	Y2	Y3	Y4	Y5	Y6	
0	Y3	X3=14	0	5/3	1	-2/3	0	0	

0	Y5	X5=5	0	-1/3	0	1/3	1	-1	
6	Y1	X1= 8	1	2/3	0	1/3	0	0	
Zj-Cj			0	0	0	2	0	0	

Since all Zj –Cj >=0, an optimum solution has reached. Thus an optimum basic feasible solution to the LPP is

X1=8

X2=0

MAX Z =48

Example-2

Use big M method to solve a given LPP.

Minimize Z = 5X1-6X2-7X3

Subject to constraints

X1+5X2-3X3>=15

5X1-6X2+10X3<=20

X1+X2+X3=5

X1, X2 & X3 >= 0

Solution

Introducing slack variables X4>=0 to the first equations in order to convert <= type to equality and add surplus variable X5>=0 to the second equation in order to convert >= type to equality.

Then the standard form of LPP is

MIN Z=5X1-6X2-7X3+X4-X5

Subject to constraints

5X1-6X2+10X3+X4=20

X1+5X2-3X3-X5=15

X1+X2+X3=5

Clearly there is no initial basic feasible solution. So two artificial variables A1>=0 and A2>=0 are added in the second and third equation. Now the standard form will be

MIN Z=5X1-6X2-7X3+X4-X5+A1+A2

Subject to constraints

5X1-6X2+10X3+X4=20

X1+5X2-3X3-X5+A1=15

X1+X2+X3+A2=5

Simplex Table-1

			5	-6	-7	0	0	-M	-M	Xbi/
Cb	**YBa**	**Xba**	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	**Y6**	**Y7**	**Yi1**
0	Y4	X4=20	5	-6	10	1	0	0	0	-
M	Y6	X6=15	1	5	-3	0	-1	1	0	15/5 = 3
M	Y7	X7= 5	1	1	1	0	0	0	1	5/1=5
Zj-Cj			2M-5	6M+6	2M+7	0	-M	0	0	

From the calculations related to entering column and leaving variable, which is summarized in the above table, it is clear that introduces X2 and drop X6 from the basis. The new simplex table is given below;

Simplex Table-2

			5	-6	-7	0	0	M	M	Xbi/
Cb	**YBa**	**Xba**	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	**Y6**	**Y7**	**Yi1**
0	Y4	X4=38	31/5	0	32/5	1	-6/5	6/5	0	190/31
-6	Y2	X6=3	1/5	1	-3/5	0	-1/5	1/5	0	-
M	Y7	X7= 2	4/5	0	8/5	0	1/5	-1/5	1	10/8
Zj-Cj			(-31+ 4M)/5	0	3+8 M/5	0	6+ M/5	-	-	

Now, we see that we have to include X3 and drop X7 from the basis.

The new modified table- 3 is given below

-1/5 +32/5*1/5*5/8

Simplex Table-3

			5	-6	-7	0	0	M	M	Xbi/Yi1
Cb	**YBa**	**Xba**	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	**Y6**	**Y7**	
0	Y4	X4=30	31/5	0	0	1	-2	2	-4	
-6	Y2	X2=15/4	1/5	1	0	0	-1/8	1/8	3/8	
-7	Y3	X3= 5/4	0	0	1	0	-1/8	1/8	5/8	
Zj-Cj			-31/5	0	0	0	-1/8	-	-	

Since all Zj –Cj <=0, an optimum solution has reached. Thus an optimum basic feasible solution to the LPP is

X3=5/4

X2=15/4

MIN Z= (5*0) + (-6*15/4) + (-7* 5/4)

= (-90/4) + (-35/4)

= -125/4

2.7 Duality – Introduction

The term dual in the general sense means implies two or double. In the context of linear programming duality implies that each programming problem can be analyzed in two different ways but having equivalent solutions. Associated with every linear programming problem whether it is maximization or minimization type there exist another linear programming problem, which is based upon same data, and having same solution. The original problem is called the primal problem while the associated one is called its dual problem.

The concept of duality is based on the fact that any linear programming must be first put in its standard form before solving the problem by simplex method. Since all the primal-dual computations are obtained directly from the simplex table, it is logical that we define the dual that may be constituent with the standard form of the primal.

The format of the Simplex method is such that solving one type of problem is equivalent to solving the other simultaneously.

Thus, if the optimal solution to one is known, then optimal solution of the other can also be read from the final simplex table.

In some cases, considerable computing time can be saved by solving the dual.

The importance of the duality concept is due to two main reasons.

- Firstly, if the primal contains a large number of constraints and a smaller number of variables, converting it to the dual problem and then solving it can considerably reduce the labor of computation.
- Secondly, the interpretation of the dual variables from the cost or economic point of view proves extremely useful in making future decisions in the activities being programmed.

Steps In Formulating The Dual Problem

Various steps involved in formulation of a primal-dual pair are:

Step 1

Put the given linear programming problem into its standard form. Consider it as a primal problem.

Step 2

Identify the variables to be used in the dual problem. For each constraint, assign one dual variable. The number of the variables in the dual problem will be equal to the number of constraint equations in the primal problem.

Step 3

- Write down the objective function of the dual, using the right hand side constants of the primal constraints.
- If the problem is of maximization type, the dual will be a minimization problem and vice versa.

Step 4

Make use of dual variable identified in Step 2; write down the constraints for the dual problem.

a. If the primal is a maximization problem, the dual constraints must be all of '>=' type and vice versa.
b. The column coefficients of the primal constraints become the row coefficients of the dual constraints.
c. The coefficients of the primal objective function become the right hand side constants of the dual constraints.
d. The dual variables are defined to be unrestricted in sign.

Step 5

Using steps 3 and 4, write down the dual of the L. P. P

Advantages of duality

1. It is advantageous to solve the dual of a primal having fewer constraints, because the number of constraints usually equals the number of iteration required to solve the problem.
2. It also avoids the necessary for adding surplus or artificial variables and solves the problem quickly (i.e. primal-dual algorithm). In economics, duality is useful in the formulation of the input and output systems.
3. The dual variables provide n important economic interpretation of the final solution of an LP problem.
4. It is quite useful when investigating changes in the coefficients or formulation of an LP problem (i.e. in sensitivity analysis).
5. Duality is used to solve an LP problem by the Simplex method in which the initial solution is infeasible (i.e. the dual simplex method).

Example-1

Write the dual of the following linear programming problem:

Maximize Z=x1-x2+3x3

Subject to the constraints:

x1+x2+x3<=10

2x1-0x2-x3<=2

2x1-2x2-3x3<=6

Where x1, x2, x3>=0

Solution

If W1, W2, W3 be the dual variable corresponding to the primal constraints.

Then, the dual problem will be

Minimize Z=10W1+2W2+6W3

Subject to the constraints:

W1+2W2+2W3>=1

W1-2W3>=-1

W1-W2-3W3>=3

Where W1, W2, W3 >=0

Example-2

Max Z=3x1+10x2+2x3

Subject to

2x1+3x2+2x3<=7

3x1-2x2+4x3=3

And x1, x2, x3 >=0

Solution

Given the primal LPP is

Max Z=3x1+10 x2+2x3

Subject to

$$2x_1+3x_2+2x_3 \le 7$$

$$3x_1-2x_2+4x_3=3$$

$$\text{And } x_1, x_2, x_3 \ge 0$$

Since the primal problem contains 2 constraints and 3 variables, the dual problem will contain 3 constraints and 2 dual variables Y1, Y2.

Also, since the second constraint in the primal problem is with equality, the corresponding second dual variable Y2 is unrestricted in sign.

Therefore, the dual problem is

Min $W=7Y_1+3Y_2$

Subject to

$$2Y_1+3Y_2 \ge 3$$

$$3Y_1-2Y_2 \ge 10$$

$$2Y_1+4Y_2 \ge 2$$

And $Y_1 \ge 0$, & Y_2 is unrestricted.

Important theorems' statement used in Primal -Dual context

1. The dual of the dual is primal.
2. If either the primal or the dual problem has an unbound objective function value, then the other problem has no feasible solution is known as Existence theorem.
3. If the primal or the dual has a finite optimum solution, then the other problem also possess a finite optimum solution and the optimum values of the objective functions of the two problems are equal is called Fundamental theorem of duality.
4. Each basic feasible solution in primal problem is related to a complementary basic feasible solution in dual problem.
5. Complementary slackness theorem:

a) If a primal variable is positive, then the corresponding dual constraint is an equation at the optimum and vice versa.

b) If a primal constraint is a strict inequality, then the corresponding dual variable is zero at the optimum and vice versa.

6. If dual has no feasible solution, then primal also admits no feasible solution.

Duality and Simplex Method

If primal is a maximization problem, then following are the set of rules that govern the derivation of the optimum solution:

Rule 1

Corresponding net evaluations of the starting primal variables=Difference between the left and right sides of the dual constraints associated with the starting primal variables.

Rule 2

Negative of the corresponding net evaluations of the starting dual variables equal to the difference between the left and right sides of the primal constraints associated with the starting dual variables.

Rule 3

If the primal (dual) problem is unbounded, then the dual (primal) problem does not have any feasible solution.

Example-1

Use duality to solve the following L.P.P:

Maximize Z=2X1+X2

Subject to constraints:

X1+2X2<=10

X1+X2<=6

X1-X2<=2

X1-2X2<=1

X1, X2, X3>=0

Solution

The dual problem of the given primal is

Minimize Z*=10Y1+6Y2+2Y3+Y4

Subject to constraints:

Y1+Y2+Y3+Y4>=2

2Y1+Y2-Y3-2Y4>=1

Y1, Y2, Y3, Y4>=0

Introducing surplus variables s1>=0, s2>=0 and artificial variables A1>=0, A2>=0, an initial basic feasible solution is A1=2 and A2=1.

The iterative simplex tables are:

Initial Iteration

Introduce y1 and drop y8.

			-10	- 6	- 2	-1	0	0	-M	-M
CB	**YB**	**WB**	**y1**	**y2**	**y3**	**y4**	**y5**	**y6**	**y7**	**y8**
-M	y7	2	1	1	1	1	-1	0	1	0
-M	y8	1	(2)	1	-1	-2	0	-1	0	1
	Z*	-3M	-3M+10	-2M+6	2	M+1	M	M	0	0

First iteration: Introduce y3 and drop y7.

			-10	- 6	- 2	- 1	0	0	-M	-M
CB	**YB**	**WB**	**y1**	**y2**	**y3**	**y4**	**y5**	**y6**	**y7**	**y8**

-M	y7	3/2	0	1/2	(3/2)	2	-1	1/2	1	1/2
10	y1	1/2	1	1/2	-1/2	-1	0	-1/2	0	1/2
	Z*	-3M/2-5	0	-M/2+1	-3M/2+7	-2M+11	M	-M/2+5	0	3M/2-

Second Iteration: Introduce y2 and drop y1.

			-10	- 6	-2	- 1	0	0	-M	-M
CB	**YB**	**WB**	**y1**	**y2**	**y3**	**y4**	**y5**	**y6**	**y7**	**y8**
-2	y3	1	0	1/3	1	4/3	-2/3	1/3	2/3	-1/3
-10	y1	1	1	(2/3)	0	-1/3	-1/3	-1/3	1/3	13
	Z*	-12	0	-4/3	0	-5/3	14/3	8/3	M-14/3	M-8/3

Final Iteration: Optimum Solution.

			-10	- 6	-2	-1	0	0	-M	-M
CB	**YB**	**WB**	**y1**	**y2**	**y3**	**y4**	**y5**	**y6**	**y7**	**y8**
-2	y3	1/2	-1/2	0	1	3/2	-1/2	1/2	1/2	-1/2
-6	y2	3/2	3/2	1	0	-1/2	-1/2	-1/2	1/2	1/2
	Z*	-10	2	0	0	1	4	2	M-4	M-2

Thus an optimum feasible solution to the dual problem is

w1=0, w2=3/2, w3=1/2;

Min Z*= - (- 10) =10

Chapter End Problems

1. Solve the following problem using Simplex Method

MaximizeZ = 3x1 + 2x2 + 5x3

Subject to

2X1 + 3X2 + 5X3 < 30

X1 + X2 + X3 < 11

2X1 - X2 - X3 < 8

X1, X2 and X3 > 0

2. Find the optimal solution to the LP problem given, using graphical method.

MaximizeZ = 600X1 + 400X2

Subject to

300X1 + 1000X2 ≥ 24000

1000X1 + 1000X2 ≥ 16000

2000X1 + 6000X2 ≥ 48000

X1, X2 ≥ 0

3. Using Simplex method, find non-negative values of x1 and x2 which

Maximize Z = 2X1 + X2

subject to the following constraints

X1 + 2X2 ≤ 10

X1 + X2 ≤ 6

X1 - X2 ≤ 2 and

X1 – 2X2 ≤ 1

where, X1, X2, X3 & X4 ≥ 0

4. Apply graphical method to find non-negative values of X1 and X2 which

Minimize Z = 10X1 + 25X2

subject to

$$X1 + X2 \geq 50$$
$$X1 \geq 20 \quad \text{and}$$
$$X2 \leq 40$$

5. Maximize Z = X1 + 2X2 + 3X3

Subject to the constraints

$$X1 - X2 + X3 \geq 4,$$
$$X1 + X2 + 2X3 \leq 8,$$
$$X1 - X3 \geq 2;$$
$$X1, X2, X3 \geq 0$$

6. Write down the dual of the following primal problem and solve it

Minimize Z = 4x1 +5x2+7x3

Subject to

$$X1 + X2 + X3 = 120$$
$$2X1 + 3X3 + \leq 40$$
$$4X2 + 6X3 \leq 80$$
$$X1, X2, X3 \geq 0$$

Chapter Summary

This chapter has brought out an introduction about the field of Operations Research and various features and at the end, few limitations. Also, introduction to various algorithms like simplex, Big-M Method and Duality is dealt.

Suggested Readings

Operations Research	Hillier & Lieberman	TMH
Operations Research	R. Paneerselvam	PHI

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UNIT-III

Lesson 3 - Transportation / Assignment & Inventory Management

Lesson Objectives

- To Introduce The Role Of Transportation Model Problems And Various Methods To Solve It
- To Discuss The Importance Of Assignment Problems And Solving Them
- To Introduce Various Models In The Inventory Management

Chapter Structure

This Chapter is organized in the following order

- 3.1 Transportation Problems
- 3.2 Assignment problems
- 3.3 Inventory Models

3.1 Transportation Problem

The transportation problems are special cases of the linear programming models. It deals with the situation in which a commodity / good is transported from Sources to **Destinations**. Thus, obviously, the objective is to determine the amount of commodity to be transported from each source to each destination in such a way that the total transportation cost is lowest.

Transportation model problems play very significant role in any country's economy, particularly, in the business contexts. Since, these kinds of problems are modelled with 1000s of constraints and assumptions, we consider the problem usually involves the physical movement of goods and services from various supply origins / sources /

factories to multiple demand destinations / markets / distribution centres within the given constraints of supply and demand in such a way that the total transportation cost is minimized.

Since, the transportation models are special types of linear programming (LP) problems; they can also be solved by the Simplex Algorithm. However, as a model developer, one will face severe issues of handling huge number of variables and constraints. Even small transportation problems such as 3X4 [3 origins & 4 destinations] requires 12 variables and 7 constraints. Thus, a direct application of the Simplex method may be a cumbersome process and involve huge amount of calculations, solving them by hands will be almost impossible and at times, even computers may not be useful. However a transportation problem has a special mathematical structure which permits it to be solved by a fairly efficient method known as Transportation Models.

The basic transportation problem was originally developed by F.L. Hitchcock (1941) in his study entitled "the distribution of a product from several sources to numerous locations." In the year, 1947, a mathematician by name, T.C. Koopmans published a study on "optimum utilization of the transportation system". Later, Linear Programming formulation and the associated systematic procedure for solution were developed by George B. Dantzig (1951).

Mathematical Formulation of Transportation Problems

As we discussed just above, Transportation models deals with the transportation of a product manufactured at different plants or factories (supply origins) to a number of manufactured at different warehouses (demand destinations). The objective is to satisfy the destination requirements within the plant's capacity constraints at the minimum transportation cost. Transportation models thus typically arise in situations involving physical movement of goods from plants to warehouses, warehouses to wholesalers, wholesalers to retailers and retailers to customers. Solution of the transportation models requires the determination of how many units should be transported from each supply origin to each demands destination in order to satisfy all the destination demands white minim sing the total associated cost of transportation

Consider a soft drink manufacturing firm, which has m plants located in **m** different cities. The total production is to be supplied to the retail shops in '**n**' different cities. We want to determine the transportation schedule that minimizes the total cost of transporting soft drinks from various plants to various retail shops. This situation can be formulated as a linear programming problem.

Let us consider the m-plant locations (origins) as O1, O2… Om and the n-retail markets (destination) as D1, D2… Dn respectively. Let ai ≥ 0, i= 1, 2 ….m, be the amount available at the ith plant-Oi. Let the amount required at the jth market-Dj be bj ≥ 0, j= 1,2,….n.

Let the cost of transporting one unit of soft drink form ith origin to jth destination be Cij, i= 1, 2 ….m, j=1, 2….n. If Xij ≥ 0 be the amount of soft drink to be transported from ith origin to jth destination , then the problem is to determine xij so as to Minimize the total cost of transportation, which is denoted as Z.

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij}$$

Subject to the constraint

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1,2,\ldots m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1,2,\ldots n.$$

And Xij ≥ 0, for all i and j

This linear programming formulation is also known as a ***Transportation Problem.***

The Transportation Table

The above set of constraints represents 'm+n' equations in m X n non-negative variables. Each variable Xij appears in exactly two constraints, one is associated

$$\sum_{i=1}^{m} x_{ij} = b_j \text{ and } \sum_{j=1}^{n} x_{ij} = a_i$$

with the origin and the other is associated with the destination. It allows us to put the above LPP in the matrix form, the elements of A are either 0 or 1.

	D_1	D_2			D_n	
0_1	C_{11}	C_{12}	....	C_{1n}		a_1
0_2	C_{21}	C_{22}	....	....	C_{2n}	a_2
....	....	....	....	....	....	....
0m	C_{m1}	C_{m2}	....	....	C_{mn}	a_m
	b_1	b_2			b_n	

Requirement

Assumptions in Transportation Problems

A basic assumption is that the distribution costs of units from source i to destination j is directly proportional to the number of units distributed.

Moreover, **if the demand is equal to supply in a given transportation model, it is called as a balanced problem;** in a balanced problem all the products that can be supplied are used to meet the demand. There are no slacks and so all constraints are equalities rather than inequalities.

On the other hand, **if the demand is not equal to supply, it is known as unbalanced transportation model.** However, to obtain an initial solution, we have to modify the unbalanced transportation problem to a balanced one, by adding a dummy origin or a dummy market, depend upon the shortage.

For many applications, the supply and demand quantities in the model will have integer values and implementation will require that the distribution quantities also be integers. Fortunately, the unit coefficients of the unknown variables in the constraints guarantee an optimal solution with only integer values.

Problem Formulation -Example

Consider the following example: Adani Power Limited, which is a electric power producing company in India, has three electric power plants that supply the needs of four cities. Each power plant can supply the following numbers of kWhr of electricity: Plant 1 – 35 million; Plant 2 – 50 million; Plant 3 – 40 million.

The peak power demands in these cities, which occur at the same time, are as follows (in kWhr): City 1 – 45 million; City 2 – 20 million; City 3 – 30 million; City 4 – 30 million. The costs of sending 1 million kWhr of electricity from plant to city depend on the distance the electricity must travel (see Table).

From	To				Supply
	City-1	City-2	City-3	City-4	
Plant-1	8	6	10	9	**35**
Plant-2	9	12	13	7	**50**
Plant-3	14	9	16	5	**40**
Demand	**45**	**20**	**30**	**30**	

Formulate this problem to minimize the cost of meeting each city's peak power demand.

Solution

We begin by defining a variable for each decision that Adani Power has to make.

We define, (for i = 1, 2, 3 and j = 1, 2, 3, 4)

Xij = number of (million) kWhr produced at plant i and sent to city j

Then, the objective of the Adani Power is to minimize the total transmission cost; this is achieved by allocating appropriately by linking the plant and city through the lowest cost of transmission.

As we have discussed in the Mathematical Modelling of Problems, in the Unit-2, we derive the following objective function;

Min Z = 8X11 + 6X12 + 10X13 + 9X14 + 9X21 + 12X22 + 13X23 + 7X24 + 14X31 + 9X32 + 16X33 + 5X34

This objective can be realized subject to the following constraints.

X11 + X12 + X13 + X14 ≤ 35 ------ (Supply constraint from Plant-1)

X21 + X22 + X23 + X24 ≤ 50 ------ (Supply constraint from Plant-2)

X31 + X32 + X33 + X34 ≤ 40 ------ (Supply constraint from Plant-3)

X11 + X21 + X31 ≥ 45 ----- (Demand constraint at the city-1)

X12 + X22 + X32 ≥ 20 ----- (Demand constraint at the city-2)

X13 + X23 + X33 ≥ 30 ----- (Demand constraint at the city-3)

X14 + X24 + X34 ≥ 30 ----- (Demand constraint at the city-4)

And obviously, Xij ≥ 0

This is a special case of LP problem. It can be solved by the simplex algorithm, but specialized algorithms are much more efficient.

In general, a transportation problem is specified by the following parameters:

1. A set of '**m**' supply points from which a good is shipped. Supply point '**i**' can supply, at most, $\mathbf{S_i}$ units (in the above situation, m =3, $\mathbf{S_1}$ = 35, $\mathbf{S_2}$ = 50, $\mathbf{S_3}$ = 40)
2. A set of '**n**' demand points to which the good is shipped. Demand point '**j**' must receive at least **dj** units of the shipped goods (you can see above, n = 4; d1 = 45, d2 = 20, d3 = 30, d4 = 30).

3. Each unit produced at supply point '**i**' and shipped to demand point '**j**' incurs a variable cost **cij**. (In the above case, for example, c12 = 6).

Let **Xij** = number of units shipped from supply point **i** to demand point **j**.

Thus, a general formulation is:

$$\text{Min} \sum_{i=1}^{i=m} \sum_{j=1}^{j=m} Cij.Xij$$

Subject to

$$\text{Min} \sum_{j=1}^{j=n} Xij \le S_i \; (i=1,2,\ldots\ldots\ldots..m) \text{ (supply constraints)}$$

$$\text{Min} \sum_{i=1}^{i=n} Xij \ge d_j \; (j=1,2,\ldots\ldots\ldots..n) \text{ (demand constraints)}$$

$$X_{ij} \ge 0 \; (i = 1, 2, \ldots.. m; j = 1, 2, \ldots.. n)$$

As we mentioned earlier, if $\sum S_i = \sum dj$; that is supply equal demand, it is said to be balanced transportation problem. Adani issue is a balanced transportation problem.

Thus, for a balanced transportation problem,

$$\text{Min} \sum\sum c_{ij}X_{ij}$$

Subject to

$$\sum X_{ij} = S_i \quad (i = 1, 2, \ldots.. m) \text{ (supply constraints)}$$

$$\sum X_{ij} = d_j \quad (j = 1, 2, \ldots.. n) \text{ (demand constraints)}$$

$$X_{ij} \ge 0 \; (i = 1, 2, \ldots.. m; j = 1, 2, \ldots.. n)$$

Solution of balanced transportation problem is simpler, therefore, it is desirable to formulate a transportation problem as a balanced transportation problem.

Thus, a transportation problem is specified by the supply, the demand, and the shipping costs, so the relevant data can be summarized in a transportation tableau. The cell in row i and column j corresponds to the variable X_{ij}.

If X_{ij} is a basic variable, its value is normally placed in the middle corner of the ijth cell of the tableau. The costs are normally shown in the upper right corner. The following tableau is for the Adani Power problem. (The optimal solution values are also given, which we are going to disucss shortly).

	City 1	City 2	City 3	City 4	Supply
Plant 1	8	6 10	10 25	9	**35**
Plant 2	9 45	12	13 5	7	**50**
Plant 3	14	9 10	16	5 30	**40**
Demand	**45**	**20**	**30**	**30**	

Deriving the solution for a transportation problem is done in two stages; in the first stage, we try to obtain an initial basic feasible solution; in the stage-2, we try to check, whether the solution obtained in stage-1 is optimal or not; if not optimum, we modify the allocations made [known as MODI Method / u-v method].

There are many methods for finding such a starting Basic Feasible Solution [BFS]. The easier ones are the ***northwest-corner method, the column minima method, the row-minima method and the matrix minima method (or least cost method).***

Initial Basic Feasible Solution - Northwest Corner Method

The procedure for constructing an initial basic feasible solution selects the basic variables one at a time. The North West corner rule, allocation starts to the cell, which occupies the top left-hand corner of the transportation tableau [north-west corner cell / cell- (1, 1)], and proceeds systematically along either a row or a column and make allocations to subsequent cells until the bottom right-hand corner is reached, by which time enough allocations will have been made to constitute an initial solution.

Northwest Corner Method: – ***Step-by-step procedure***

1. Start by selecting the cell in the most \North-West" corner of the table, and set X11 as large as possible (cannot be larger than s1 or d1).

2. Assign the maximum amount to this cell that is allowable based on the requirements and the capacity constraints.

- **If X11 = s1, cross out the first row**

  [That is exhausting the capacity in the origin].

- **Change d1 to (d1- s1).**

  [That is modifying the demand after making supply from origin-1 for the market-1].

- **If X11 = d1, cross out the first column**

  [That is exhausting the requirements of the market-1].

- **Change s1 to (s1 – d1).**

  [That is modifying the capacity, after making supply from origin-1 for the market-1].

3. If for any cell, supply equals demand, then the next allocation can be made in cell either in the next row or column; but don't cross out both the row as well as column. At the row or column, keep a remaining value as zero and make the allocation later to a cell; this will eliminate the process of making dummy basic cells; this issue is called as degeneracy at the initial cell. Degeneracy is reduction in availability of basic cell from the level of (m+n-1).

4. Continue applying procedure to the north-west cell that does not lie in the crossed out row or column until there is one cell left.

We will solve the problem formulated for Adani Power Limited. Consider the following transportation table.

The north-west corner is cell-(1, 1); the city requirement is 45 and plant-1can supply at most 35.

From	To				Supply
	City-1	City-2	City-3	City-4	
Plant-1	8	6	10	9	**35**
Plant-2	9	12	13	7	**50**
Plant-3	14	9	16	5	**40**
Demand	**45**	**20**	**30**	**30**	

So we decide the supply the entire 35 to city-1;

Now, the plant-1 cannot supply anymore to any cities; and the demand for city-1 will be reduced to 10, after a supply is allocated from plant-1.

This is given in the following table.

From	To				Supply
	City-1	City-2	City-3	City-4	
Plant-1	8 35	6	10	9	**35**
Plant-2	9	12	13	7	**50**
Plant-3	14	9	16	5	**40**
Demand	**45 /10**	**20**	**30**	**30**	

After removing the row-1, again we try to find the north-west corner; this time, it is the cell-(2, 1); the city requirement is 10 and plant-2 can supply at most 50. So we decided to supply the entire requirement of the city-1 [10 units]; now the city-1 requirement is completely fulfilled and removed from further considerations.

And the availability at plant-2 is reduced to 40 units, after making supply to city-1. This is given in the following table.

From	To				Supply
	City-1	City-2	City-3	City-4	
Plant-1	8 35	6	10	9	35
Plant-2	9 10	12	13	7	~~50~~/40
Plant-3	14	9	16	5	40
Demand	~~45~~ / ~~10~~	20	30	30	

After removing the column-1, again we try to find the north-west corner; this time, it is the cell-(2, 2); the city-2 requirement is 20 and plant-2 can supply at most 40. So we decided to supply the entire requirement of the city-2 [20 units]; now the city-2 requirement is completely fulfilled and removed from further considerations.

And the availability at plant-2 is reduced to 20 units, after making supply to city-2.

This is given in the following table.

From	To				Supply
	City-1	City-2	City-3	City-4	
Plant-1	8 35	6	10	9	35
Plant-2	9 10	12 20	13	7	~~50~~ / ~~40~~ / 20
Plant-3	14	9	16	5	40
Demand	~~45~~ / ~~10~~	~~20~~	30	30	

After removing the column-2, again we try to find the north-west corner; this time, it is the cell-(2, 3); the city-3 requirement is 30 and plant-2 can supply at most 20.

So we decided to supply the entire availability at plant-2 [20 units] to the city-3; now the city-2 requirement is reduced to 10 units; however plant-2 is completely exhausted its capacity and hence removed from further considerations.

And the requirement city-3 is reduced to 10 units. This is given in the following table.

From	To				Supply
	City-1	City-2	City-3	City-4	
Plant-1	8 35	6	10	9	**35**
Plant-2	9 10	12 20	13 20	7	~~50~~ / ~~40~~ / ~~20~~
Plant-3	14	9	16	5	**40**
Demand	~~45~~ / ~~10~~	~~20~~	~~30~~ / 10	30	

In the left out table, the north-west corner is cell-(3, 3); the requirement of the city-3 is 10 units and plant-3 has 40 units with it. Thus the allocation 10 units has been made to cell (3, 3) and left out cell is cell- (3, 4); the city-4 requirement is 30 units and plant-3 has 30 units left out after supplying to city-3.

From	To				Supply
	City-1	City-2	City-3	City-4	
Plant-1	8 35	6	10	9	**35**
Plant-2	9 10	12 20	13 20	7	~~50~~ / ~~40~~ / 20
Plant-3	14	9	16 10	5 30	~~40~~ / ~~30~~
Demand	~~45~~ / ~~10~~	~~20~~	~~30~~ / ~~10~~	~~30~~	

Now the allocation obtained from north-west corner method is summarized below;

$X11 = 35$; $X21 = 10$; $X22 = 20$; $X23 = 20$ $X33 = 10$; $X34 = 30$

Total transportation cost through the north-west corner method

TC = (35 X 8) + (10 X 9) + (20 X 12) + (20 X 13) + (10 X 16) + (30 X 5)

= 280 + 90 + 240 + 260 + 160 + 150

= 1180

Note: The cells, which are getting allocated is called as basic cells. The number of basic cells in any transportation problem should be equal to (row+column-1). The cells, which are not getting any allocation is known as non-basic cells.

Advantages and disadvantages of North-west corner Method

The major advantage of this method is its simplicity to use; however, north-west method does not consider costs, thus, it may lead to more iteration before optimal solution is reached. We will solve the above problem by matrix-minima / least cost method, which gives importance for lowest cost for making allocations.

Initial Basic Feasible Solution - Matrix-Minima / Least Cost Method

Matrix minimum (Least cost) method is a method for computing a basic feasible solution of a transportation problem, where the basic variables are chosen on the basis of lowest unit cost of transportation. This method is very useful because it reduces the computation and the time required to determine the optimal solution. The following steps summarize the approach

Matrix-Minima / Least Cost Method: – ***Step-by-step procedure***

1. In this method, we begin allocation by considering the minimum cost of transportation. Find the cell /variable with the smallest transportation cost (say C_{ij}).
2. Then assign X_{ij} its largest possible value, minimum $\{s_i, d_j\}$.
3. Cross out row i or column j and reduce the supply or demand of the

non-crossed out row or column by the value of X_{ij}.

4. Then choose from the cells that do not lie in a crossed out row or column the cell with the minimum cost and repeat procedure.
5. Continue until there is one cell that can be chosen.

For our better understanding of the method, we solve the same Adani Power limited problem which introduced and solved by North-west corner method.

The lowest transportation cost among all the costs, is found in the cell-(3, 4), which is 5; the city requirement is 30 and plant-3 can supply at most 40.

From	To				Supply
	City-1	City-2	City-3	City-4	
Plant-1	8	6	10	9	**35**
Plant-2	9	12	13	7	**50**
Plant-3	14	9	16	5	**40**
Demand	**45**	**20**	**30**	**30**	

So we decide the supply the entire 30 to city-4 from plant-3.

Now, the plant-3 can supply only 10 units to any cities and the demand for city-4 is completely exhausted. This is given in the following table.

From	To				Supply
	City-1	City-2	City-3	City-4	
Plant-1	8	6	10	9	**35**
Plant-2	9	12	13	7	**50**

Plant-3	14	9	16	5 30	~~40~~ / 10
Demand	45	20	30	~~30~~	

Now the lowest transportation cost among all the costs, after removing the column-4 from consideration is found in the cell-(1, 2), which is 6; the city-2 requirement is 20 and plant-1 can supply at most 35. So we decide the supply the entire 20 to city-2 from plant-1. Now, the plant-1 can supply only 25 units to any cities and the demand for city-2 is completely exhausted.

This is given in the following table.

From	To				Supply
	City-1	City-2	City-3	City-4	
Plant-1	8	6 20	10	9	~~35~~ / 15
Plant-2	9	12	13	7	50
Plant-3	14	9	16	5 30	~~40~~ / 10
Demand	45	~~20~~	30	~~30~~	

Now the lowest transportation cost among all the costs, after removing the column-2 from consideration is found in the cell-(1, 1), which is 8; the city-1 requirement is 45 and plant-1 can supply at most 15. So we decide the supply the entire 15 to city-1 from plant-1. Now, the plant-1 is completely exhausted its availability and removed from further consideration; city-1 requirement is reduced to 30.

This is given in the following table.

From	To: City-1	City-2	City-3	City-4	Supply
Plant-1	8, 15	6, 20	10	9	~~35~~ / ~~15~~
Plant-2	9	12	13	7	50
Plant-3	14	9	16	5, 30	~~40~~ / 10
Demand	~~45~~ / 30	~~20~~	30	~~30~~	

Now the lowest transportation cost among all the costs, after removing the row-1 from consideration is found in the cell-(2, 1), which is 9; the plant-2 can supply at most 50 and the city-1requirement is 30. So we decide the supply the entire 30 to city-1 from plant-2. Now, the plant-2 availability is reduced to 20 and the city-1 is supplied completely the need. This is given in the following table.

From	To: City-1	City-2	City-3	City-4	Supply
Plant-1	8, 15	6, 20	10	9	~~35~~ / ~~15~~
Plant-2	9, 30	12	13	7	50 / 20
Plant-3	14	9	16	5, 30	~~40~~ / 10
Demand	~~45~~ / ~~30~~	~~20~~	30	~~30~~	

Now we left out with only one column, city-3, which needs 30 units; it is supplied from plant-2 [20 units] and plant-3 [10 units].

From	To: City-1	City-2	City-3	City-4	Supply
Plant-1	8, 15	6, 20	10	9	~~35~~ / ~~15~~
Plant-2	9, 30	12	13, 20	7	~~50~~ / 20

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Plant-3	14	9	16 10	5 30	**~~40~~ / 10**
Demand	**~~45~~ / ~~30~~**	**~~20~~**	**30**	**~~30~~**	

Thus, we made allocation by considering the lowest cost in the cost matrix. The initial basic feasible solution is given below;

X11 = 15; X12= 20; X21= 30; X23= 20 X33= 10; X34= 30

Total transportation cost through the least cost / matrix minima method

TC = (15 X 8) + (20 X 6) + (30 X 9) + (20 X 13) + (10 X 16) + (30 X 5)

= 120 + 120 + 270 + 260 + 160 + 150

= 1080

You can note that the total cost of allocation through the North West corner method is 1180, whereas the cost of allocation through least cost method is 1080, which is a superior basic feasible solution than a solution obtained through north-west corner method.

Note: You may note that the number of basic cells in any transportation problem should be equal to (row+column-1).

Initial Basic Feasible Solution – Vogel's Approximation Method [Vam]

The Vogel Approximation Method [VAM] is an iterative procedure for computing an initial basic feasible solution of a transportation problem. This method is preferred over the two methods discussed in the previous sections, because the initial basic feasible solution obtained by this method is either optimal or very nearer to the optimal solution. Therefore the amount of time required to arrive at the optimum solution is greatly reduced.

Vogel's Approximation Method [Vam]:– ***Step-by-step procedure***

Step 1

Compute a penalty for each row in the transportation table. The penalty for a given row is the difference between the smallest cost and the next smallest cost in that particular row; write this difference (penalty) along the side of the table against the corresponding row.

Step 2

Compute a penalty for each column of the transportation table. Identify the cell having minimum and next to minimum transportation cost in each column and write the difference (penalty) against the corresponding column.

Step 3

- Identify the row or column with the largest penalty; make a mark in the penalty [circle the respective penalty for future references]
- In this identified row or column, choose the cell, which is having the lowest transportation cost
- Allocate the maximum possible quantity to the cell having lowest cost in that row or column
    - Exhaust either the supply at a particular source or satisfy demand at a warehouse.
- If there is a tie between / among the penalties computed, select that row/column which has minimum cost.
- If there is a tie in the minimum cost also select that row/column which will have maximum possible assignments. It will considerably reduce computational work.
- Also, you have an option of breaking the tie between / among the penalties corresponding to two or more rows or columns by breaking the tie arbitrarily

Step 4

Reduce the row supply or the column demand by the amount assigned to the cell.

Step 5

- If the row supply is now zero, eliminate the row;
- If the column demand is now zero, eliminate the column
- If both the row supply and the column demand is zero, eliminate either the row or column.
- After eliminating the row or column, in the left out row or column, enter the balance as zero [this will help in avoiding degeneracy, which is an issue while solving the problem for optimality check]
- In future iterations, if the row or column with zero is selected, follow the same procedure, as if you are allocating a quantity; only the difference is, you will allocate zero as quantity to a cell. This will avoid number of other steps, which are slightly complicated.

Step 6

Recompute the row and column difference for the reduced transportation table, which has obtained by omitting rows or columns crossed out in the preceding step.

Step 7

Repeat the above procedure until the entire supply at factories are exhausted to satisfy demand at different warehouses.

For our better understanding of the method, we solve the same Adani Power limited problem which introduced and solved by North-west corner method as well as Least Cost Methods.

- The lowest and next lowest for each row is located and penalty is computed and marked against the respective rows.
- Similarly, the lowest and next lowest is located and penalty is computed for each column and marked against the respective columns.

Table-1: First Iteration

From	To				Supply	Row Penalty-1
	City-1	City-2	City-3	City-4		
Plant-1	~~8~~	~~6~~	10	9	35	2
Plant-2	~~9~~	12	13	~~7~~	50	2
Plant-3	14	~~9~~	16	~~5~~ 30	~~40~~ 10	4
Demand	45	20	30	~~30~~		
Column Penalty-1	1	3	3	2		

- The maximum among all the penalties [rows as well as columns together] is marked;
  - In case of a tie, you may choose the row or column that is having the lowest transportation cost cell;
- That row or column should be allocated first; the allocation is made to the cell, which is having lowest transportation cost; in case of tie between cells, you can break it arbitrarily.

In the above table, it is 4, appearing in the 3rd row; the lowest transportation cost 5 appears in the cell (3, 4).

Availability at the plant-3 is 40 units and city-4 requires 30 units; so we supply the entire 30 units from plant-3;

Now the city-4 is eliminated from further consideration and plant-3 availability is reduced to 10.

The transportation table is modified with the above changes and placed below; in that table, again the process is repeated [while doing future calculations, we will not consider costs in the column-4 anymore] and row/ column penalty-2 is computed.

Table-2: Second Iteration

From	To				Supply	Row Penalty-1	Row Penalty-2
	City-1	City-2	City-3	City-4			
Plant-1	~~8~~	~~6~~	10	9	35	2	2
Plant-2	~~9~~	~~12~~	13	~~7~~	50	2	3
Plant-3	~~14~~	~~9~~ **10**	16	~~5~~ **30**	~~40~~ ~~10~~	4	5
Demand	~~45~~	~~20~~ 10	30	~~30~~			
Column Penalty -1	*1*	*3*	*3*	*2*			
Column Penalty -2	*1*	*3*	*3*	-			

In the above table, highest penalty is 5, appearing again in the 3rd row; the lowest transportation cost 9 appears in the cell (3, 2).

Availability at the plant-3 is 10 units and city-2 requires 20 units; so we supply the entire 10 units from plant-3;

Now the plant-3 is eliminated from further consideration and city-2 requirement is reduced to 10.

The transportation table is modified with the above changes and placed below; in that table, again the process is repeated [while doing future calculations, we will not consider costs in the column-4 & Row-3 anymore] and row/ column penalty-3 is computed.

Table-3: Third Iteration

From	To				Supply	Row Penalty-1	Row Penalty-2	Row Penalty-3
	City-1	City-2	City-3	City-4				
Plant-1	~~8~~	~~6~~ 10	10	9	~~35~~ 25	2	2	2
Plant-2	~~9~~	~~12~~	13	~~7~~	50	2	3	3
Plant-3	~~14~~	~~9~~ 10	16	~~5~~ 30	~~40~~ ~~10~~	4	5	-
Demand	~~45~~	~~20~~ ~~10~~	30	~~30~~				
Column Penalty-1	1	3	3	2				
Column Penalty-2	1	3	3	-				
Column Penalty-3	1	6	3					

In the above table, highest penalty is 6, appearing again in the 2nd column; the lowest transportation cost 6 appears in the cell (1, 2).

Availability at the plant-1 is 35 units and city-2 requires 10 units; so we supply the entire 10 units from plant-1;

Now the city-2 is eliminated from further consideration and plant-1 requirement is reduced to 25.

The transportation table is modified with the above changes and placed below; in that table, again the process is repeated [while doing future calculations, we will not consider costs in the column-4, column-2 & Row-3 anymore] and row/ column penalty-4 is computed.

Table-4: Fourth Iteration

From	To				Supply	Row Penalty -1	Row Penalty -2	Row Penalty -3	Row Penalty -4
	City-1	City-2	City-3	City-4					
Plant-1	~~8~~	~~6~~ 10	10	9	~~35~~ 25	2	2	2	2
Plant-2	~~9~~ 45	~~12~~	13	~~7~~	~~50~~ 5	2	3	3	4
Plant-3	~~14~~	~~9~~ 10	16	5 30	~~40~~ ~~10~~	4	5	-	-
Demand	~~45~~	~~20~~ ~~10~~	30	~~30~~					
Column Penalty-1	1	3	3	2					
Column Penalty-2	1	3	3	-					
Column Penalty-3	1	6	3	-					
Column Penalty-4	1	-	3	-					

In the above table, highest penalty is 6, appearing again in the 2nd column; the lowest transportation cost 6 appears in the cell (1, 2).

Availability at the plant-1 is 35 units and city-2 requires 10 units; so we supply the entire 10 units from plant-1;

Now the city-2 is eliminated from further consideration and plant-1 requirement is reduced to 25.

Now we left out with only one column, city-3. This is serviced by plant-1, 25 units and plant-2, by 5 units.

Thus, we fulfil the requirements of all the cities and from the plants. We made the initial allocation through Vogel's approximation method.

Table-5: Fourth Iteration

From	To				Supply	Row Penalty -1	Row Penalty -2	Row Penalty -3	Row Penalty -4
	City-1	City-2	City-3	City-4					
Plant-1	~~8~~	~~6~~ 10	10 25	9	~~35~~ ~~25~~	2	2	2	2
Plant-2	~~9~~ 45	~~12~~	13 5	~~7~~	~~50~~ ~~5~~	2	3	3	4
Plant-3	~~14~~	~~9~~ 10	16	~~5~~ 30	~~40~~ ~~10~~	4	5	-	-
Demand	~~45~~	~~20~~ ~~10~~	~~30~~	~~30~~					
Column Penalty-1	1	3	3	2					
Column Penalty-2	1	3	3	-					
Column Penalty-3	1	6	3	-					
Column Penalty-4	1	-	3	-					

Now the allocations are:

X12 = 10 X13= 25
X21 = 45 X23 = 5
X32 = 10 X34 = 30

The total transportation cost

= (10 X 6) + (25 X 10) + (45 X 9) + (5 X 13) + (10 X 9) + (30 X 5)
= 60 + 250 + 405 + 65 + 90 + 150
= 1020

You can note that the total cost of allocation through the North West corner method is 1180; least cost method is 1080, whereas through VAM, we get 1020, which is a superior basic feasible solution than a solution obtained through other methods.

Note: You may note that the number of basic cells in any transportation problem should be equal to (row+column-1).

Optimality Checking Of The Initial Basic Feasible Solution: Modi [Modified Distribution Method] Method

You would have noticed that the initial basic feasible solution obtained by various methods provide different starting solution. Once you obtain the initial solution by any of the method, the next step is to check the optimality of the solution obtained and if not, improve the solution and obtain the optimal solution. Two popularly known methods are 'Stepping Stone Method' and 'MODI Method or u-v method.

In the stepping stone method, we have to draw as many closed paths as equal to the unoccupied cells for their evaluation. To the contrary, in MODI method, only closed path for the unoccupied cell with highest opportunity cost is drawn. Here, we will discuss only the MODI Method, which is more scientific and approaches the optimal solution faster.

Modi [Modified Distribution Method] Method - ***Step-by-step procedure***

Step1

Determine an initial basic feasible solution using any one of the three methods:

North West Corner Rule
Matrix Minimum Method
Vogel Approximation Method

Step2

Each row, assign one 'dual' variable, say- u1, u2, u3…; for each column, assign on dual variable – say, v1, v2, v3…

Now using the basic cells [which are assigned through any one of the three methods], and the transportation costs of those basic cells -Cij, we will determine the values of these u_i and v_j.

Determine the values of dual variables, ui and vj, using $u_i + v_j = Cij$
Since the net evaluation is zero for all basic cells, it follows that
$z_{ij} - c_{ij} = u_i + v_j - C_{ij}$, for all basic cells (i, j). So we can make use of this relation to find the values of u_i and v_j

Step3

Compute the opportunity cost, for those cells, which are not allocated for any goods to be transported [known as non-basic cells], using $(u_i + v_j)$ - Cij

Step4

- Check the sign of each opportunity cost.
- If the opportunity costs of all these unoccupied cells / non-basic cells are either negative or zero, the given solution is the optimal solution.
- On the other hand, if one or more unoccupied cell has positive entry / opportunity cost, it is an indication that the given solution is not an optimal solution; it can be improved and further savings in transportation cost are possible.

Step5

- Select the unoccupied cell with the highest positive opportunity cost as the cell to be included in the next solution.
- This cell has been left out / missed out by the initial solution method.
- If this cell is allocated, it will bring down the overall transportation cost

Step6

Draw a closed path or loop for the unoccupied cell selected in the previous step. A loop in a transportation table is a collection of basic cells and the cell, which is to be converted as basic cell. It is formed in such a way that, it has only even number cells in any row or column.

- After identifying the entering variable X_{rs}, form a loop; this loop starts at the non-basic cell (r, s) connecting only basic cells.
- Assign alternate plus and minus signs at the unoccupied cells on the corner points of the closed path with a plus sign at the cell being evaluated
- Determine the maximum number of units that should be shipped to this unoccupied cell.
- The smallest value with a negative position on the closed path indicates the number of units that can be shipped to the entering cell.
- Now, **add** this quantity to all the cells on the corner points of the closed path marked with plus signs, and **subtract** it from those cells marked with minus signs.
- In this way, an unoccupied cell becomes an occupied cell.
- Other basic cells, the quantity allocated are modified, in such a way that without affecting the row availability and column / market requirements
- Such a closed path exists and is unique for any non-degenerate solution.

Please note that the right angle turn in this path is permitted only at occupied cells and at the original unoccupied cell.

Step7

Repeat the whole procedure until an optimal solution is obtained.

To understand the MODI Method, let us consider the solution obtained by Least Cost Method.

Step1

From	To				Supply
	City-1	City-2	City-3	City-4	
Plant-1	8 15	6 20	10	9	35

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Plant-2	9 **30**	12	13 **20**	7	**50**
Plant-3	14	9	16 **10**	5 **30**	**40**
Demand	**45**	**20**	**30**	**30**	

Step2

For each row, we assign u_i and for each column, we will assign v_j
Let us assume any one of the ui or vj as zero; preferably a row or column having maximum number of basic cells

Since there is a tie, we break it arbitrarily.
Let us assume, U1= 0
We know that U1+V1= 8 [ui+vj= Cij for all the basic cells]

$$0+ V1= 8 \Longrightarrow V1= 8$$

Similarly

U1+V2= 6 $\Longrightarrow$ V2 = 6

V1+ U2 = 9 $\Longrightarrow$ 8+ U2 = 9 $\Longrightarrow$ U2 = 1

U2 + V3 = 13 $\Longrightarrow$ 1+ V3 = 13 $\Longrightarrow$ V3 = 12

V3 + U3 = 16 $\Longrightarrow$ 12 + U3 = 16 $\Longrightarrow$ U3 = 4

U3 + V4 = 5 $\Longrightarrow$ 4 + V4 = 5 $\Longrightarrow$ V4 = 1

These findings are incorporated in the following table

Table-1

From	**To**				**Supply**	
	City-1	**City-2**	**City-3**	**City-4**		
Plant-1	8 15	6 20	10	9	35	**U1=0**
Plant-2	9 30	12	13 20	7	50	**U2=1**
Plant-3	14	9	16 10	5 30	40	**U3=4**

Demand	45	20	30	30	
	V1=8	**V2=6**	**V3=12**	**V4=1**	

Step3

Let us compute the opportunity cost, for those cells, which are not allocated for any goods to be transported [known as non-basic cells], using $(u_i + v_j)$ - Cij

For example, cell (1, 3)

Net evaluation for cell (1, 3)

= ui+vj-Cij = u1+v3-C13 = [0 + 12-10] = 2

Net evaluation for cell (1, 4)

= ui+vj-Cij = u1+v4-C14 = [0 + 1-9] = -8

Similarly for all other cells, the net evaluation / opportunity cost is computed and placed in left hand top corner of each non-basic cell; the table is placed below;

Table-2

From	**To**				**Supply**	
	City-1	**City-2**	**City-3**	**City-4**		
Plant-1	8 15	6 20	$\boxed{2}$ 10	$\boxed{-8}$ 9	35	**U1=0**
Plant-2	9 30	$\boxed{-5}$ 12	13 20	$\boxed{-5}$ 7	50	**U2=1**
Plant-3	$\boxed{-2}$ 14	$\boxed{1}$ 1 9	16 10	5 30	40	**U3=4**
Demand	45	20	30	30		
	V1=8	**V2=6**	**V3=12**	**V4=1**		

Step4

Since all the net evaluation / opportunity costs are not negative, it is an indication of the solution is not in the optimum stage; it can be improved

Among the positive entries, the maximum positive appears in the cell (1, 3), which is 2. So this cell to be converted as a basic cell.

Step5

- After identifying the entering variable X_{rs}, form a loop; this loop starts at the non-basic cell (r, s) connecting only basic cells.
- Assign alternate plus and minus signs at the unoccupied cells on the corner points of the closed path with a plus sign at the cell being evaluated
- Determine the maximum number of units that should be shipped to this unoccupied cell.
- The smallest value with a negative position on the closed path indicates the number of units that can be shipped to the entering cell.
- Now, add this quantity to all the cells on the corner points of the closed path marked with plus signs, and subtract it from those cells marked with minus signs.
- In this way, an unoccupied cell becomes an occupied cell.

Other basic cells, the quantity allocated are modified, in such a way that without affecting the row availability and column / market requirements. The loop formed is shown below;

Table-3 Forming The Loop

From	To				Supply	
	City-1	City-2	City-3	City-4		
Plant-1	8 15 (-)	6 20	$\boxed{2}$ 10 (+)	$\boxed{-8}$ 9	35	**U1=0**
Plant-2	9 30 (+)	$\boxed{-5}$ 12	13 20 (-)	$\boxed{-5}$ 7	50	**U2=1**

Plant-3	$\boxed{-2}$ 14	$\boxed{1}$ 9	16 10	5 30	40	**U3=4**
Demand	45	20	30	30		
	V1=8	**V2=6**	**V3=12**	**V4=1**		

In the loop, between the cells with negative sign, lowest quantity allocated is 15, in the cell (1, 1); it should be shifted to the cell (1, 3), where there is a positive sign.

Table-3 Forming The Loop

From	**To**				**Supply**	
	City-1	**City-2**	**City-3**	**City-4**		
Plant-1	8	6 20	$\boxed{2}$ 10 15	$\boxed{-8}$ 9	35	**U1=0**
Plant-2	9 45	$\boxed{-5}$ 12	13 5	$\boxed{-5}$ 7	50	**U2=1**
Plant-3	$\boxed{-2}$ 14	$\boxed{1}$ 9	16 10	5 30	40	**U3=4**
Demand	45	20	30	30		
	V1=8	**V2=6**	**V3=12**	**V4=1**		

Thus, wherever there is a positive sign, the quantity is added; in the negative signed cell, it is subtracted.

Also, when we move the quantities, we should not move to 3 cells in a row or column; the movement of goods will be always in terms of even numbers.

You may notice in the above table, while moving the goods, we skipped the basic cell (1, 2) from the loop. Other allocations remain as such; we will now compute the cost of this allocation.

Now the allocations are:

X12 = 20 X23= 5

X13 = 15 X33 = 10

X21 = 45 X34 = 30

The total transportation cost

= (20 X 6) + (15 X 10) + (45 X 9) + (5 X 13) + (10 X 16) + (30 X 5)

= 120 + 150 + 405 + 65 + 160 + 150

= 1050

You may note that Least Cost method which has an initial cost of allocation as 1080, reduced by Rupees 30 by MODI / u-v method.

Once again, we check, whether the solution is optimum or not; we start again to compute the ui and vj for each row and column;

Table-4 Re-computing ui and vj

From	To				Supply	
	City-1	City-2	City-3	City-4		
Plant-1	8	6 20	10 15	9	35	U1=10
Plant-2	9 45	12	13 5	7	50	U2=13
Plant-3	14	9	16 10	5 30	40	U3=16
Demand	45	20	30	30		
	V1=-4	V2=4	V3=0	V4=11		

We assume V3 = 0, since it has 3 basic cells in the column and repeat the same process.

Now we will compute the Net evaluation / opportunity cost for the non-basic cells and try to form the loop, if there are positive opportunity cost in any of the cells.

Table-5 Re-computing opportunity cost

From	To				Supply	
	City-1	City-2	City-3	City-4		
Plant-1	-2 8	6 20 (-)	10 15 (+)	-10 9	35	U1=10

Plant-2	9 45	$\boxed{-3}$ 12	13 5	$\boxed{-5}$ 7	**50**	**U2=13**
Plant-3	$\boxed{-2}$ 14	$\boxed{3}$ 9 (+)	16 10 (-)	5 30	**40**	**U3=16**
Demand	45	20	30	30		
	V1=-4	**V2= -4**	**V3=0**	**V4=-11**		

The maximum positive appears in the cell (3, 2); it should be converted as basic cell and the loop is shown in the above table. The modified distribution is shown below and the cost is computed.

Table-5 Modified distribution

From	**To**				**Supply**	
	City-1	**City-2**	**City-3**	**City-4**		
Plant-1	8	6 **10**	10 **25**	9	35	**U1=**
Plant-2	9 **45**	12	13 **5**	7	50	**U2=**
Plant-3	14	9 **10**	16	5 **30**	40	**U3=**
Demand	45	20	30	30		
	V1=-	**V2=**	**V3=**	**V4=**		

You may note that the cost of allocation is

=(10X 6) + (25X 10) + (45 X 9) + (5 X 13) + (10 X 9) + (30 X 5)

= 60 + 250 + 405 + 65 + 90 + 150

= 1020

Thus, the total cost is reduced by modifying the distribution / allocations made earlier. Now once again, we will check whether the solution is optimum or not.

Table-6 Finding ui / vj and checking optimality of the solution

From	To				Supply	
	City-1	City-2	City-3	City-4		
Plant-1	$\boxed{-2}$ 8	6 **10**	10 **25**	$\boxed{-7}$ 9	35	**U1=0**
Plant-2	9 **45**	$\boxed{-3}$ 12	13 **5**	$\boxed{-2}$ 7	50	**U2=3**
Plant-3	$\boxed{-5}$ 14	9 **10**	$\boxed{-3}$ 16	5 **30**	40	**U3=3**
Demand	45	20	30	30		
	V1= 6	**V2=6**	**V3=10**	**V4=2**		

Since, the net evaluation / opportunity cost is negative for all the non-basic cells, it is indication that the solution is in the optimum stage.

Variations in the Transportation Models

Unbalanced Transportation Models: If the demand is not equal to supply, the transportation problems are known as unbalanced transportation problems.

You have to add a dummy row / column with zero as transportation cost with the shortfall (difference between supply and demand) as the quantity available in the origin or marketing requirements.

Then, in the usual manner, using any one of the initial solution methods will be applied and initial solution will be obtained.

Practice Problems

Solve the transportation problem for which the cost, origin availabilities and destination requirements are given below.

	D1	D2	D3	D4	D5	D6	ai
O1	1	2	1	4	5	2	30
O2	3	3	2	1	4	3	50
O3	4	2	5	9	6	2	75

O4	3	1	7	3	4	6	20
bj	20	40	30	10	50	25	175

Solve the transportation problem with unit transportation costs demands and supplies as given below.

		Distribution Centre				
		D1	**D2**	**D3**	**D4**	**Supply**
	F1	3	3	4	1	100
Factory	**F2**	4	2	4	2	125
	F3	1	5	3	2	75
Demand		120	80	75	25	

Solve the following TPP through Vogel's approximation method.

Origin / Destination	**Destination**				
	D1	**D2**	**D3**	**D4**	**Supply**
A	21	16	25	13	**11**
B	17	18	14	23	**13**
C	32	17	18	41	**19**
Demand	**6**	**10**	**12**	**15**	**43**

****

Lesson 3.2 - Assignment Problems

Assignment problems are special case of linear programming problems. The task is to assign 'i' resources (i= 1, 2, 3, 4 …m) to 'j' tasks (j= 1, 2, 3, 4…n) in such a way that the total cost of performing the tasks is lowest. The cost associated in performing resource-i by task-j is denoted by 'Cij'. Moreover, each resource should be exactly performed by one task. Thus, the assignment problem is a special type of Linear Programming Problems, where assignees are being assigned to perform tasks. In general, in the assigning people to jobs is a common application; however, the assignees need not be people; they can also be machines, projects, location or plants.

Classic examples, like assigning the machine operator to the most suitable job, assigning the set of project managers to most suitable projects and assigning the subject to suitable faculty typically fall under the category of assignment problems.

In the machine operator- job assignment, the objective of the firm may be reducing the overall time taken or reducing the wastages by appropriately assigning the operators to the suitable job; in assigning the project managers to suitable projects, the firm may have an objective of reducing the overall lead time, or cost of operation; in the example of subject-faculty allocation, the university education administrator may be interested in reducing the total number of 'failures' in a class.
Thus, a typical assignment problem falls under the category of minimization model; of course, maximization models can also be solved by the Assignment algorithms after suitable modifications. The mathematical expression / formulation of the assignment problem shall be written as:

$$\text{Minimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} Cij * Xij$$

Subject to

$$\sum_{i=1}^{n} Xij = 1 \qquad \text{for every 'j'}$$

$$\sum_{j=1}^{n} Xij = 1 \qquad \text{for every 'i'}$$

And $\quad Xij = 0$ or 1

Thus, the assignment problems are zero-one problems, where the decision variables Xij take the value of either zero or one only. The assignment problems can also be consider as special case of 'Transportation Models', where, 'm= n'; that is the number of sources is equal to number of destinations and ai=1 and bj=1 for all 'i' and 'j' values.

Even though the assignment problems are considered as special case of 'transportation models' or 'linear programming' models, it is seldom solved by these methods. We use a different algorithm, more efficient algorithm, which is evolved by a group of 'Hungary' mathematicians and named after them as 'Hungarian Algorithm'.

Assumptions in Assignment Problems

1. The number of assignees and the number of tasks are equal in number.
2. Each assignee is to be assigned to exactly one task.
3. Each task is to be performed by exactly one assignee.
4. There is a cost Cij associated with assignee-i (i= 1, 2, 3 … n) performing a task j (j= 1, 2, 3 … n). This cost can also be estimated with moderate degree of precision always.
5. The objective is to determine how all the 'n' assignments should be made to minimize the total cost.

Hungarian Algorithm

The Hungarian Algorithm is applicable only for minimization models. In case the problem is not in minimization form, convert the problem in to a minimization model; this is done in the following steps.

1. Identify the largest number in the entire matrix / table.
2. Subtract all the entries from the highest number and write down the new reduced table.
3. This reduced table is in the form of 'opportunity cost' of not allocating a resource to a task; thus, the aim of the problem, which is in the new-reduced format, after subtracting all the entries from the highest, can be viewed as a minimization model.
4. Then the Hungarian method is applied, which is given in the following steps. If the given problem is a minimization model, you can skip this initiation process.

Also, on occasion, the number of assignees to be assigned to perform tasks will not be equal; that is, the number of rows will not equal to the number of columns. This kind of assignment models are known as unbalanced assignment problems. To apply the Hungarian Algorithm, you have to convert the problem into a balanced assignment problem.

An Assignment problem is known as balanced one, if the number of rows equals to the number of columns. In case, if the given problem is unbalanced one, you have to add appropriate number of rows or columns with zero as assignment cost [dummy row or dummy column] and make the problem as balanced assignment model.

Then the Hungarian method is applied to this modified table, which is given in the following steps. If the given problem is a balanced model, you can skip this initiation process.

Step-1

Subtract the smallest number in each row from every number in the row; this is known as row reduction. Enter the results in a new table.

Step-2

Subtract the smallest number in each column of the new table obtained in Step-1 from every number in the respective column; this is known as column reduction. Enter the results in the new table.

Step-3

Check whether an optimal set of assignments can be made. This is done in the following procedure;

- Draw minimum number of straight lines to cover all the zeros in the table obtained after column reduction.
- Straight lines can be drawn horizontally or vertically but not diagonally.
- Choose the row or column that contains the maximum number of zeros and draw the first line; in case of tie breaker, break it arbitrarily.
- In the rest of columns / row, find a row or column that contains maximum number of zeros and draw the next line [in case a zero is already covered by a straight line, do not count while drawing the new line].
- Repeat the above steps till all the zeros are covered by straight lines.
- If the minimum of number of straight lines equal to number of rows, an optimal set of assignment is possible; otherwise, go to the Step-4.

Step-4

If the number of straight lines is not equal to the number of rows, then do the following operations in the table, which is just now covered by straight lines.

- Subtract the smallest uncovered number [not covered by any straight lines] from every uncovered number in the table.
- Add this smallest uncovered number to the numbers at intersections of covering lines.
- Do not make any changes for those numbers which are covered by straight lines but not crossed out by lines.
- Repeat the step -3 and check the optimal number of allocations; if not repeat the step-4 again till you gets optimal number of straight lines.

Step-5

If the number of straight lines is equal to the number of rows / columns, then it is an indication of optimum solution can be arrived by using the zero elements.

- Make the assignments one at a time in positions that have zero elements.
- Search with rows or columns that have unique zero; since each row (resource) and column (task) needs to receive exactly one assignment, you may strike out the row and the column involved after making the assignment.
- Then move on to the other rows and columns that are not crossed out; continue until every row and every column has exactly one assignment.

This procedure will provide the complete set of assignments and an optimum solution for the problem.

Example-1

Find the assignment of operator to appropriate job with lowest possible time to complete the jobs.

Job	Operator				
	1	**2**	**3**	**4**	**5**
J1	5	6	8	6	4
J2	4	8	7	7	5
J3	7	7	4	5	4
J4	6	5	6	7	5
J5	4	7	8	6	8

Since the given problem is a minimization model and balanced one, we apply the Hungarian algorithm straight away on the given table.

Step-1

Subtract the minimum value in each row from all other respective row values:

Job/	Operator				
	1	**2**	**3**	**4**	**5**
J1	1	2	4	2	0
J2	0	4	3	3	1
J3	3	3	0	1	0
J4	1	0	1	2	0
J5	0	3	4	2	4

Step-2

Subtract the minimum value in each column from all other respective column values:

Job/	Operator				
	1	**2**	**3**	**4**	**5**
J1	1	2	4	1	0
J2	0	4	3	2	1
J3	3	3	0	0	0
J4	1	0	1	1	0
J5	0	3	4	1	4

Step-3

Check whether an optimal set of assignments can be made. Draw minimal number straight lines to cover all the zeros.

Job/	Operator				
	1	**2**	**3**	**4**	**5**
J1	1	2	4	1	0
J2	0	4	3	2	1
J3	3	3	0	0	0
J4	1	0	1	1	0
J5	0	3	4	1	4

- The maximum number of zeros is found in row-3 and column-5 [Three zeros each]; the first line is drawn arbitrarily at row-3

Job/	Operator				
	1	**2**	**3**	**4**	**5**
J1	1	2	4	1	0
J2	0	4	3	2	1
J3	3	3	0	0	0
J4	1	0	1	1	0
J5	0	3	4	1	4

- In the new matrix, after drawing the line, maximum number of zeros is found in row-4, column-1 and column-5 [Two zeros each]; thus the tie is broken arbitrarily and the second line is drawn at column-5.

Job/	Operator				
	1	**2**	**3**	**4**	**5**
J1	1	2	4	1	0
J2	0	4	3	2	1
J3	3	3	0	0	0

J4	1	0	1	1	0
J5	0	3	4	1	4

- In the new matrix, after drawing the lines, maximum number of zeros is found in, column-1 [Two zeros each]; thus the tie third line is drawn at column-1.

Job/	Operator				
	1	2	3	4	5
J1	1	2	4	1	0
J2	0	4	3	2	1
J3	3	3	0	0	0
J4	1	0	1	1	0
J5	0	3	4	1	4

- In the new matrix, after drawing the lines, maximum number of zeros is found in, column-2 / row-4 [one zero, which is same]; thus the fourth line is drawn at column-2 arbitrarily.

Job/	Operator				
	1	2	3	4	5
J1	1	2	4	1	0
J2	0	4	3	2	1
J3	3	3	0	0	0
J4	1	0	1	1	0
J5	0	3	4	1	4

Now the maximum number straight lines needs to cover all the zeros are four, which is less than the number of rows in the table. Thus, it is an indication of optimum number of zeros is not there in the table to make allocation.

Step-4

Since the number of straight lines is not equal to the number of rows, then we do the following operations in the table, which is just now covered by straight lines.

- Subtract the smallest uncovered number [not covered by any straight lines] from every uncovered number in the table; in the above table, the smallest uncovered number is one.
- We have to add one to the numbers at intersections of covering lines; and should not make any changes for those numbers which are covered by straight lines but not crossed out by lines.

Job/	Operator				
	1	**2**	**3**	**4**	**5**
J1	1	2	3	0	0
J2	0	4	2	1	1
J3	4	4	0	0	1
J4	1	0	0	0	0
J5	0	3	3	0	4

Now in the new matrix, which is obtained above, we repeat the same process described in the Step-3 to draw straight lines.

The first line is drawn at Row-4, which is having 4 zeros; then the second line is drawn at Column-4, which is having 3 zeros; the third line is drawn at column-1, which is having 2 zeros; the next line is drawn at row-3 and the last line is drawn at column-5

Job/	Operator				
	1	**2**	**3**	**4**	**5**
J1	1	2	3	0	0
J2	0	4	2	1	1
J3	4	4	0	0	1

J4	1	0	0	0	0
J5	0	3	3	0	4

Now the minimum number of straight lines needs to cover all zeros is equal to the number of rows/ columns, it is an indication of optimum number of allocation can be made with available zeros.

Step-5

We now proceed to make optimum allocation with available zero entries; in the new matrix, jot down all the zeros in their respective positions.

- Search row wise for a unique zero; if one such zero is found mark it and strike the respective column [zeros]
- Repeat the above step; in case unique zero is not available, repeat the above search column wise; search column wise unique zero and if one such zero is found mark it and strike the respective row [zeros]
- Repeat the above step, until all the zeros are marked. In case of tie breaker, break the tie arbitrarily.

The first unique zero is found in Row-2 and it is marked; then the column-1 is strike out; the next unique zero is found in row-5 and the same process is repeated. Third unique zero was found again in the row search at row-1; fourth unique zero was found at row-3 and finally the last unique zero was found in row-4.

Now this is an indication of optimum allocation has been made. The respective zero allocation positions are considered as optimum allocation for the given problem.

Job/	**Operator**				
	1	**2**	**3**	**4**	**5**
J1				0	⊗

J2	⊗				
J3				0	
J4			0	0	0
J5	0				⊗

Job/	Operator allocated	Cost of allocation (taken from the original cost table)	Total allocation cost
J1	O-5	4	
J2	O-1	4	
J3	O-3	4	23
J4	O-2	5	
J5	O-4	6	

Variations in Assignment Problem- *Case 1: Maximization Models*

Even though the assignment algorithm is primarily a minimization model, it can be suitably modified and used for maximization models also; whenever the objective of the firm / decision maker is to maximize the return of the allocation, you have to modify the problem in the initial table and use the same Hungarian Algorithm to solve. We will solve one example here.

Example-1

The student internship director needs as much coverage as possible in the office next week. The more hours that can be put in each day, the better. She has asked the students to provide a list of how many hours they are available each day of the week. Each student can be there on only one day and there must be a student in the office each day of the week. Use the table below and the tables provided to determine the schedule that gives the most coverage. *[Note that the objective of this problem is to maximize the hours worked, not minimize].*

Student	Mon	Tue	Wed	Thurs	Fri
A	2	4	8	4	6
B	3	2	7	3	2
C	6	8	6	5	4
D	7	4	3	6	8
E	4	5	3	1	4

Solution

As you know, the Hungarian Algorithm is primarily developed to handle minimization models; we need to modify the given problem as an 'equivalent' minimization model. We will convert the given problem as a minimization model in the following steps.

- You have to identify the largest number in the entire matrix / table – [**which is 8**]
- Subtract **all the entries** from this **highest number-8** and write down the new reduced table.
- This reduced table is in the form of 'opportunity time' of not allocating a day to a student; thus, the aim of the problem, which is in the new-reduced format, after subtracting all the entries from the highest, can be viewed as a minimization model.
- Then the Hungarian method is applied, which is given in the following steps.

1. Convert to a minimization problem by subtracting each value from 8

Student	Mon	Tue	Wed	Thurs	Fri
A	6	4	0	4	2
B	5	6	1	5	6
C	2	0	2	3	4
D	1	4	5	2	0
E	4	3	5	7	4

2. Subtract the minimum value in each row from all other row values

Student	Mon	Tue	Wed	Thurs	Fri
A	6	4	0	4	2
B	4	5	0	4	5
C	2	0	2	3	4
D	1	4	5	2	0
E	1	0	2	4	1

3. Subtract the minimum column value in each column from all other column values

Student	Mon	Tue	Wed	Thurs	Fri
A	5	4	0	2	2
B	3	5	0	2	5
C	1	0	2	1	4
D	0	4	5	0	0
E	0	0	2	2	1

4. Now conduct the line test: draw minimum number of straight lines to cover all the zeros

- The first line is drawn at Row-4, which is having maximum number zeros (3 zeros are there in the row)

Student	Mon	Tue	Wed	Thurs	Fri
A	5	4	0	2	2
B	3	5	0	2	5
C	1	0	2	1	4
D	0	4	5	0	0
E	0	0	2	2	1

- The second line is drawn at Row-5, which is having maximum number zeros after removing Row-4 from further consideration (2 zeros are there in the row)

Student	Mon	Tue	Wed	Thurs	Fri
A	5	4	0	2	2
B	3	5	0	2	5
C	1	0	2	1	4
~~D~~	~~0~~	~~4~~	~~5~~	~~0~~	~~0~~
~~E~~	~~0~~	~~0~~	~~2~~	~~2~~	~~1~~

- Now the third line is drawn at Column-3, which is having maximum number zeros after removing Row-3, Row-4 from further consideration (2 zeros are there in the column)

Student	Mon	Tue	Wed	Thurs	Fri
A	5	4	0	2	2
B	3	5	0	2	5
C	1	0	2	1	4
~~D~~	~~0~~	~~4~~	~~5~~	~~0~~	~~0~~
~~E~~	~~0~~	~~0~~	~~2~~	~~2~~	~~1~~

The last line is drawn at Row-3 / Column-2; you may draw the line in any manner; here, we consider the Row-3

Student	Mon	Tue	Wed	Thurs	Fri
A	5	4	0	2	2
B	3	5	0	2	5
~~C~~	~~1~~	~~0~~	~~2~~	~~1~~	~~4~~
~~D~~	~~0~~	~~4~~	~~5~~	~~0~~	~~0~~
~~E~~	~~0~~	~~0~~	~~2~~	~~2~~	~~1~~

Since, the number of straight lines=4, which is less than the number of row=5, it is an indication of sufficient number of zeros are not available in the last table to do the optimum allocation. Thus, we do the following steps, in the last table obtained.

- Subtract the smallest uncovered number, which is here, 2 in the above table, [not covered by any straight lines] from every uncovered number in the table.
- We have to add this two to the numbers at intersections of covering lines; and should not make any changes for those numbers which are covered by straight lines but not crossed out by lines.
- By doing so, we get the following table.

Student	Mon	Tue	Wed	Thurs	Fri
A	3	2	0	0	0
B	1	3	0	0	3
C	1	0	4	1	4
D	0	4	6	0	0
E	0	0	4	2	4

We do the line test in the above table.

Student	Mon	Tue	Wed	Thurs	Fri
A	3	2	0	0	0
B	1	3	0	0	3
C	1	0	4	1	4
D	0	4	6	0	0
E	0	0	4	2	4

We now proceed to make optimum allocation with available zero entries; in the new matrix, jot down all the zeros in their respective positions.

- The first unique zero is found in Row-3 and it is marked; then the column-2 is strike out;
- The next unique zero is found in row-5 and marked; column-1 is removed
- Since, there is no unique zero in any row or any column, you may randomly assign any zero as allocation; we allocate randomly Row-1 to Column-5. The Row-1 to Column-5 is removed from further considerations
- Again doing, row search, unique zeros are found in Row-2 and Row-4 and marked. Thus, optimum allocation is arrived.

Third unique zero was found again in the row search at row-1; fourth unique zero was found at row-3 and finally the last unique zero was found in row-4.

Student	Mon	Tue	Wed	Thurs	Fri
A			0	0	0
B			0	0	
C		0			
D	0			0	0
E	0	0			

Thus, the optimum allocation is made by taking the times from original table.

Solution	Mon	Tue	Wed	Thurs	Fri	Hours Covered
	Student-E	Student-C	Student-B	Student-D	Student-A	31

While selecting third allocation, there was no unique zero, we randomly broken the tie; so there is a possibility of more than one solution. If you choose some other zero, you may get different allocation, but the total hours will be the same as 31. You may try and check the answers, which is given below.

Solution	Mon	Tue	Wed	Thurs	Fri	Hours Covered
1	E	C	A	B	D	31
2	E	C	B	A	D	31
3	E	C	B	D	A	31

Variations in Assignment Problem- *Case 2: Unbalanced Models*

- An **Assignment problem** is known as **balanced one**, if the **number of rows equals to the number of columns.**
- When the **number of rows is not equal to number of columns,** the assignment models are called as **unbalanced assignment problems.**
- In case, if the given problem is unbalanced one, you have to add appropriate number of rows or columns with zero as assignment cost [dummy row or dummy column] and make the problem as balanced assignment model.

Exercise-1

A university wants to allocate the four subjects and six teachers claim that they have the required competencies!! /knowledge!! to teach all the subjects. The dean believes that the failure in the course is reflection of faculty member's performance! Allocate the subject to appropriate faculty members.

Subjects

	A	B	C	D
F1	25	44	33	35
F2	33	40	40	43
F3	40	35	33	30
F4	44	45	28	35
F5	45	35	38	40
F6	40	49	40	46

Solution

You may notice that the number of rows (6) not equal to the number of columns (4); thus, the given problem is an example of unbalanced assignment models. We add two dummy columns (2 dummy subjects) with zero as number failures. The modified problem becomes a balanced one and the initial table is given below;

Subjects

	A	B	C	D	E	F
F1	25	44	33	35	0	0
F2	33	40	40	43	0	0
F3	40	35	33	30	0	0
F4	44	45	28	35	0	0
F5	45	35	38	40	0	0
F6	40	49	40	46	0	0

Now we can use Hungarian algorithm, which requires a balanced one to apply. The following table is obtained after column reduction, since row reduction will give the same table above.

Subjects

	A	B	C	D	E	F
F1	0	9	5	5	0	0
F2	8	5	12	13	0	0
F3	15	0	5	0	0	0
F4	19	10	0	5	0	0
F5	20	0	10	10	0	0
F6	15	14	12	16	0	0

- The first straight line is drawn at Column-5 and second is at Column-6, which are having six zeros each.
- The next line is drawn at Row-3
- Fourth line is drawn at Row-1; fifth at Row-4; the last line at Row-5
- Since the number of straight lines equal to number of rows/columns,

optimum number of zeros is available to make the final allocation.

Subjects

	A	B	C	D	E	F
F1	0	9	5	5	0	0
F2	8	5	12	13	0	0
F3	15	0	5	0	0	0
F4	19	10	0	5	0	0
F5	20	0	10	10	0	0
F6	15	14	12	16	0	0

The next step is to consider the zeros and making the allocations. Since, every row is having zeros; we do the column wise search;

- Column-1, Column-3 & Column-4 are having unique zero, we mark them for allocation and cross out the respective rows in which the zero appear.
- In the process, we get unique zero in Column-2 and strike out the Row-5
- Now we have tie at Row-2 and Row-6, which can be broken arbitrarily.
- We allocate Row-2 to Column 5 (E) and Row-6 to Column-6 (F).

Subjects						
	A	B	C	D	E	F
F10	⊗				0	0
F2					⊗	0
F3		0		⊗	0	0
F4			⊗		0	0
F5		⊗			0	0
F6					0	⊗

Thus, the optimum allocation is

Subjects

	A	B	C	D	E	F	Total failure
Allocation	F1	F5	F4	F3	F2	F6	118
	25	35	28	30	0	0	

Chapter End Problems

Exercise-1

A set of 5 crews operates in a construction firm; the ability / skill sets of the crews differ considerably and hence have different execution time in completing the five projects, which are expected to be taken soon by the firm. The following table summarizes the executive time for each projects by each crews. Assign the crews to appropriate projects. [Project execution times in days]

Projects

	A	B	C	D	E
C1	20	30	25	15	35
C2	25	10	40	12	28
C3	15	18	22	32	24
C4	29	8	34	10	40
C5	35	23	17	26	45

Exercise-2

A cricket team faces the issues of inconsistent performance of the top order batsmen in the recent times. In the recent 10-innings average for the top 4 batsmen are given in the following table. Consider the problem of assigning four top-order batsmen to four different batting positions, so that the total runs gathered by the team is at its best.

Batsman

	A	B	C	D
Position-1	5	11	8	9
Position-2	5	7	9	7
Position-3	7	8	9	9
Position-4	6	8	11	12

Exercise-3

A consulting firm estimated the sales revenues [in millions INR per month] for 5 types of departmental stores in four locations of a city for a chain store operator. What type of departmental store is more suitable for a location so that the total profit is maximized for the firm?

Locations

	A	B	C	D
Shoe	10	6	12	8
Toy	15	18	5	11
Auto	17	10	13	16
House ware	14	12	13	10
Video	14	16	6	12

Exercise-4

A company wishes to assign 4 salesmen to 4 districts the volume of sales matrix is given below. Make the optimal assignment which results in maximum volume of sales

Salesman	District			
	A	B	C	D
1	250	300	420	400
2	350	400	200	250
3	500	375	400	350
4	400	350	420	300

Exercise-5

Department Head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. His estimates of time each man would take to perform each task is given in the Matrix below. Allocate the appropriate persons to the suitable tasks.

Men / Tasks	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

****

Lesson 3.3 - Inventory Management – an Introduction

Introduction

Inventory is the single largest Investment in the assets for most of the 'Production dominated Industries', 'Wholesalers', 'Retail Chain Stores/ Superstores', 'Stockist/Dealers' and Retailers. Serving from 'Mass market to Niche Market or otherwise known as STP Marketing (Segmentation – Targeting – Positioning) leads to proliferation of products, resulted in high level of inventory holdings or Stock Keeping Units (SKU's) both Raw Material as well as Finished Goods.

In firms like HLL, P&G, Cavin Kare, the amount invested in inventory is competing with their own product lines. These companies almost stretched line, with range of products like Detergents, Soaps, Cosmetics, Packaged Foods etc are competing with each other line in terms of returns, investments etc., Companies like HLL is having negative working capital.1

High holding cost, the Opportunity Cost on Inventory, the Penalty cost of not meeting the orders, Procuring from Various countries/sources through various modes of transport, increasing Transportation Cost are few crucial facets of Inventory Management to be studied carefully.

The Savings in Inventory holdings should be compared with ordering cost and Cost of Transportation. Most of the firms, with a conventional accounting to not proper accounting mechanism, make it difficult to find out any of the cost(s) and reliable cost estimates. The impact of Inventory Management in corporate profitability should be assessed very carefully. Profitability can be improved by increasing the sales volume or cutting the cost associated with various operations. One of such operations is keeping the Inventory Cost under control. The policy should be very specific in terms of required level of inventory, re-order policy subject to fulfilling the customer's expected satisfaction level.

1.1 Reasons for Holding Inventory

Any firm would like to hold inventory at specific level to meet smooth, uninterrupted production schedule or to cater consumer demand in case of wholesalers or Retailers or to achieve the economies of scale from operations or protect the image from uncertainties or fluctuating demand as the case may be. A recent study on very popular 85 FMCG brands such as RIN, Nestle, Sunrise, Ariel, LUX, etc, which constitute roughly 20% sales of the FMCG potential (estimated annual sales Rs.40, 000 Crores), the stock outs occurs nearly 20% times 2. This study revealed that even though conducted at metros and especially super chain stores, lack efficiency of the channel members and the policy of the organizations to keep the lower inventory levels.

1.1.1 Avoiding Uncertainties

Raw Materials Inventory: Today managers are dealing with Multi product – Multi plant with a multiple sources/warehouses/countries. Fluctuation in price and demand seasonally of product nature forces the organization to hold inventory against these kinds of uncertainties.

WIP Inventory: Between the manufacturing operations, within the plant to avoid shutdown of main units, because of the critical part/ spare is out of stock or the line has broken down, normally certain amount of Work – in – Progress inventory is always maintained. In some cases, some of the spares/components may be produced in batches to achieve the efficiency or it can be produced with little effort and money. Thus the WIP Inventory is part and parcel of inventory management.

Finished Goods Inventory: In case of fast moving consumer goods, as the demand is fluctuating over time, it is always better to keep/hold finished goods to meet the demand against uncertainties. If the raw material is seasonal or the demand is seasonal it is necessary to keep finished goods inventory to meet the demand as in the cases like Sugarcane, Cement, Soft drinks etc.,

1.1.2 Achieving Economies of Scale

In case of transportation, it is obvious rather than transporting 'Less than a Truckload' or 'Less than a Car load' (LCL) of quantities, full load definitely reduces the transportation cost per unit per kilometer. This would applicable to both finished goods inventory as well as raw materials. In case of bulk purchases, the manufacturer/buyer firm definitely would try to explore the Quantity Purchase Schemes (QPS) associated with bulk purchases or price break options against various quantities.

Some of the firms produce large quantities at a time to avoid frequent changes in the production line. In such cases, not only we have to hold large quantity buy also must compare holding cost, cost of obsolescence, and cost of capital against the preparation cost of production. On the other hand, smaller batch size would reduce inventory holdings; capital locked in inventory but will result in frequent setting or high set up cost.

1.1.3 Meet the Buffer

In the distribution channel, or in a supply chain, the participants are scattered geographically. Sometimes, the time between placing the order and getting the stock (Replenishment time) may vary due to several reasons. It is a compulsion to hold inventory at various stages in order to meet the demand.

1.1.4 Balancing Seasonally in Demand and Supply

Sometimes there may be steep increase in demand. For example during the festival seasons, there is sharp rise in demand of clothes/suiting/shirting/sugar/oil etc., It is very much required to hold higher inventory to meet/explore the opportunity.

Products such as cements, steel, building material normally the sales is sluggish during winter. Soft drinks are picking up the trend during summer. To meet these kind of seasonally in demand, again the firm is forced to keep higher level of inventories.

1.2 Types of Inventory

Based on the reasons, for which the inventory is accumulated, it can be classified into the following categories.

1.2.1 Cycle Stock

A lot size or batch size is that quantity 'a stage of supply chain' either produces or purchases at any given point of time. A firm, which sells 5 personal computers per day normally, would stock/order for 50/100 personal computers. The firm is holding an average stock of 10 days or 20 days. Cycle Inventory is the average inventory builds up in the supply chain. A stage of supply chain purchases or produces more than the demand for the product; the concept of cycle inventory is in picture.

Therefore, Cycle Inventory = Lot Size/2

The role of cycle inventory would be used well if we know some idea about 'Average Flow Time'.

Average Flow Time = Average Inventory (or CycleInventory)/Demand = Q/(2D)

Hence, the Average flow time for the computer firm is 10 days. (100/2*5 = 100/10)

Therefore the larger cycle inventory will result in the longer lag time between purchases/production to sales, which would in turn affect the working capital requirements of the firm.

1.2.2 Raw Material Inventory

The raw material inventory is a relative term to the industry/ application of the component/spare/parts/material. For an automobile manufacturer, even the Tyres /mirrors constitute the raw material inventory, which are finished goods inventory of the respective firms which manufacture these components.

A sugar factory with crushing capacity of 75 MT sugarcane operating at 100% capacity utilization, would require holding at least 75 MT of sugarcane on any day. If the policy is to keep at least 10days requirements, then they have to hold 750MT on any time.

A seasonal nature of raw material may force the firm to keep the inventory at certain levels. Some occasion it will be beneficial for the firm to purchase in bulk quantity.

1.2.3 Work – in – Progress Inventory

A passenger car manufacturer if takes 5 days to roll out a car from the production run, the value addition in every day is kept as a WIP inventory. The gearbox which is the fulcrum of cars normally purchased/ produced in larger quantities so as to avoid any shortfalls or unexpected breakdown in that systems. In general WIP Inventory is held at various stages for smooth and efficient flow of operations.

1.2.4 Finished Goods Inventory

The finished goods inventory is held at various stages of the supply chain. For example, the production lot, which is just then produced, will be kept for packaging/transportation to warehouses or Carrying & Forwarding Agents, the Dealers/stockiest are carrying inventory to meet the requirement, the wholesalers and retailers carrying certain amount of products to meet the demand.

Sometimes, the company policy may state such as 'you have to hold atleast 2.5 weeks of inventory at any point of time' etc. On the other occasions such as taking advantages of price reductions or making use of new sales promotional schemes, which may push the demand curve higher, the inventory is kept at various levels across the spectrum.

1.2.5 In transit Inventories

For example, if we consider the Passenger car giant, Maruti Udyog Limited, which is getting Gear Boxes from Suzuki Motor Corporation, Japan, at any point of time, the gear boxes, that are en route, forms the 'In transit Inventories' for MUL. Again depends upon the application, the 'In transit Inventory' may be Raw material or WIP or Finished goods.

A Study by bar-coding authority EAN India limited, roughly Rs.5000/- Crore of goods of the FMCG products are always in transit, which is equivalent to one eighth of the FMCG industry's produce 3.

Summary

Inventory means a physical stock of goods, which are kept in hand, for smooth and efficient running of future affairs of an organization. Almost every business must carry out some inventory for smooth running. The problem is to take decisions that how much should be stocked and when should be ordered for stocking.

An overstock would require higher invested capital per unit of time, but less frequent occurrences of shortages and placements of orders. An under stock on the other hand would decrease the invested capital per unit of time, but would increase the frequency of ordering as well as the risk of running out of stock.

These two extremes are costly. Decisions regarding the quantity ordered and the time at which it is ordered may thus be based on the minimization of an appropriate cost function which balances the total cost resulting from over stocking and under stocking.

1.3 Terms associated with Inventory Management

In this section, let us gain some idea about various costs and terminology associated with Inventory Management.

1. Set up Cost (or) Ordering cost:(Cs)

For example, consider a production unit, which is manufacturing Instant Food mixes, using the same grinding and packing facilities for both 'Idli-mix' and 'Kesari Mix'. Before they change the production run it is necessary to clean the grinder with left outs of previous runs. Some time, they may use some common materials to clear or using cotton waste to clearing the residues or some special chemicals to clean it. It may consume material and/or men or there is some amount of cost associated with the operations.

On the other hand, if we have to place an order as in the case of a Stockist/Dealer, the cost may range from simple clerical plus stationers plus postage to complex estimates such as placing a quotation. Here again it consumes labor and/or material, and finally can be bringing down to a cost element associated with it.

Thus, this is the cost incurred with the placement of an order or with the initial preparation of production facility such as resetting the equipment for production. The set up cost is usually independent of the quantity ordered or size of the production run.

2. Production cost (or) Selling Price.(C)

It is the actual price; an item is produced or purchased (sold). In case of production it is the cost of producing an item and it may be a constant or variable one.

3. Holding cost (or) Storage cost (or) Carrying cost.(C1)

This represents the cost of carrying inventory in storage. It includes the interest on invested capital, storage space cost, insurance and handling cost. Holding costs are usually assumed to vary directly with the level of inventory as well as the length of the time the item is held in stock.

Holding cost consists of so many components with it and the type of storage such as own warehouses to rental warehouses, makes things much more complicated than expected. Above all, the accounting practices of many organizations may not support or sound enough to decide the cost estimates. Even though, the company owns the storage space, electricity expenses are met, the policies may not be very clear to arrive at the opportunity cost forgone by owning these facilities.

4. Shortage cost. (C2)

These are the penalty cost incurred as a result of running out of stock, when the commodity is needed. It generally includes the costs, due to loss of production, cost of idle equipment, the loss of goodwill of customers and the penalty of missing the delivery schedule.

In a study on FMCG segments, the stock out percentages of 85 prime brands, were estimated roughly around 25%, which means, out of these 85 fast moving brands, on an average, nearly 20 brands will be out of stock 4. These 85 brands, was in a position to control roughly 20% market share of the FMCG segment. The amount of revenue loss and loss of good will and ultimately loss of the customer base are going to be the consequences of shortages.

Recent survey reports of NCEAR say that brand switching is a common phenomenon in most of the FMCG product lines. Above all, the companies are trying for the same market share through the variety of consumer promotion tools. During the year 2001, almost 25-30 consumer promotions were offered in a month against the average of 5-6 5. In the case of rural consumers, the rate of brand switching is much higher than the urban chaps 6.

Some of the companies while purchasing make it compulsory to include a class on treatment on missing the scheduled delivery. Most of the times, very huge penalty such as the loss estimated as the case of missed scheduled will be entirely borne by the supplier and the missed scheduled used to estimate the performance of the vendor/seller/suppliers.

5. Demand. (D)

Demand is the number of units required per period and may be known exactly or known in terms of probabilities.

A Classic Case of Demand Forecasting

...A look at the company's existing supply chain (which links the suppliers, factories, storage depots and retailers) showed that the primary problem was inventory. There was too much of it. In 1995, inventory levels at HLL's personal products (PP) division were 20% of the divisional turnover. In the same year, inventory levels at its detergents division were 24% of its divisional turnover. Together the divisions accounted for approximately 55% of HLL's sales (Rs 1,851 crore out of Rs 3,366 crore in 1995).

That was one hell of a problem. The way out: demand forecasting, to ensure that the factories do not roll out stuff the consumer doesn't want. Once HLL got a hang of it, the inventory pile-up shrunk dramatically.

The starting point was the customer. How much was he buying? Distributors like Nemichand Shah & Co, which covers the Nainital area, were asked to send their men out on different routes covering different retail outlet everyday. Birju, one of Shah's men, dutifully notes every unit sold by each retailer and the numbers are sent back to HLL's distribution Centre for the area. Similarly, other distribution centers also get this information from other stockiest. Demand is then aggregated for the week.

This data is sent back to its 60-odd factories. In turn, they inform their suppliers. The system came into effect in 1997. With the forecasting process in place, HLL was in a position to aggregate daily demand. This is how the system worked: if stockists around Calcutta report that 5,000 units of Lifebuoy soap were sold that day, HLL's Garden Reach factory in Calcutta will have to ensure that a similar number reach the distribution Centre the next day. Similarly, a packaging supplier would have to keep 5,000 wrappers of Lifebuoy ready for delivery to the factory.

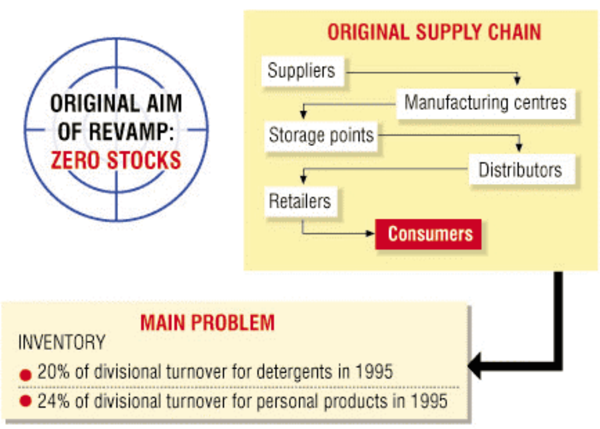
(Source: See, 'Supplying Success', Business World, July 19, 1999)

Problems in which demand is known and fixed are called deterministic problems.

Problems in which demand is assumed to be a random variable are called stochastic problems. The demand is invariably probabilistic in nature for many real time situations. For some of the products, the demand may be seasonal also, such as soft drinks, cement etc.

6. Lead-time

When an order is placed it may be of instantaneous delivery or it may require some time before delivery is effected. The time between the placement of order and its receipt is called lead-time. Again the lead-time may also follow probability distribution. I.e., the lead-time is certain or uncertain. Consider the example of a FMCG Distribution channel, which faced lot of problems with high amount of inventory and distribution efficiency.



Figure

If the distributor is placing order on, say February 1, the order may be processed at the Storage point by 4or 5 the month, he will get the stocks by end of 8 or 9. This will hold well if everything goes as expected. If there is stock out of one or two of the products, then the lead-time may vary further. If we refer the picture, the company has suffered with high amount of inventory, which is almost 50% of the annual sales of the company. The problem is estimating the demand and lead-time with certain amount of accuracy.

7. Stock Replenishment

Getting stocks again is called replenishment. Instantaneous replenishment occurs when the stock is purchased from outside. Uniform replenishment may occur when the procured from local manufacturer.

For example, an automobile manufacturer purchasing Tyres from the local Original Equipment Manufacturer, used to get stocks at some fixed intervals say everyday one truckload or 1000 Tyres instead of getting the entire estimated demand of 30,000 Tyres in a month at once. Some of the parts such as screws and bolts etc can be purchased at bulk and stored. Here the entire demand for the month is supplied at once.

8. Inventory control with known demand

Inventory control problem in which demand is assumed to be fixed and completely known is called Economic Order Quantity (EOQ) problems. If it is the context of production, the batch size to be produced is fixed and completely determined in advance, and then it is known as Economic Lot Size problems.

9. Buffer Stock or Safety Stock

Generally demand is uncertain and cannot be pre determined completely. Lead-time is not always known exactly. The revenue forgone by not keeping adequate inventory i.e., cost of under stocking and sometimes payment of penalty for not meeting the delivery schedule are consequences of inadequate inventory levels. To overcome the situations of uncertainty in demand and lead-time, some extra stock is advisable so that shortages may not occur. This extra stock is known as buffer stock.

References

1. See "Supplying Success", Business World, July 1999.
2. See "Operation Streamline", Business World, February 18, 2002, pp20-26.
3. See "Operation Streamline", Business World, February 18, 2002, pp20-26.
4. See "Operation Streamline", Business World, February 18, 2002, pp20-26.
5. See "A Bait for the Buyer", Business World, February 25, 2002, pp48-49.
6. See "Supplying Success", Business India, July 18, 1999,
7. Douglas M.Lambert, James R. Stock, Strategic Logistics Management, 3rd Edition (New York: Irwin McGraw Hill Inc.1993) pp398-446

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Lesson 3.4 - Determining Economic Order Quantities- Deterministic Models – Purchase Order Quantities without shortages

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As we have already classified the economic order quantity or economic batch size problems into two types, based on the demand position. If the demand is known and certain, those problems are called Inventory management problems under certainty in demand and other types, where the demand or lead time is known in terms of probabilities, known as stochastic problems, inventory control problems with uncertain demand.

Model I

Determination of the optimum quantity ordered (or produced) and the optimum interval between successive orders (or production) if the demand is known and uniform with no shortages permitted

An Illustration: Assume that you are required (as a student) Rs. 100/- to meet your daily requirements such as food, stay, refreshment etc. In a month, your demand is Rs. 3000/-. Whenever you are contacting parents, they are in a position to give your requirements in single stroke on the day itself. To get the money, you have to go to your native which costs you say Rs. 50/- Instead of keeping the money in your room, if you keep it in a bank, you may get an interest of Rs.20/- month. If this is the situation given to you, how much you have to get from your parents such that your holding/opportunity cost and ordering/procurement cost is, get balanced?

Here the problem is how much you have to get your parents every time so that the costs associated with holding the amount is get balanced? Here the example which may be seems to obvious, instead, if we take a car manufacturer, whose requirement is say 3000 Gear boxes per month, who is producing 100 cars in a day, finds that, keeping a gear box in warehouse costs him Rs. 200/- per month (excluding cost of gear box) and placing the order would costs Rs. 500 per order. In this situation, how much gear boxes he has to order, how much should be the order size so that the cost associated such as ordering and holding get balanced and total costs are minimized?

If we summarize the assumptions, we are approaching the problems through Economic Order Quantity or Economic Lot Size problems, or in general known as EOQ problems.

Solution

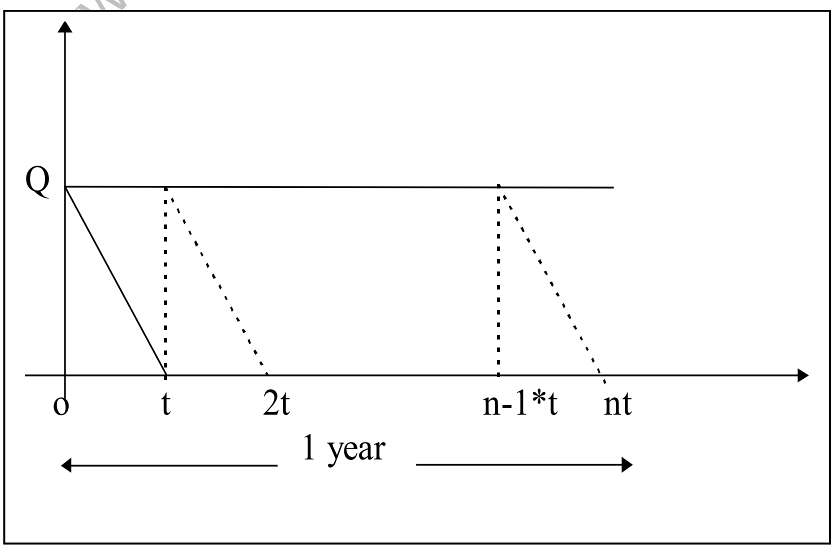
Let us make the following assumptions:

1. Let the demand is known and uniform. Let D be the demand for a period say 1-year.
2. Shortages are not permitted.
3. Let the replenishment of items be instantaneous.
4. Lead-time is zero.
5. Let Q be the Economic Order Quantity for every cycle.
6. Let Cs be the Set up cost for every cycle
7. Let C1 be the inventory holding cost per unit per unit of time.

Let us divide the one-year into n equal parts each of duration't'

$$\text{Therefore } n * t = 1 \text{ or } t = 1/n$$

Let Q be the economic order quantity for every cycle.
The graph of this model is given by,



 www.FirstRanker.com

Therefore $n * Q = D$ or

$t = Q/D$

Inventory for the time period t = Area of the triangle Qot

$= ½\ t * Q$

Therefore the Average inventory for one unit of time

$= [½\ t * Q]/ t = Q/2$

Since all the triangles are similar, the average inventory for the whole year = ½ Q

Therefore the annual inventory holding cost = ½ Q * C1 -------- (1)

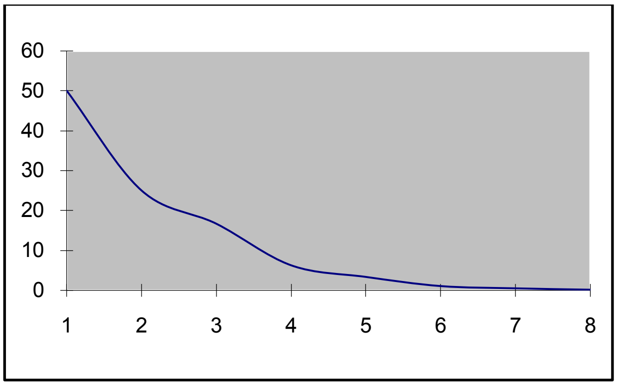
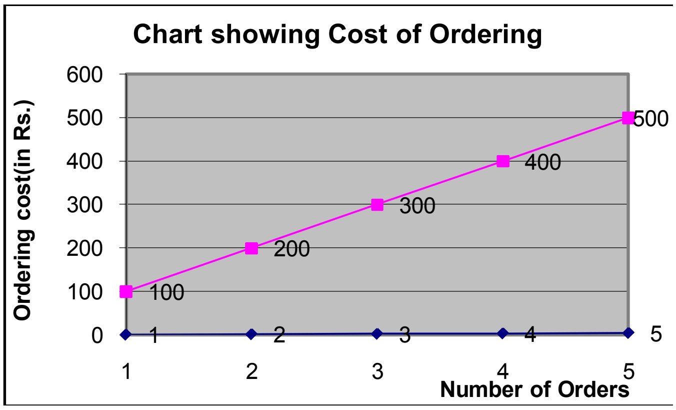


Diagram Showing the Holding cost function

And Annual Set up cost = n * Cs ---------- (2)

= D/Q * Cs (since n * Q = D)



Therefore Annual Total Cost Ca = ½ Q * C1 + D/Q * Cs ---- (3)

In addition, C_a is a function on Q. For maximization or minimization, the first derivative is found out and equated with zero. If the second derivative is greater than zero indicates the function attains its minimum value when the first derivative is equated with zero.

$$d(C_a)/dQ = 0$$

$$d(C_a)/dQ = ½ C1 - DC_s/Q2 = 0$$

Or $$D * C_s / Q2 = ½ C1$$

$$Q2 = 2 * D * C_s / C1$$

$$Q = \sqrt{[2 * D * C_s / C1]}$$

Moreover, the second derivative is strictly greater than zero.

By substituting the value of Q, which has derived just now, which is possessing the characteristic of balancing the ordering cost and holding cost, in the equation for total cost function Ca, we get

$$Ca = ½ C1 * \sqrt{[2 * D * Cs / C1]} + D* Cs * \sqrt{[C1 / 2D Cs]}$$

On simplification, we get $Ca = \sqrt{[2*D*Cs* C1]}$

Therefore $E.O.Q = \sqrt{[2 * D * Cs / C1]}$

$$n = D/Q$$

And $t = 1/n$

Total Cost of Inventory (Excluding cost of Materials) Ca

$$= \sqrt{[2*D*Cs* C1]}$$

And Total Cost of Inventory including cost of Material,

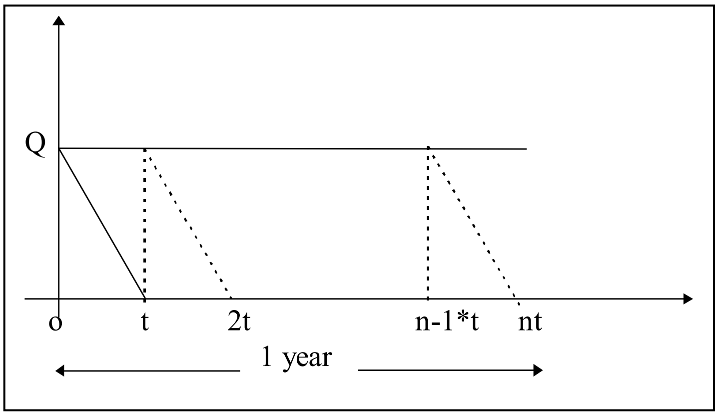
$$Ca = [2*D*Cs* C1]^{½} + (D* C)$$

Where 'c' is the unit cost of the material/component

Some Special Cases

Case 1

Let us assume that the year we divided into n parts say, t1, t2, t3, etc., which are not equal.



The total inventory over time t1 = ½ Q * t1

Total inventory over time t2 = ½ Q * t2 and so on.

Therefore total inventory over 1 year = ½ Q * [t1 + t2 + … +tn]

= ½ Q * t

And Average inventory = ½Q

Therefore Annual inventory holding cost = ½ Q * C1

Since annual set up cost is the same, we will get the same Ca hence; we will get the same EOQ and other related issues.

Therefore,

E.O.Q = [2 * D * Cs / C1] ½

n = D/Q

And t = 1/n

Case 2

Let the Setup cost depend on the # of units that are being ordered or produced.

Since the set up cost for a cycle = Cs + Q * b, where 'b' is the cost of ordering one unit.

Therefore Ca = [½ * Q* C1] + D/Q (Cs + Q *b)

And d (Ca)/ d Q = 0 implies,

d (Ca)/ d Q = ½ C1 - DCs/ Q2 =0

Or $D * Cs / Q2 = ½ C1$

$Q2 = 2 * D * Cs / C1$

$Q = [2 * D * Cs / C1] ½$

And, $Ca = ½ C1 * [2 * D * Cs / C1] ½ + D^* Cs *[C1 / 2D Cs] ½ + D^* b$

On simplification, we get $Ca = [2^*D^*Cs^* C1] ½ + D^* b$

Note: The only change is addition of **D*b** term in the cost.

Example-1

SMS Limited uses annually 24,000 Paper Boxes, which costs Rs. 1.25/- per unit. Placing each order cost Rs. 22.50/- and the carrying cost is 5.4% per year of the average inventory. Find the total cost including the cost of Boxes.

Solution

$D = 24,000$ $Cs = Rs. 22.50/-$

And $C1 = 1.25 * 5.4\% = 0.0675$/Boxes/year.

Therefore, EOQ, $Q = [2 * D * Cs / C1] ½ = 4000$ Boxes

And 'n' = D/Q = 24000/ 4000 = 6 i.e. we have to make 6 orders

$t = Q/D$

$= 4000/24000$

$= 0.16666$ years or

$= 0.16666^* 12$

$= 2$ months

Total Cost Ca, $= [2 * D * Cs * C1] ½$

$= 270/-$

Total Cost including cost of raw material = 270 + [24000 * 1.25] = 30270/-

Example-2

M/s Shriram Industries has to supply 600 industrial fans per year. The firm never permitted shortages to occur. Moreover, the storage cost amounts to Rs. 0.60/ unit/ year. The set up cost per production run is Rs. 80/-. Find the optimum order quantity, number of orders to place in a year and average yearly cost.

Solution

From the given problem, it is clear that,

Demand, $D = 600$ units/year

Carrying cost, $C1 = 0.60$/unit/year

Cost of set up, $Cs = Rs. 80/-$

Therefore, $EOQ = Q = \sqrt{[2 * D * Cs / C1]}$

$= \sqrt{[(2 * 600 * 80)/0.6]}$

On simplifying, we will get, EOQ, Q = 400

Number of order, n = D/Q

n= 600/400 = 1.5 orders per year

Time between any two successive orders, **t = 1/n** = [1/1.5]

= 0.6666 years or 8 months

Total cost of holding the inventory, Tc

$= \sqrt{[2 * D * Cs * C1]} = \sqrt{[2 * 600*80* 0.60]} = 240/-$

Example-3

Preethi Computers purchases 22,000 silicon chips every year and each unit cost Rs. 22/-, as they are purchasing in bulk quantity such a low price is possible. Cost of each order is Rs. 350/-. Its inventory carrying cost is 18% of average inventory. What should be EOQ. What is the optimum number of day's supply for optimum order? What is the annual cost on inventory including cost of the material?

Solution

From the given problem, it is clear that,

Demand, $D = 22000$ units/year

Cost of material, $C=$ Rs. 22 per unit

Carrying cost, $C1 = 18\% * (22) = 3.96$

Cost of set up, $Cs = Rs. 350/-$

Therefore, $EOQ = Q = \sqrt{[2 * D * Cs / C1]}$

$= \sqrt{[(2 * 22000 * 350)/ (3.96)]}$

On simplifying, we will get, EOQ, Q = 1972

Number of order, n = D/Q

n= 22000/1972 = 11.156 orders per year

Time between any two successive orders, **t = 1/n** = [1/11.156]

= 0.08964 years or 1.07567 months

= 33 days (approx)

Total Cost Ca, $= \sqrt{[2 * D * Cs * C1]}$ ½

$= \sqrt{[2 * 22000 * 350*3.96]}$

= **7809**

Total Cost including cost of raw material

=7809 + (22,000* (22))

= 4, 91,809 including cost of raw materials)

Example-4

PRG Engineering Company is a distributor for water pumps. The company's sales amount to 50,000 units of Water Pumps per year. The order receiving/processing and handling cost are Rs. 3 per order while the trucking cost is Rs. 12 per order. Further, the interest cost is Rs. 0.06 per unit per year. Deterioration cost is Rs. 0.004 per unit per year. Storage cost is Rs. 1000/year for 50000 units. Find the EOQ; also find the time between orders and number of orders to be placed.

Solution

From the given problem, it is clear that,

Demand, D = 50000 pumps /year

Carrying cost, C1 = 0.006 + 0.004 + 1000/50000 = 0.084

Cost of set up, Cs = Rs. 3+5= 15 per order

Therefore, EOQ = Q

$= \sqrt{[2 * D * Cs / C1]}$

$Q = \sqrt{[(2 * 50000 * 15)/ (0.084)]}$

On simplifying, we will get, EOQ, Q = 4226 (approx)

Number of order, n = D/Q

n= 50000/4226 = 11.83 orders per year

Time between any two successive orders, **t = 1/n** = [1/11.83]

= 0.08452 years or 1.01424 months

= 30.427 days (approx)

Model 2

Determination of the optimum batch quantity [EBQ] if the demand is known and uniform with a finite rate of replenishment

An Illustration: Consider the example, which is given in the Model 1. Assume that you are requiring Rs. 100 per day and in a month your demand is Rs. 3000/- to manage your requirements. Again, you may get your required amount immediately. Only the change is, instead of giving your demand Rs. 3000/- in a single stroke, your parents decided to give the sum in such a way that first 20 days, you will be given with Rs. 150/- per day out of which Rs. 100/- per day is incurred and every day you have to save Rs. 50/- for the first twenty days, and you will not be given any amount after 20th. The remaining days in the month, you will start using the 'savings'.

If we consider companies such as Hyundai, Maruti and other bigger automobiles, they use to follow this kind of 'constant rate of replenishment' instead of instantaneous replenishment. If a truck manufacturer who is placing an order of 10,000 Tires would appreciate the concept of finite rate of fulfilling the orders rather than getting all the 10,000 Tires in a single stroke. He may save a lot in keeping smaller warehouse(s) capital locked in inventory is minimal. For the supplier also, he finds it convenient to sending the quantities in batches rather than bulk dispatches.

Solution

Let us have the following assumptions:

1. Demand is known and uniform
2. Let 'r' be the rate of demand per unit of time.
3. Shortages are not permitted.
4. Inventory is replenished at the rate of 'k' units per unit of time.
5. Let the supply of the items be instantaneous.
6. Lead-time is zero.

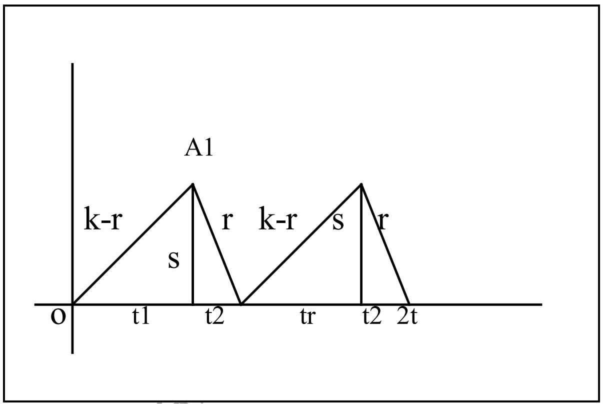
7. Let Q be the EBQ for each cycle.
8. Let Cs be the set up cost per cycle.
9. Let C1 be the holding cost per unit, per unit

Let us divide the total time say 1-year into 'n' equal parts, each of duration't'.

Therefore n * t = 1

Let us further divide time 't' into 2 parts, say t1 and t2 such that (1) Inventory is building up at a constant rate of 'k-r' units per unit time during t1. (2) There is no replenishment during time t2 and the inventory is decreasing at the rate of 'r' units per unit time.

At the end of time t1, let S be the level of the inventory and at the end of time t2, zero is the level of inventory. The graph is as follows:



Since S = (k-r)*t1

= r * t2

Therefore t1 = **S/ (k-r)**

& t2 = **S/r**

t1+ t2 = S * [1/(k-r) + 1/r] = S [k/ r *(k-r)]

That is, t = S* k / r * (k-r) --------- (a)

And Q = **k * t1**

= **k * S / (k-r)**

Therefore S = (k-r) * Q / k & t = S * k / r * (k-r)

= [(k-r) * Q/ k] * [k/ r * (k-r)]

= **Q/r**

Total inventory during time period t = area of the triangle OA1t = ½ t* S

Average Inventory = [½ t* S] / t = **S/2**

Since all the triangles are similar,

Average inventory for the whole year = ½ S

Annual inventory holding cost $= ½ * S * C1 = \frac{(k-r) * Q * C1}{2k}$

And Annual set up cost for 'n' cycles = **n* Cs**

$= 1/t * Cs = [r * Cs] / Q$

Annual total cost Ca = Annual Holding cost + Annual ordering/Setup cost

$$= \frac{(k-r) * C1 * Q}{2k} + (r/Q) * Cs \quad \text{------------ (1)}$$

And for maximization OR Minimization,

$$\frac{dCa}{dQ} = \frac{(k-r)C1}{2k} - \frac{r * Cs}{Q2} = 0$$

This implies,

$$\frac{(k-r)*C1}{2k} \quad \frac{- r * Cs}{Q2} => 0$$

$$Q2 = (2 k * r * Cs) / [(k-r) * C1]$$

Therefore, EBQ, $Q = \sqrt{\frac{[2 k * r * Cs]}{(k-r) * C1}}$

And, t = Q/r

n = r/Q

Therefore the total cost $= \sqrt{2 * Cs * C1 * (k-r) * r / k}$

By substituting the EBQ in the above said cost function (1).

Example-1

Gee. Vee. Bearing Ltd has to supply 10000 bearings per day to a SMS Automobiles Ltd., who is assembling some of the components of an automobile company. Mr. Stephen, who is production in-charge, finds that when he starts a production run, he can produce 25000 bearings per day. The cost of holding a bearing in stock per year is 12 paise and the set up cost of a production run is Rs. 1800/= How frequently should the production run be made, and what is the optimum quantity to be produced.

Solution

The given data is summarized in the following table.

r = 10000 B's/day

k = 25000 B's/day

Cost of Set up, Cs = 1800/-

Holding cost, C1 = Rs. 0.12/day

Therefore, EBQ,

$$Q= \sqrt{[(2* 25,000 * 10,000 * 1800) / (15,000 * 0.12)]}$$

= 22360 bearings per run

t = Q/r

= 22,360/10,000 = 2.23 days

I.e. once in 2.23 day, he has to produce 22,360 bearings.

N= Number of production batches in a day = r/Q

= 10,000/22,360=0.4472 batches per day.

(Since all the units of measurements are in days, n is also computed per day)

Example-2

S.G. Computers is assembling 50 personal computers per day. The demand occurs at the rate of 25 per day. If the procurement cost of components costs the firm Rs. 250 per batch and the holding cost of a quality checked and packaged computers in warehouse is Rs. 10 per unit per day. Find the economic batch size for one run, assuming the shortages are not permitted. Find the minimum total cost for one run if the cost of the one personal computer is Rs. 15000.

Solution

Production rate, k = 50 PC's/day

Rate of demand, r = 25 PC's /day

Procuring cost, Cs = 250

Holding cost per unit, C1 = 10 per day

Therefore, EBQ,

$$Q = \sqrt{\frac{[2\ k^*\ r^*\ Cs]}{(k-r)\ *\ C1}}$$

$$Q = \sqrt{[(2^*\ 50\ *\ 25\ *\ 250)\ /\ (25^*\ 10]}$$

Q = 50 PC per batch

t = Q/r = 50/25 = 2 days

I.e. once in 2 days, he has to produce 50 PCs.

And the total cost = $\sqrt{2\ *\ Cs\ *\ C1\ *\ (k-r)\ *\ r/\ k}$

And the total cost = $\sqrt{2\ *\ 250\ *\ 10\ *\ (25)\ *\ 25/\ 50}$

= Rs. 250/-

Total cost, including the material cost, Ca =

= 250 + (15,000 * 50)

= 7, 50, 250/-(including RM)

Example-3

Shriram Industries, a manufacturer of television cabinets, has to supply his customers 24000 units of television cabinets (shells alone) per year. And, the company can produce the shells at a rate of 3000 per month. The cost of one preparation of the equipments for the production run is Rs. 500/- and the holding cost of one cabinet per month is 15 paisa. Determine the optimum manufacturing quantity, optimum interval between the set ups and total cost, if the cost of one cabinet is Rs. 775/-

Solution

Production rate, k = 3000 shells/month

Rate of demand, r = 2000 PC's /month

Procuring cost, Cs = 500

Holding cost per unit, C1 = 15 paisa per shell per month

Cost of a shell, C = Rs. 775

Therefore, EBQ,

$$Q = \sqrt{\frac{[2\ k^*\ r^*\ Cs]}{(k\text{-}r)\ *\ C1}}$$

$$Q = \sqrt{[(2^*\ 3000\ *\ 2000\ *\ 500)\ /\ (1000^*\ 0.15]}$$

Q = 6325 shell per batch (approx)

t = Q/r = 6325/2000 = 3.1625 days

I.e. once in 3 days, he has to produce 6325 shells

And the total cost = $\sqrt{2 * Cs * C1 * (k\text{-}r) * r/ k}$

And the total cost = $\sqrt{2 * 500 * 0.15 * (1000) * 2000/ 3000}$

= Rs. 316.28/-

Total cost, including the material cost, Ca

= Rs. 316.28 + (24000 * 775)

= Rs. 1, 86, 00, 316/- (including RM)

Example-4

Ram Publications finds that demand for their Operations Research books is 24000 books per year. The book publisher can print 4500 copies of the book in a month. The set up cost per set up is Rs. 350/- The rental for the warehouse is Rs. 700 per month. For the books, he pays Rs. 150 as insurance premium per month. He appointed a Storekeeper who can handle all the transactions as well as physical handling of the books, for which he pays Rs. 1500 as salary. Find the EOQ for the above production schedule.

Solution

Production rate, k = 4500 books/month

Rate of demand, r = 2000 books /month

Procuring cost, Cs = 3500

Holding cost per unit, C1 = [700 + 150 +1500] = 2350

Therefore, EBQ,

$$Q = \sqrt{\frac{[2\ k^*\ r^*\ Cs]}{(k-r)\ *\ C1}}$$

$$Q = \sqrt{[(2^*\ 4500\ *\ 2000\ *\ 3500)\ /\ (2500^*\ 2350]}$$

$$Q = \sqrt{104 \text{ books per batch (approx)}}$$

t = Q/r

= 104/2000

= 0.052 months

= 0.052 X 30

= 1.56 days

I.e. once in 2 days, he has to produce 104 books.

Example-5

SMS Motor Company, the 2-wheeler manufacturer producing 2,50,000 units of mopeds per annum. The company is having its own tyre-manufacturing unit, which is producing tyres for internal consumption as well as acting as 'Original Equipment Manufacturer' for several 2-wheelers manufacturers in India. The tyre-manufacturing unit is in a possession to manufacture 20,000 tyres per day as far as any model is concerned and, will be in a position to produce 8,500 tyres per day as far as internal requirements are concerned. Whenever, the production operations starts on one specific type or model, again to produce a new model we have to re set the equipments which will costs Rs. 2500/ per setting. To hold one tyre in inventory, the company has to incur Rs. 100 per year. Assuming 250 working days per year, what will be the optimum manufacturing quantity?

Solution

Production rate, k = 8500 tyres /day

Rate of demand, r = 1000 tyres /day

Procuring cost, Cs = 2500

Holding cost per unit, C1 = [100/250] = 0.40 per day

Therefore, EBQ,

$$Q = \sqrt{\frac{[2\ k^*\ r^*\ Cs]}{(k-r)\ ^*\ C1}}$$

$$Q = \sqrt{[(2^*\ 8500\ ^*\ 1000\ ^*\ 2500)\ /\ (7500^*\ 0.40]}$$

$$Q = \sqrt{3764 \text{ tyres per batch (approx)}}$$

$$t = Q/r$$

$$= 3764/1000$$

$$= 3.764 \text{ days}$$

I.e. once in 4 days, he has to produce 3764 tyres

Model – 3

E.B.Q with known demand, shortages is permitted and replenishment of inventory is instantaneous

Illustration: Consider the example, discussed in the Model – 1. Assume that you are required (as a student) Rs. 100/- to meet your daily requirements such as food, stay, refreshment etc. In a month, your demand is Rs. 3000/-. Whenever you are contacting parents, they are in a position to give your requirements in single stroke on the day itself. To get the money, you have to go to your native which costs you say Rs. 50/- Instead of keeping the money in your room, if you keep it in a bank, you may get an interest of Rs.20/- month. Every month, 25th assume your roommate used to get Rs. 500/- from you. Because of this practice, you find that you are running out cash on every 25th. However, you can borrow the amount from neighbor/friends at nominal interest rates. If this is the situation given to you, how much you have to get from your parents such that your holding/opportunity cost and ordering/ procurement cost is, get balanced?

Again, consider the example of a car manufacturer, whose requirement is say 3000 Gear boxes per month, who is producing 100 cars in a day, finds that keeping a gear box in warehouse costs him Rs. 200/- per month (of course, excluding cost of gear box) and placing the order would costs Rs. 500 per order. In addition to this, he included a clause in the contract, stating that in case of non-compliance of the supply schedule, the supplier has to pay a penalty of Rs. 1000/ per gear box per day. In this situation, how much gear boxes he has to order, how much should be the order size so that the cost associated such as ordering and holding get balanced and total costs are minimized? Whenever he places an order the entire order size is satisfied in, single stroke/replenishment is instantaneous.

Solution

Let us have the following assumptions.

1. Demand is known and uniform. Let D be the total demand for one unit of time.
2. Shortages are permitted, and let C2 is the shortage cost per quantity per unit of time.
3. Production of the item or procurement is instantaneous.
4. Lead -time is zero.
5. Let C1 be the inventory holding cost per unit. Per unit of time
6. Let Cs be the set up or ordering cost for every production run (or cycle).

Divide the given total time period into 'n' units each of time't'

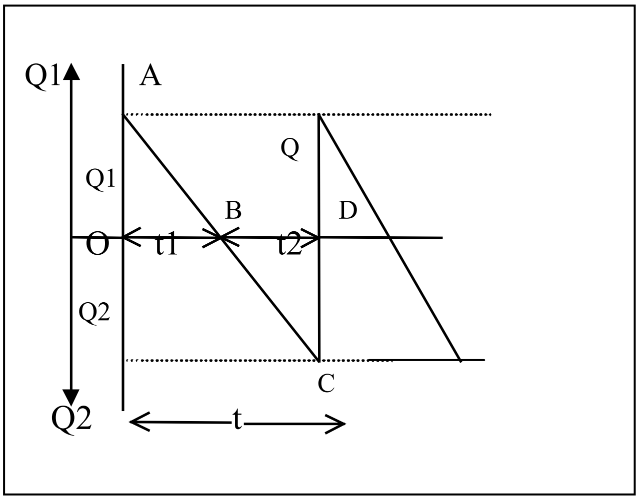
$$n * t = 1$$

Let Q be the EBQ for every run

The total demand $D = n * Q$

Let us assume that each production run consists of 2 parts say t1 and t2 such that during the interval t1 items are drawn from the inventory as needed and during t2, orders for the item are being accumulated, but not filled. And at the end of interval ' t ' an amount Q is ordered, the amount Q is divided into Q1 and Q2 such that Q1 + Q2 = Q where Q1 denotes the amount that goes into the inventory and Q2 denotes the amount that is immediately taken to satisfy unfilled demands.

The Graph of this model is given below.



Total inventory during the time period t = Area of the Triangle AOB

= ½ * t1 * Q1

Therefore the Inventory holding cost during time

t= [½ * t1 *Q1] * C1

Total shortages during the time period

t = Area of the triangle BCD

= ½ * t2 * Q2

Shortage cost for time t = [½ * t2 * Q2] * C2

Ordering cost or set up cost during time t = Cs

Therefore,

Total cost during time t = Holding cost + Penalty Cost + Preparation/Set up cost

= [½ * t1 *Q1 * C1] + [½ * t2 * Q2 * C2] + Cs

Hence,

Total cost for one unit of time

= 1/t * {[½ * t1 *Q1 * C1] + [½ * t2 * Q2 * C2] + Cs} --- (A)

Since the Triangles OAB and BCD are similar,

This implies,

Q1/Q2 = t1/t2 {When the triangles are similar, the ratio of Corresponding sides will be equal. }

Q1+Q2/Q2 = t1 +t2 /t2

Q/Q2 = t/t2 or

t2 /t = Q2/Q

Similarly, t1/t = Q1/Q

And $D = n * Q$, which implies,

$n = D/Q$

By substituting the values, in (A), we get,

(A) becomes,

$$Ca = [½ Q1 * C1 * Q1/Q] + [½ Q2 * C2 * Q2/Q] + Cs * D/Q$$

$$= [½ C1 * Q12 /Q] + [½ C2 (Q - Q1) 2 / Q] + D * Cs /Q$$

For maximization or minimization, dCa/ dQ1

$$= [C1 * Q1 /Q] - [C2 * (Q-Q1) / Q] = 0$$

$$[C1 * Q1] - [C2 * Q] + C2 * Q1 = 0$$

Or $(C1 + C2) Q1 = [C2 * Q]$

$$Q1 = [C2 * Q] / [C1 + C2]$$

$$Q2 = [C1 * Q] / [C1 + C2]$$

Using these values in the Ca function, we get,

$$Ca = [½ C1 * Q12 /Q] + [½ C2 (Q - Q1) 2 / Q] + D * Cs /Q$$

$$= \frac{½ C1* C22Q2}{Q * [C1 + C2] 2} + \frac{½ C2 * C12 * Q2}{Q * [C1 + C2] 2} + D * Cs /Q$$

$$= \frac{½ C1 * C2 * Q * [C1 + C2] \quad + D * Cs /Q}{[C1 + C2] 2}$$

$$= \frac{½ C1 * C2 * Q}{C1+ C2} + D * Cs /Q$$

Further $d Ca/ dQ = \frac{C1 * C2}{C1+ C2} - D * Cs / Q2 = 0$

On simplification we get Q = [2 D * Cs * [C1 + C2] / C1 C2] ½

Using this value of Q in the minimum cost function Ca, we get the total minimum cost.

Minimum Cost, (Excluding cost of Raw materials,)

$$\mathbf{Ca = \sqrt{[2 * D * Cs *C1 * C2 / (C1+ C2)]}}$$

E.O.Q, $\mathbf{Q = \sqrt{[2 D * Cs * [C1 + C2] / C1 C2]}}$

Time between the orders, $t = Q / D$ and

Number of orders, $n = D / Q$

Example-1

Raja Motors, a contractor undertakes to supply diesel engines to a truck manufacturer at the rate of 25 engines per day. There is a class in the contract for penalizing him for missing the scheduled delivery date. Rs 1000/- per engine per day is the penalty charged on Raja Motors. He finds that the cost of holding a completed engine in stock is Rs. 120/- per month. His production process is such that he can produce enough engines within a short time. Determine how often he can make a production run, and what size it should be, if he has to incur Rs. 10000 every time, whenever a production run is made.

Solution

The given data can be summarized in the following table.

D = 25 e/day	C2 = Rs. 1000/e/day	Cs = 1000	C1 = 120/30 /e/day = Rs. 4/engine/day

Substituting these values in the equation for EOQ, we get,

E.O.Q, $Q = \sqrt{[2 D * Cs * [C1 + C2] / C1\ C2]}$

$= \sqrt{[2 * 25 * 1000 * (1004) / 1000 * 4]}$

= 112 engines per run.

Time between the orders,

t = Q / D

= 112/25

= 4.48 days

Number of orders, n = D / Q

= 25/112 =0.2232 order per day.

Alternatively, in a year, we have place 81 orders. (0.2232 * 365)

Total Cost, Ca $= \sqrt{[2 * D * Cs * C1 * C2 / (C1+ C2)]}$

$$= \sqrt{[2 * 25 * 1000 * 1000 * 4 / (1004)]}$$

= Rs. 446.32

Example-2

The demand for soap is 50 units per month and the products are withdrawn uniformly. The expenses incurred while purchasing for each time is Rs. 200/-. The cost of each soap is Rs 20/- per item and the inventory holding cost of Rs. 4 per item per month. In addition, a profit of Rs 2 per item per month is gained from selling the soap. Determine how often to make purchases and what size it should be such that it will minimize the total inventory cost?

Solution

From the given problem, we identify the following information /costs;

Demand for the item, D = 50/month

Holding cost C1, = Rs. 4/month

Shortage Cost C2, = Rs 2/month/per unit

Set up cost Cs, = Rs. 200

Substituting these values in the equation for EOQ, we get,

$$\text{E.O.Q,} \quad Q = \sqrt{[2\,D * Cs * [C1 + C2] / C1\,C2]}$$

$$= \sqrt{[2 * 50 * 200 * (6) / 4 * 2]}$$

= 122 soaps per order

$$\text{Total Cost, Ca} = \sqrt{[2 * D * Cs * C1 * C2 / (C1 + C2)]}$$

$$= \sqrt{[2 * 50 * 200 * 2 * 4 / (2+4)]}$$

= Rs. 163.30

Time between the orders,

$$t = Q / D$$
$$= 122/50$$
$$= 2.44 \text{ months}$$

Number of orders,

$$n = D / Q$$
$$= 50/122$$
$$= 0.41 \text{order per month.}$$

Alternatively, in a year, we have placed 4.92 orders. (0.41 * 12)

Example-3

The demand for Gem Clips is uniform at a rate of 200 boxes /month. The fixed cost Rs 10 is incurred each time while purchase is made. The cost of each box is Rs. 10 per item and the inventory carrying cost is Rs 0.25 per box per month. The shortages are penalized at the rate of Rs. 1.25 per box per month; determine what should be the purchase cycle what size it should be?

Solution

From the given problem, we identify the following information / costs;

Demand, D = 200 boxes/month

Penalty Charges, C2 = 1.25/month/box

Purchasing cost, Cs = Rs. 10

Holding cost, C1 = 0.25/box/month

Substituting these values in the equation for EOQ, we get,

$$\text{E.O.Q,} \quad Q = \sqrt{[2 D * Cs * [C1 + C2] / C1\ C2]}$$

$$= \sqrt{[2 * 200 * 10 * (1.5) / (1.25 * 0.25)]}$$

$$= 139 \text{ boxes per order}$$

Time between the orders,

$$t = Q / D$$

= 139/200

= 0.695 months

= (0.695*30 days)

= 21 days

Number of orders,

n = D / Q

= 200/139

= 1.45 order per month.

Example-4

SMS Electronics Ltd., which is manufacturing Color televisions also, produces its own speakers, which are used in television sets. Since the company produces 8 models of its color televisions with product codes as (TFR-CTV/01 to TFR-CTV/08). The television sets are assembled on a continuous production line at a rate of 8000 per month. The speakers are produced in batches because they do not warrant setting up a continuous production line and relatively large quantities can be produced in a short time. Discuss the problem of the company, determining when and how much to produce given the following data.

1. Each time a batch is produced, a set up cost of Rs. 12000/- is incurred, as the models are differing in technologies and appearances and component requirements.
2. The cost of keeping a quality checked speakers (QC-Passed Speakers) in stock is 30 paisa per day.
3. The production cost of a single speaker excluding the set up cost is Rs. 1000 and can be assumed a unit cost.
4. Shortage is going to stop the production and estimated that Rs. 1.10 month whenever a speaker is not available.

Solution

From the given problem, we identify the following information / costs;

Demand, D = 8000

Set up cost, Cs = 12000

Holding cost, C1 =9.90/month/unit

Shortage Cost, C2 = 1.10/month/unit

Substituting these values in the equation for EOQ, we get,

$$\text{E.O.Q,}\quad Q = \sqrt{[2 D * Cs * [C1 + C2] / C1\, C2]}$$

$$= \sqrt{[2 * 8000 * 12000 * (11) / (9.9 *1.1)]}$$

= 13926 speakers per order

Time between the orders,

t = Q / D

= 13926/8000

= 1.741 months

= (1.741*30 days)

= 52 days

Number of orders, n = D / Q

= 8000/13926

= 0.574order per month.

= 0.574 * 12 months

= 6.89 orders per year

Model 4

Finding the EOQ with shortages and the replenishment of inventory is at a finite rate

An Illustration: Consider the example, which is given in the Model 1. Assume that you are requiring Rs. 100 per day and in a month your demand is Rs. 3000/- to manage your requirements. Again, you may get your required amount immediately. Only the change is, instead of giving your demand Rs. 3000/- in a single stroke, your parents decided to give the sum in such a way that first 20 days, you will be given with Rs. 150/- per day out of which Rs. 100/- per day is incurred and every day you have to save Rs. 50/- for the first twenty days, and you will not be given any amount after 20th. The remaining days in the month, you will start using the 'savings'. Assume that on every 22nd your room mate/ close friend use to borrow Rs. 500 from you. You will run out of money after 25th, which you are in a position to manage through borrowings from local firms at a nominal rate. (Indirectly you are paying a penalty by way of interest for the shortages.)

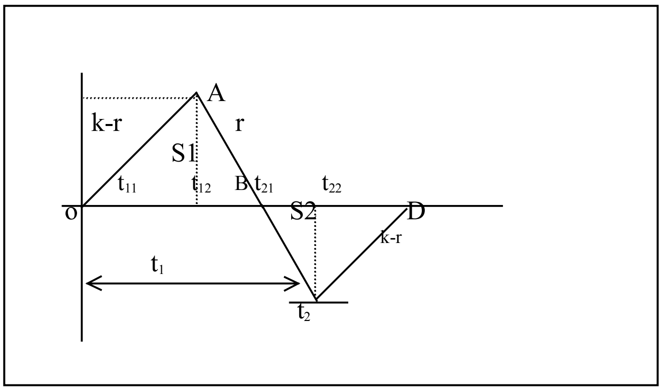
If we consider companies such as Hyundai, Maruti and other bigger automobiles, they use to follow this kind of 'constant rate of replenishment' instead of instantaneous replenishment. If a truck manufacturer who is placing an order of 10,000 Tyres would appreciate the concept of finite rate of fulfilling the orders rather than getting all the 10,000 Tyres in a single stroke. He may save a lot in keeping smaller warehouse(s), capital locked in inventory is minimal. For the supplier also, he finds it convenient to sending the quantities in batches rather than bulk dispatches. But all these relaxation have a binding such that the quantities should be supplied on specified date and time, any violation should dealt very severally by levying large penalties.

Solution

Let us have the following assumptions:

1. Demand is known and uniform. Let 'r' be the rate of demand for one unit of time.
2. Let 'k' be the rate of replenishment per unit of time.
3. Shortages are permitted and let C2 be the shortage cost per unit per unit of time.
4. Lead-time is zero.
5. Let C1 be the holding cost per unit of inventory per unit of time.
6. Let Cs be the set up cost per production run or ordering cost per cycle.

Assume that each production run of length't' consists of 2 parts say t1 and t2. In turn, they are sub divided into t11, t12 and t21, t22 as shown in the figure.



From the figure, S1 = (k-r) * t11 & also S1 = r * t12

And S2 = t21 * r & also S2 = (k-r) * t22

Therefore t11 = S1 / (k-r) & t12 = S1 / r

t21 = S2 / r & t22 = S1 / r

t1 = t11 + t12 = S1 [(k-r +r) / r * (k-r)] = S1 * k / r * (k-r)

Similarly, t2 = t21 + t22 = S2 *k / r * (k-r)

Therefore, t = t1 + t2 = (S1 + S2) *k / r * (k-r)

But we know that, t = Q / r

Therefore, Q / r = (S1 + S2) *k / r * (k-r)

$\Longrightarrow$ S1 + S2 = (k-r) * Q / k

Or S2 = (k-r) * Q / k - S1 ------- (1)

Therefore Total Inventory during time t, = Area of the triangle OAB

= ½ t1 * S1 = ½ * S1 * S1 * k / r * (k-r) = ½ k * S1 2 / r * (k-r)

Therefore Total inventory holding cost

= [½ k * S1 2 / r * (k-r)] * C1

And total shortages during the time t = Area of the triangle BCD

= ½ t2 * S2

= ½ S2 * S2 * k / r * (k-r)

= ½ S22 * k / r* (k-r)

Therefore total Shortage cost during the time

t = [½ S22 * k / r* (k-r)] * C2

And the set up cost for a cycle or ordering cost per order = Cs

Hence the total cost incurred during time

t = Holding cost + Shortage cost + Ordering cost

= [{½ k * S1 2 / r * (k-r)} * C1] + [{½ S22 * k / r* (k-r)} * C2] + Cs

Therefore Average Total cost for one unit of time

= 1/t {[{½ k * S1 2 / r * (k-r)} * C1] + [{½ S22 * k / r* (k-r)} * C2] + Cs}

But t = Q/r, we get

Average Cost

= 1/Q {[½ k * S1 2 * C1/ (k-r)] + [{½ S22 * k * C2 / (k-r)] + r *Cs}

Since

S1 = (k-r) * Q / k - S2, substituting this in the average cost function,

Ave. Cost

= 1/Q {½ k * C1 *[(k-r) * Q / k - S2] 2 / (k-r)} + {½ k * C2 * S22 / (k-r)} + r *Cs

For Maximization or minimization

dC / dS2 = 0 and d C / d Q = 0

dC / d S2 = 1/Q {½ k * C1 * { (-)2 * [(k-r) * Q / k - S2] + ½k * C2 * 2 * S2 / (k-r) } = 0

$$k / (k-r) * Q \left\{ C2 * S2 - \frac{C1 * (k-r) * Q}{k} + C1 * S2 \right. = 0$$

$$\text{i.e. } (C1 + C2) * S2 = \frac{C1 * (k-r) * Q}{k}$$

$$S2 = \frac{C1}{(C1 + C2)} * \frac{(k-r) * Q}{k}$$

Calculate the value of S1 by substituting the value of S2 we get,

$$S1 = \frac{(k-r)}{k} *Q - S2$$

$$= \frac{(k-r) *Q}{k} - \frac{C1}{(C1 + C2)} * \frac{(k-r) * Q}{k}$$

$$S1 = \frac{C2}{C1 + C2} * \frac{(k-r)}{k} * Q$$

Substituting the S1 and S2 values in Cost function Ca, we get

$$Ca = \frac{1/2\, C1C * (k-r)*Q}{(C1+C2) * k} + Cs*r/Q$$

$dCa / d Q = 0$ implies

$$Q2 = \frac{2 r *Cs * (C1 + C2) * k}{C1 * C2 * (k-r)}$$

$$Q2 = \left[\frac{2 r *Cs * (C1 + C2) * k}{C1 * C2 * (k-r)}\right]^{1/2}$$

And substituting the value of Q in the cost function we get the minimum cost value for the cycle.

$$Ca = \left[\frac{2 r * Cs * C1* C2 (k-r)}{(C1 + C2) * k}\right]^{1/2}$$

As usual, the number of orders 'n' = r/ Q
And time between orders = Q/r

Example

The demand for an item in a company is 12000 per year and the company can produce the item at the rate of 3000 per month. The cost of one set up is Rs. 500 and the holding cost of one unit per month is Rs 0.15. The shortage cost of one unit is Rs. 20 per year. Determine the optimum manufacturing quantity and the # of production runs and time between the production runs?

Solution

The given quantities can be summarized in the following table.

r = 1000	k= 3000	C1 =0.15/m	C2 = 5/3m	Cs= 500

Q = (2 * 1000 * 500 * [0.15 + 5/3] * 3000 *3) / (0.15 * 5 * 2000)

= 330 units.

And the number of shortages = S2 = [C1/(C1+C2)] * [(k-r)/k] * Q

= 0.15 * 2000 * 330 / 1.82 *3000

= 181 units.

Manufacturing time = Q / n = 330/3000 = 1.1 months

Time between set up = Q /r = 330 / 1000 = 3.3 months

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UNIT- IV

Network Problems

Introduction

A network consists of several destinations or jobs which are linked with one another. A manager will have occasions to deal with some network or other. Certain problems pertaining to networks are taken up for consideration in this unit.

Lesson 1 - Shortest Path Problem

Lesson Outline

- The description of a shortest path problem.
- The determination of the shortest path.

Learning Objectives

After reading this lesson you should be able to

- Understand a shortest path problem
- Understand the algorithm for a shortest path problem work out numerical problems

The Problem

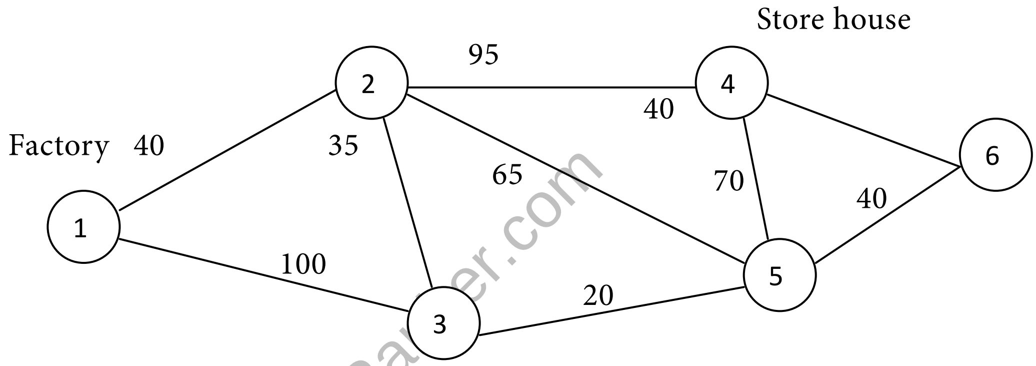
Imagine a salesman or a milk vendor or a post man who has to cover certain previously earmarked places to perform his daily routines. It is assumed that all the places to be visited by him are connected well for a suitable mode of transport. He has to cover all the locations. While doing so, if he visits the same place again and again on the same day, it will be a loss of several resources such as time, money, etc. Therefore he shall place a constraint upon himself not to visit the same place again and again on the same day. He shall be in a position to determine a route which would enable him to cover all the locations, fulfilling the constraint.

The shortest route method aims to find how a person can travel from one location to another, keeping the total distance traveled to the minimum. In other words, it seeks to identify the shortest route to a series of destinations.

Example

Let us consider a real life situation involving a shortest route problem.

A leather manufacturing company has to transport the finished goods from the factory to the store house. The path from the factory to the store house is through certain intermediate stations as indicated in the following diagram. The company executive wants to identify the path with the shortest distance so as to minimize the transportation cost. The problem is to achieve this objective.



Linkages from Factory to Store house

The shortest route technique can be used to minimize the total distance from a node designated as the **starting node or origin** to another node designated as the **final node.**

In the example under consideration, the origin is the factory and the final node is the store house.

Steps In The Shortest Route Technique

The procedure consists of starting with a set containing a node and enlarging the set by choosing a node in each subsequent step.

Step 1

First, locate the origin. Then, find the node nearest to the origin. Mark the distance between the origin and the nearest node in a box by the side of that node.

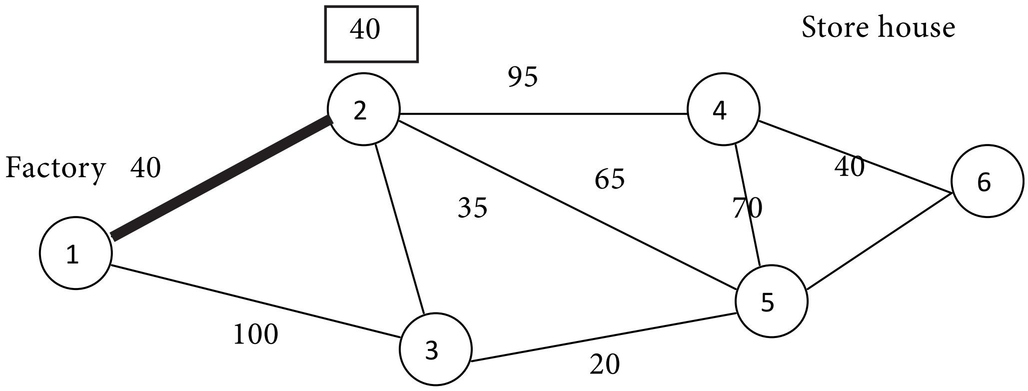
In some cases, it may be necessary to check several paths to find the nearest node.

Step 2

Repeat the above process until the nodes in the entire network have been accounted for. The last distance placed in a box by the side of the ending node will be the distance of the shortest route. We note that the distances indicated in the boxes by each node constitute the shortest route to that node. These distances are used as intermediate results in determining the next nearest node.

Solution For The Example Problem

Looking at the diagram, we see that node 1 is the origin and the nodes 2 and 3 are neighbours to the origin. Among the two nodes, we see that node 2 is at a distance of 40 units from node 1 whereas node 3 is at a distance of 100 units from node 1. The minimum of {40, 100} is 40. Thus, the node nearest to the origin is node 2, with a distance of 40 units. So, out of the two nodes 2 and 3, we select node 2. We form a set of nodes {1, 2} and construct a path connecting the node 2 with node 1 by a thick line and mark the distance of 40 in a box by the side of node 2. This first iteration is shown in the following diagram.



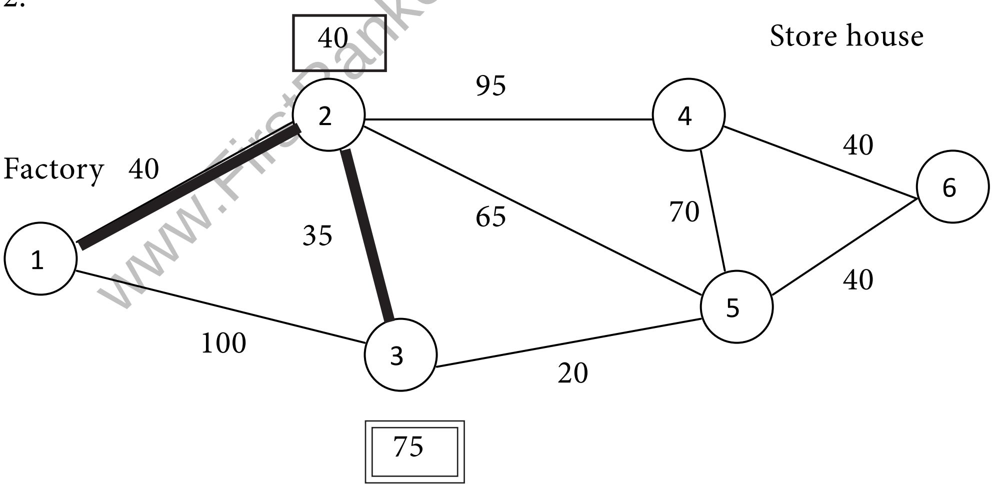
Iteration No. 1

Now we search for the next node nearest to the set of nodes {1, 2}. For this purpose, consider those nodes which are neighbours of either node 1 or node 2. The nodes 3, 4 and 5 fulfill this condition. We calculate the following distances.

The distance between nodes 1 and 3 = 100.
The distance between nodes 2 and 3 = 35.
The distance between nodes 2 and 4 = 95.
The distance between nodes 2 and 5 = 65.
Minimum of {100, 35, 95, 65} = 35.

Therefore, node 3 is the nearest one to the set {1, 2}. In view of this observation, the set of nodes is enlarged from {1, 2} to {1, 2, 3}. For the set {1, 2, 3}, there are two possible paths, viz. Path 1 → 2 → 3 and Path 1 → 3 → 2. The Path 1 → 2 → 3 has a distance of 40 + 35 = 75 units while the Path 1 → 3 → 2 has a distance of 100 + 35 = 135 units.

Minimum of {75, 135} = 75. Hence we select the path 1 → 2 → 3 and display this path by thick edges. The distance 75 is marked in a box by the side of node 3. We obtain the following diagram at the end of Iteration No. 2.



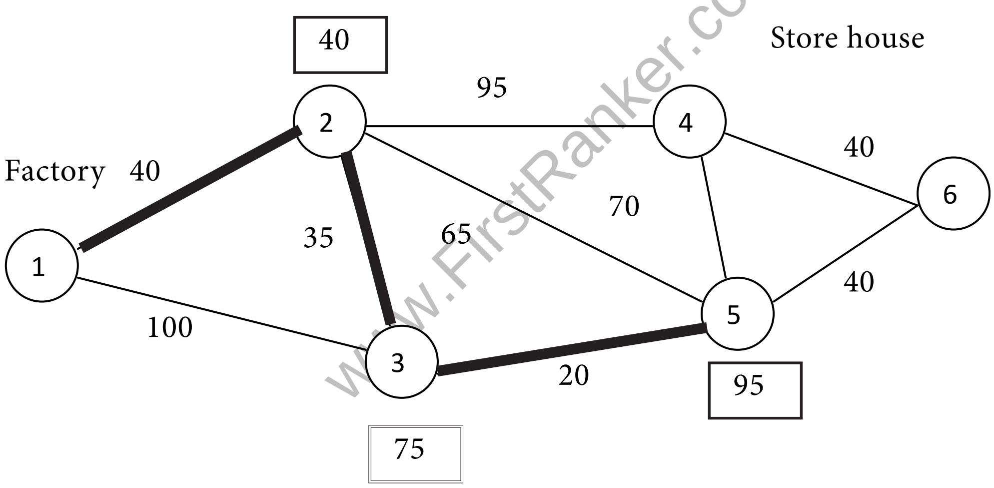
Iteration No. 2

Repeating The Process

We repeat the process. The next node nearest to the set {1, 2, 3} is either node 4 or node 5.

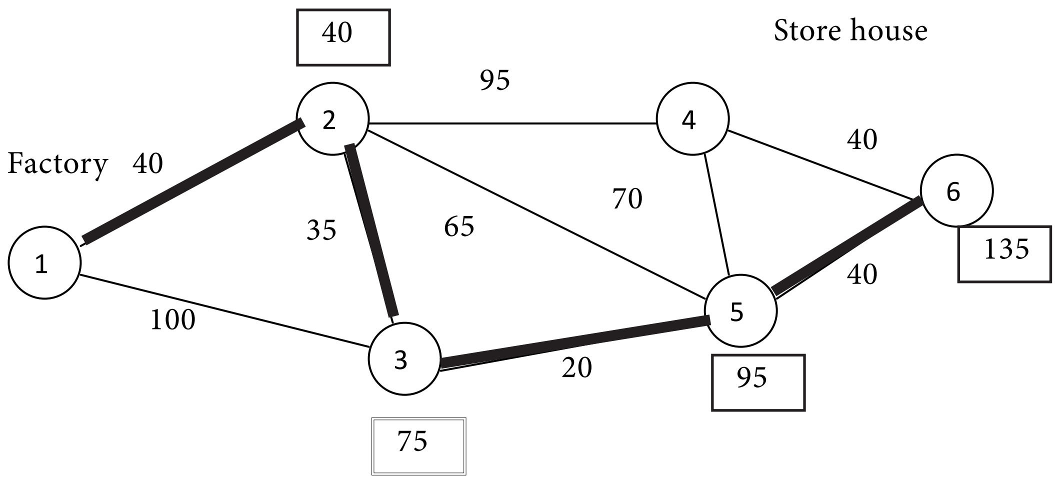
Node 4 is at a distance of 95 units from node 2 while node 2 is at a distance of 40 units from node 1. Thus, node 4 is at a distance of 95 + 40 = 135 units from the origin.

As regards node 5, there are two paths viz. 2 → 5 and 3 → 5, providing a link to the origin. We already know the shortest routes from nodes 2 and 3 to the origin. The minimum distances have been indicated in boxes near these nodes. The path 3 → 5 involves the shortest distance. Thus, the distance between nodes 1 and 5 is 95 units (20 units between nodes 5 and 3 + 75 units between node 3 and the origin). Therefore, we select node 5 and enlarge the set from {1, 2, 3} to {1, 2, 3, 5}. The distance 95 is marked in a box by the side of node 5. The following diagram is obtained at the end of Iteration No. 3.



Iteration No. 3

Now 2 nodes remain, viz., nodes 4 and 6. Among them, node 4 is at a distance of 135 units from the origin (95 units from node 4 to node 2 + 40 units from node 2 to the origin). Node 6 is at a distance of 135 units from the origin (40 + 95 units). Therefore, nodes 4 and 6 are at equal distances from the origin. If we choose node 4, then travelling from node 4 to node 6 will involve an additional distance of 40 units. However, node 6 is the ending node. Therefore, we select node 6 instead of node 4. Thus the set is enlarged from {1, 2, 3, 5} to {1, 2, 3, 5, 6}. The distance 135 is marked in a box by the side of node 6. Since we have got a path beginning from the start node and terminating with the stop node, we see that the solution to the given problem has been obtained. We have the following diagram at the end of Iteration No. 4.



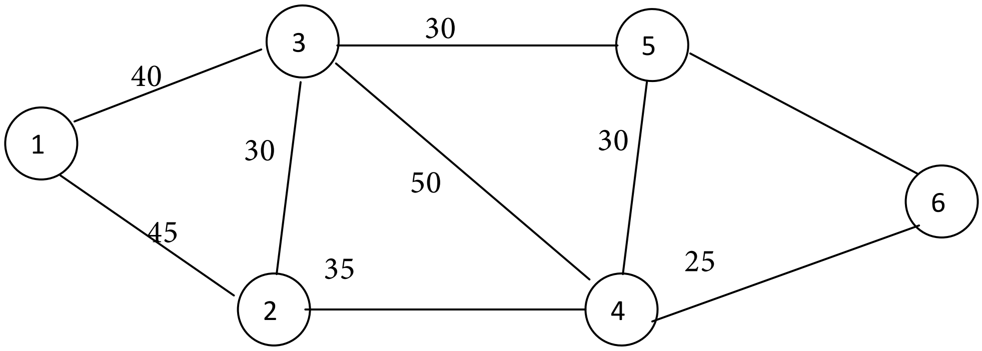
Iteration No. 4

Minimum Distance

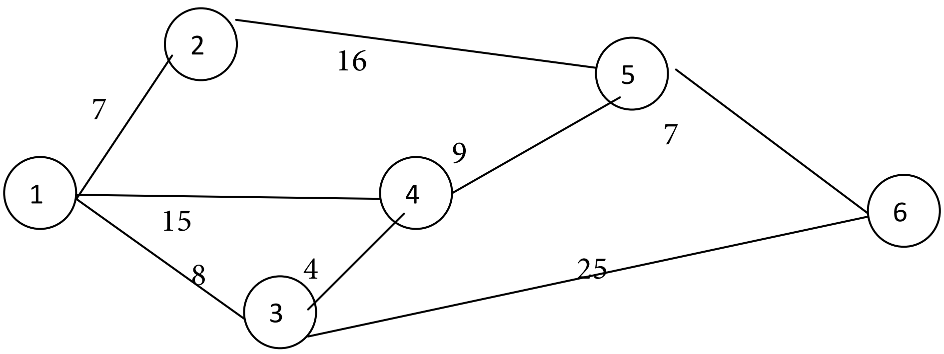
Referring to the above diagram, we see that ***the shortest route*** is provided by the path 1 → 2 → 3 → 5 → 6 with a minimum distance of 135 units.

QUESTIONS

1. Explain the shortest path problem.
2. Explain the algorithm for a shortest path problem
3. Find the shortest path of the following network:



4. Determine the shortest path of the following network:



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Lesson 2 Minimum Spanning Tree Problem

Lesson Outline

- The description of a minimum spanning tree problem.
- The identification of the minimum spanning tree.

Learning Objectives

After reading this lesson you should be able to

- Understand a minimum spanning tree problem
- Understand the algorithm for minimum spanning tree problem
- Locate the minimum spanning tree
- Carry out numerical problems

Tree: A minimally connected network is called a tree. If there are n nodes in a network, it will be a tree if the number of edges = n-1.

Minimum spanning tree algorithm

Problem : Given a connected network with weights assigned to the edges, it is required to find out a tree whose nodes are the same as those of the network.

The weight assigned to an edge may be regarded as the distance between the two nodes with which the edge is incident.

Algorithm

The problem can be solved with the help of the following algorithm. The procedure consists of selection of a node at each step.

Step 1

First select any node in the network. This can be done arbitrarily. We will start with this node.

Step 2

Connect the selected node to the nearest node.

Step 3

Consider the nodes that are now connected. Consider the remaining nodes. If there is no node remaining, then stop. On the other hand, if some nodes remain, among them find out which one is nearest to the nodes that are already connected. Select this node and go to Step 2.

Thus the method involves the repeated application of Steps 2 and 3. Since the number of nodes in the given network is finite, the process will end after a finite number of steps. The algorithm will terminate with step 3.

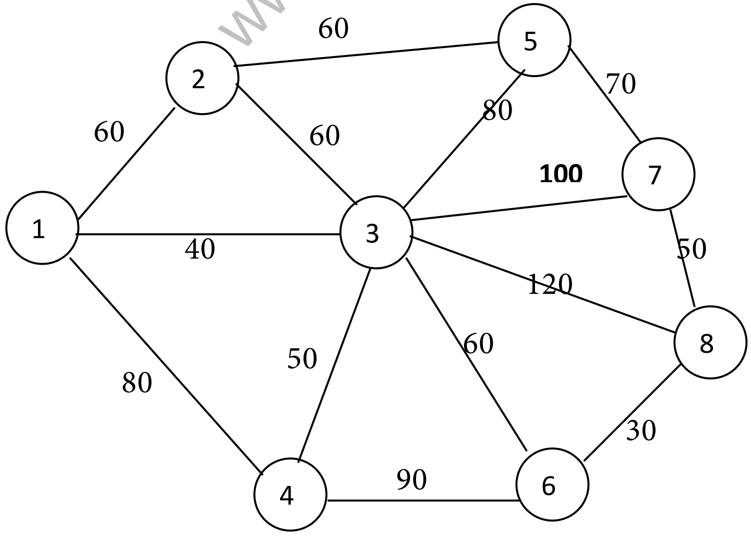
How to break ties

While applying the above algorithm, if some nodes remain in step 3 and if there is a tie in the nearest node, then the tie can be broken arbitrarily.

As a consequence of tie, we may end up with more than one optimal solution.

Problem 1

Determine the minimum spanning tree for the following network.



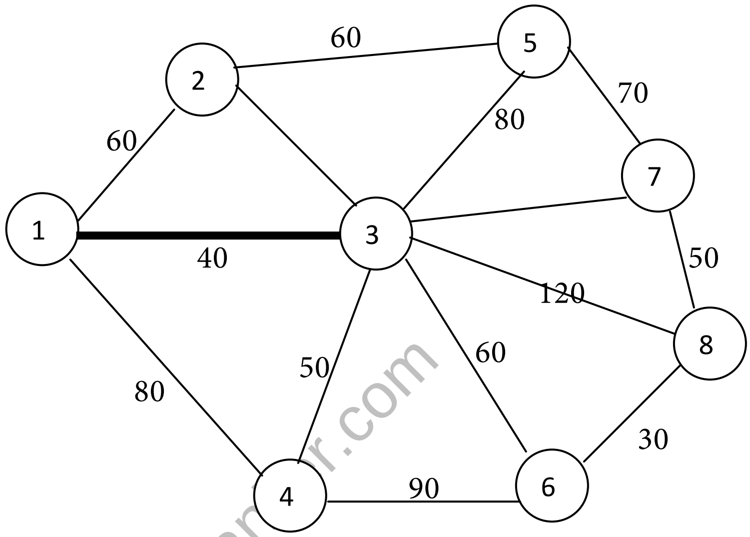
Solution

Step 1

First select node 1. (This is done arbitrarily)

Step 2

We have to connect node 1 to the nearest node. Nodes 2, 3 and 4 are adjacent to node 1. They are at distances of 60, 40 and 80 units from node 1. Minimum of {60, 40, 80} = 40. Hence the shortest distance is 40. This corresponds to node 3. So we connect node 1 to node 3 by a thick line. This is Iteration No. 1.

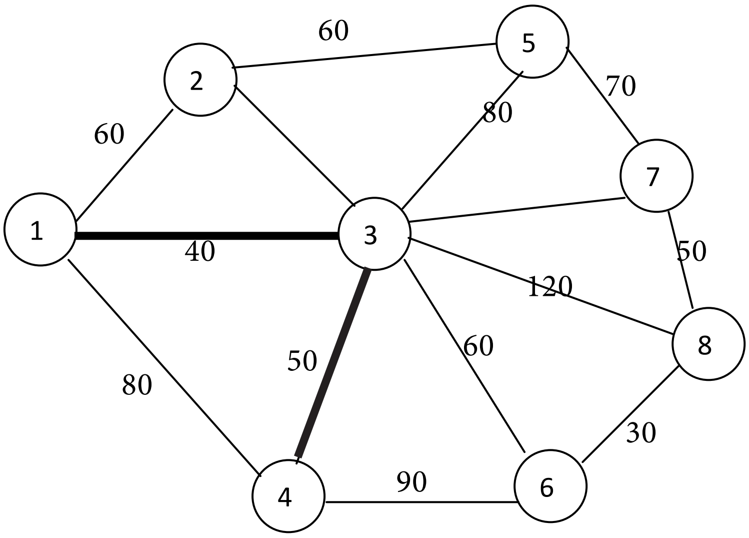


Iteration No. 1

Step 3

Now the connected nodes are 1 and 3. The remaining nodes are 2, 4, 5, 6, 7 and 8. Among them, nodes 2 and 4 are connected to node 1. They are at distances of 60 and 80 from node 1. Minimum of {60, 80} = 60. So the shortest distance is 60. Next, among the nodes 2, 4, 5, 6, 7 and 8, find out which nodes are connected to node 3. We find that all of them are connected to node 3. They are at distances of 60, 50, 80, 60, 100 and 120 from node 3. Minimum of {60, 50, 80, 60, 100, 120} = 50. Hence the shortest distance is 50.

Among these nodes, it is seen that node 4 is nearest to node 3.
Now we go to Step 2. We connect node 3 to node 4 by a thick line. This is Iteration No.2.

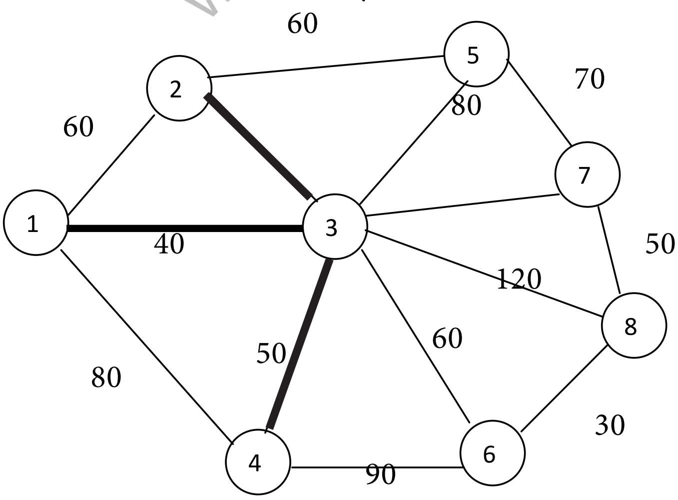


Iteration No. 2

Next go to step 3.

Now the connected nodes are 1, 3 and 4. The remaining nodes are 2, 5, 6, 7 and 8. Node 2 is at a distance of 60 from node 1. Nodes 5, 6, 7 and 8 are not adjacent to node 1. All of the nodes 2, 5, 6, 7 and 8 are adjacent to node 3. Among them, nodes 2 and 6 are nearer to node 3, with equal distance of 60.

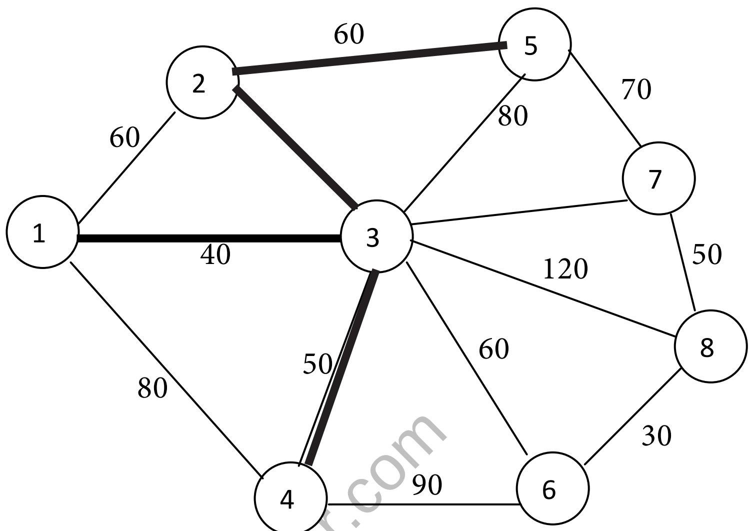
Node 6 is adjacent to node 4, at a distance of 90. Now there is a tie between nodes 2 and 6. The tie can be broken arbitrarily. So we select node 2. Connect node 3 to Node 2 by a thick line. This is Iteration No. 3.



Iteration No. 3

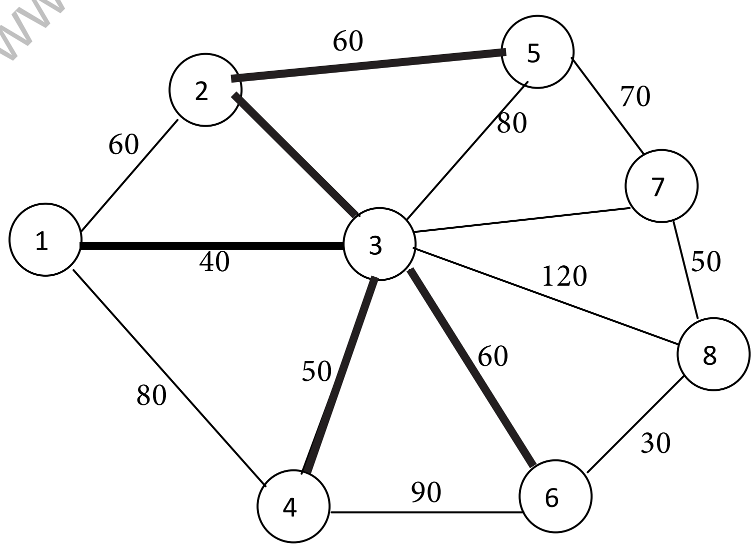
We continue the above process

Now nodes 1, 2, 3 and 4 are connected. The remaining nodes are 5, 6, 7 and 8. None of them is adjacent to node 1. Node 5 is adjacent to node 2 at a distance of 60. Node 6 is at a distance of 60 from node 3. Node 6 is at a distance of 90 from node 4. There is a tie between nodes 5 and 6. We select node 5. Connect node 2 to node 5 by a thick line. This is Iteration No. 4.



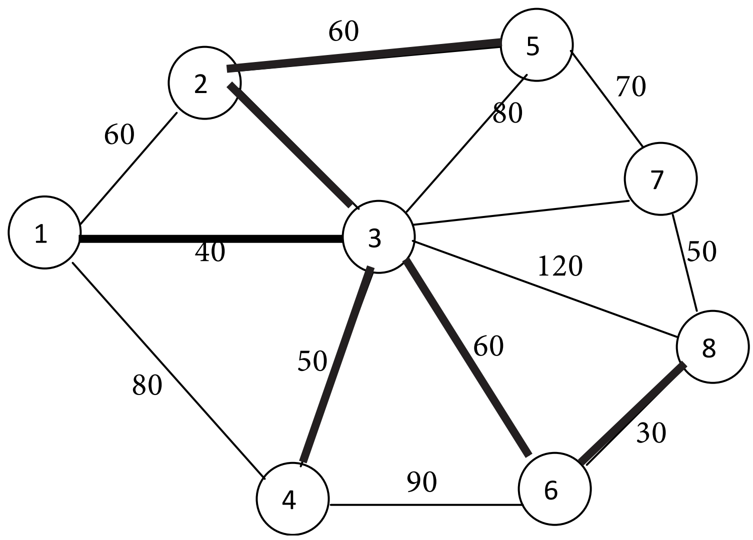
Iteration No. 4

Now nodes 1, 2, 3, 4 and 5 are connected. The remaining nodes are 6, 7 and 8. Among them, node 6 is at the shortest distance of 60 from node 3. So, connect node 3 to node 6 by a thick line. This is Iteration No. 5.



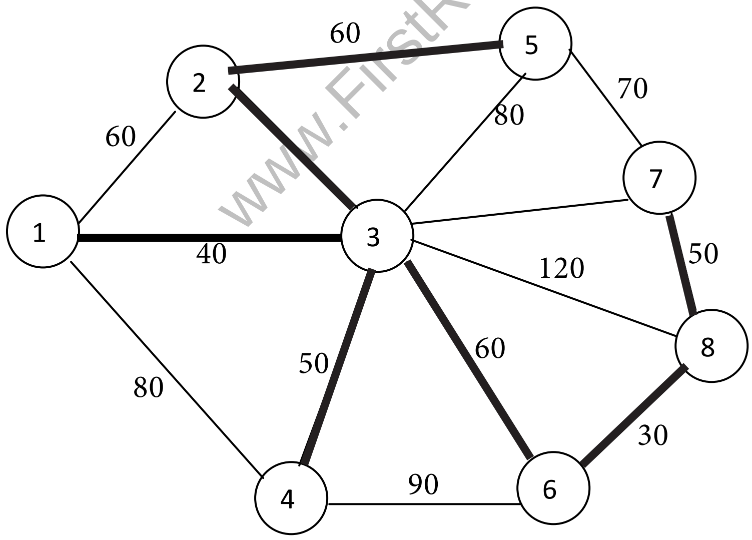
Iteration No. 5

Now nodes 1, 2, 3, 4, 5 and 6 are connected. The remaining nodes are 7 and 8. Among them, node 8 is at the shortest distance of 30 from node 6. Consequently we connect node 6 to node 8 by a thick line. This is Iteration No. 6.



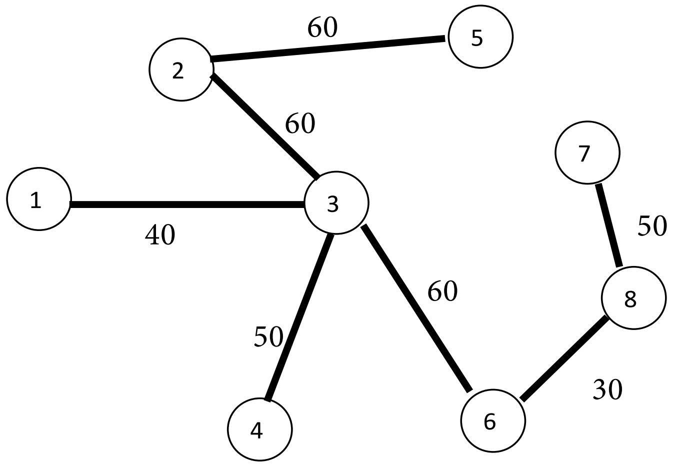
Iteration No. 6

Now nodes 1, 2, 3, 4, 5, 6 and 8 are connected. The remaining node is 7. It is at the shortest distance of 50 from node 8. So, connect node 8 to node 7 by a thick line. This is Iteration No.7.



Iteration No. 7

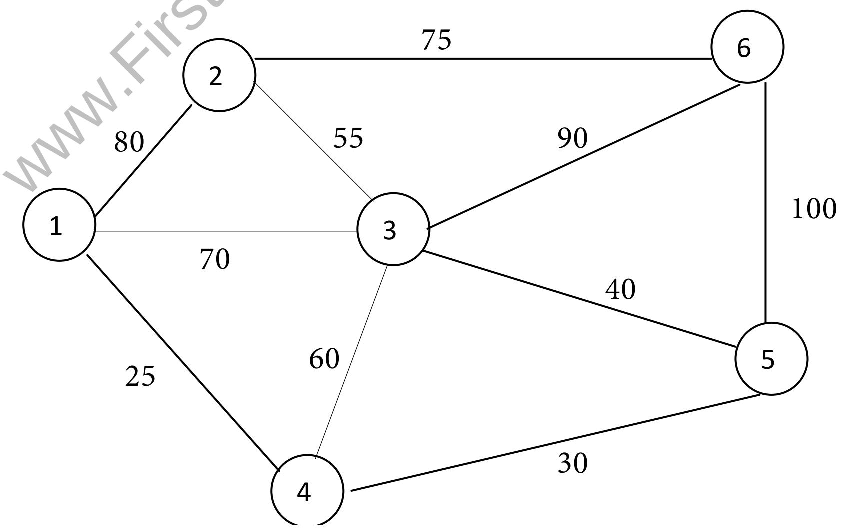
Now all the nodes 1, 2, 3, 4, 5, 6, 7 and 8 are connected by seven thick lines. Since no node is remaining, we have reached the stopping condition. Thus we obtain the following minimum spanning tree for the given network.



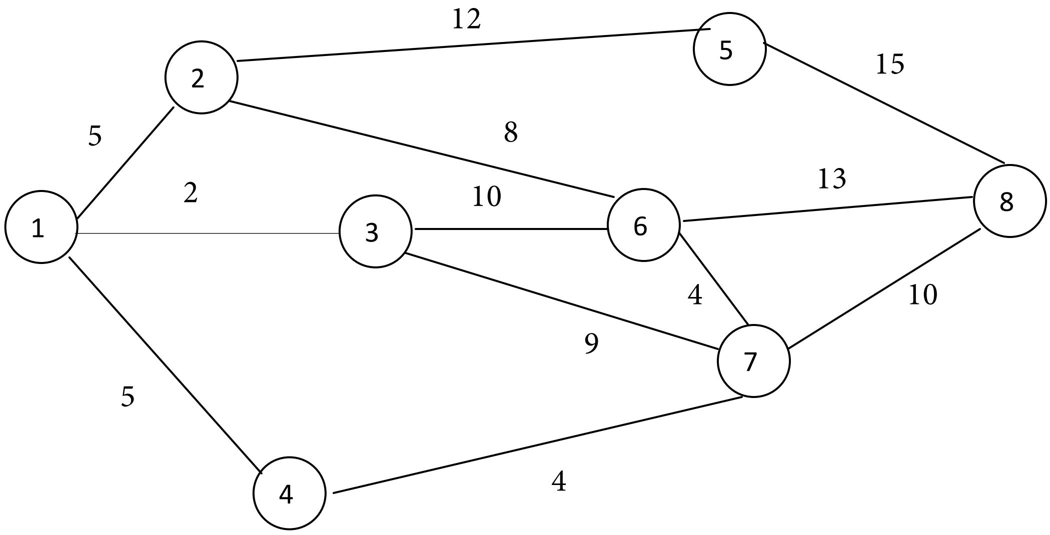
Minimum Spanning Tree

Questions

1. Explain the minimum spanning tree algorithm.
2. From the following network, find the minimum spanning tree.



3. Find the minimum spanning tree of the following network:



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Lesson 3 - Project Network

Lesson Outline

- The key concepts
- Construction of project network diagram

Learning Objectives

After reading this lesson you should be able to

- Understand the definitions of important terms
- Understand the development of project network diagram
- Work out numerical problems

Key Concepts

Certain key concepts pertaining to a project network are described below:

1.Activity

An activity means a work. A project consists of several activities. An activity takes time. It is represented by an arrow in a diagram of the network. For example, an activity in house construction can be flooring. This is represented as follows:

$$\xrightarrow[\text{flooring}]{\hspace{5cm}}$$

Construction of a house involves various activities. Flooring is an activity in this project. We can say that a project is completed only when all the activities in the project are completed.

2.Event

It is the beginning or the end of an activity. Events are represented by circles in a project network diagram. The events in a network are called the **nodes.**

Example

Starting a punching machine is an activity. Stopping the punching machine is another activity.

3.Predecessor Event

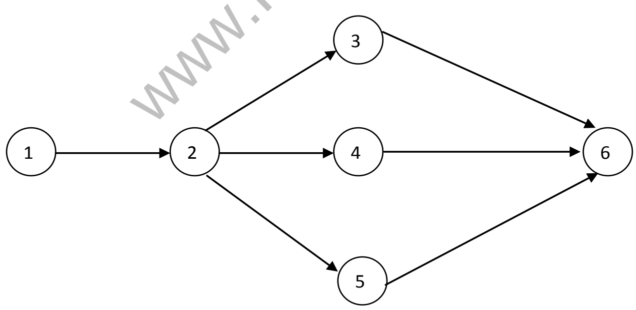
The event just before another event is called the predecessor event.

4.Successor Event

The event just following another event is called the successor event.

Example:

Consider the following.



In this diagram, event 1 is predecessor for the event 2.
Event 2 is successor to event 1.
Event 2 is predecessor for the events 3, 4 and 5.
Event 4 is predecessor for the event 6.
Event 6 is successor to events 3, 4 and

5.**Network**

A network is a series of related activities and events which result in an end product or service. The activities shall follow a prescribed sequence. For example, while constructing a house, laying the foundation should take place before the construction of walls. Fitting water tapes will be done towards the completion of the construction. Such a sequence cannot be altered.

6.Dummy Activity

A dummy activity is an activity which does not consume any time. Sometimes, it may be necessary to introduce a dummy activity in order to provide connectivity to a network or for the preservation of the logical sequence of the nodes and edges.

7.Construction of a Project Network

A project network consists of a finite number of events and activities, by adhering to a certain specified sequence. There shall be a **start event and an end event (or stop event)**. All the other events shall be between the start and the end events. The activities shall be marked by directed arrows. An activity takes the project from one event to another event.

An event takes place at a point of time whereas an activity takes place from one point of time to another point of time.

Construction Of Project Network Diagrams

Problem 1

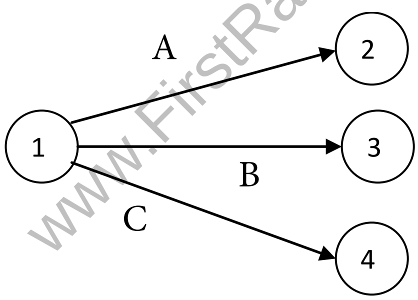
Construct the network diagram for a project with the following activities:

Activity Event Event	Name of Activity	Immediate Predecessor Activity
1→2	A	-
1→3	B	-
1→4	C	-
2→5	D	A
3→6	E	B
4→6	F	C
5→6	G	D

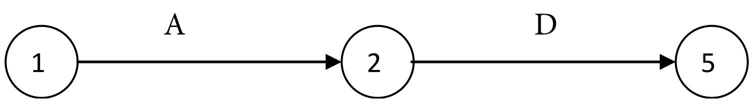
Solution

The start event is node 1.

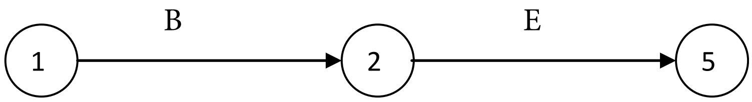
The activities A, B, C start from node 1 and none of them has a predecessor activity. A joins nodes1 and 2; B joins nodes 1 and 3; C joins nodes 1 and 4. So we get the following:



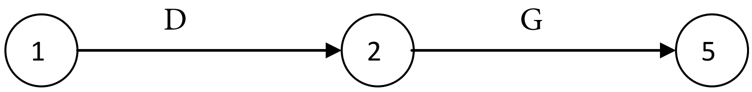
This is a part of the network diagram that is being constructed. Next, activity D has A as the predecessor activity. D joins nodes 2 and 5. So we get



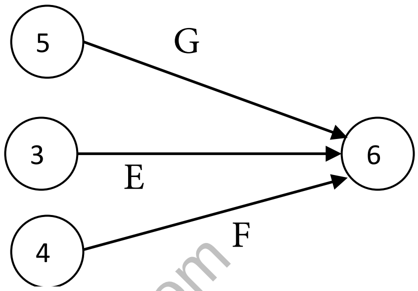
Next, activity E has B as the predecessor activity. E joins nodes 3 and 6. So we get



Next, activity G has D as the predecessor activity. G joins nodes 5 and 6. Thus we obtain

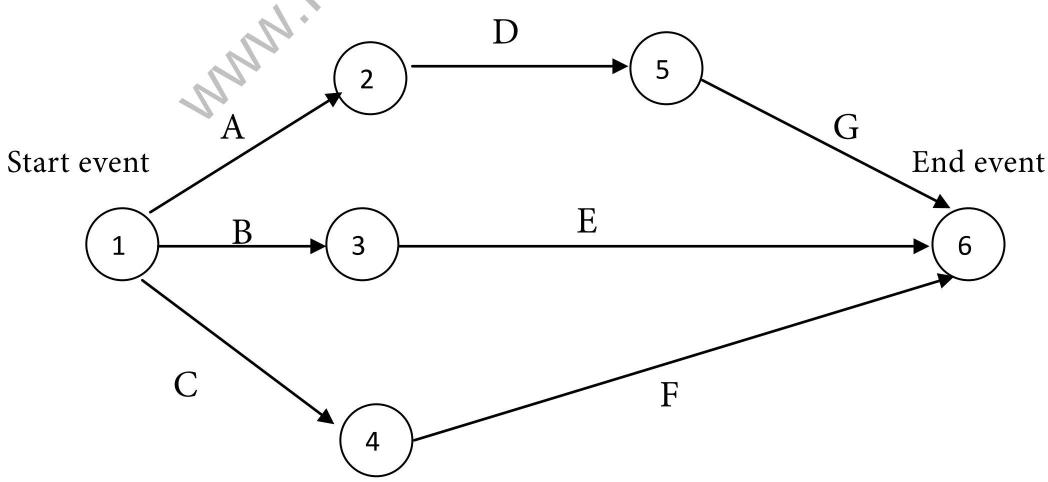


Since activities E, F, G terminate in node 6, we get



6 is the end event.

Combining all the pieces together, the following network diagram is obtained for the given project:



We validate the diagram by checking with the given data.