Model Question Paper-2 with effect from 2018-19
(CBCS Scheme)

USN | | | | | | | | | | | **18MAT11**

First Semester B.E. Degree Examination
Calculus and Linear Algebra

(Common to all Branches)

Time: 3 Hrs Max.Marks: 100

Note: Answer any FIVE full questions, choosing at least ONE question from each module.

Module-1

1. (a) With usual notation, prove that $\tan\phi = r[d\theta/dr]$. **(06 Marks)**

   (b) Find the radius of curvature at the point $(3a, 3a)$ on the curve $x^3 + y^3 = 3axy$. **(06 Marks)**

   (c) Show that the evolute of the ellipse: $x^2/a^2 + y^2/b^2 = 1$ is $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$ **(08 Marks)**

OR

2. (a) Find the pedal equation of the curve : $l/r = 1 + e\cos\theta$. **(06 Marks)**

   (b) Find the radius of curvature for the curve $\theta = \left[\sqrt{r^2 - a^2}/a\right] + \cos^{-1}[a/r]$ at any point on it. **(06 Marks)**

   (c) Show that the angle between the pair of curves: $r^2\sin 2\theta = 4$ & $r^2 = 16\sin 2\theta$ is $\pi/3$. **(08 Marks)**

Module-2

3. (a) Using Maclaurin's series, prove that $\log(\sec x + \tan x) = x + \frac{x^3}{6} + \frac{x^5}{24} + \ldots$ **(06 Marks)**

   (b) Evaluate (i) $\lim_{x\to a}[2 - (x/a)]^{\tan\left(\frac{\pi x}{2a}\right)}$ (ii) $\lim_{x\to 0}[(1/x)]^{2\sin x}$ **(07 Marks)**

   (c) Examine the function $f(x, y) = 2(x^2 - y^2) - x^4 + y^4$ for its extreme values. **(07 Marks)**

OR

4. (a) If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, show that $(1/2)u_x + (1/3)u_y + (1/4)u_z = 0$ **(06 Marks)**

   (b) If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$, show that $J[(x, y, z)/(r, \theta, \phi)] = r^2\sin\theta$. **(07 Marks)**

   (c) A rectangular box, open at the top, is to have a volume of 32 cubic ft. Find the dimension of the box requiring least material for its construction. **(07 Marks)**

Module-3

5. (a) Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$ by changing into polar coordinates. **(06 Marks)**

   (b) Find the volume the tetrahedron bounded by the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and the coordinate planes, using double integration. **(07 Marks)**

   (c) Show that $\int_0^\infty \frac{e^{-x^2}dx}{\sqrt{x}} \times \int_0^\infty \sqrt{x}\, e^{-x^2}dx = \frac{\pi}{2\sqrt{2}}$ **(07 Marks)**

OR

6. (a) Evaluate: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$ **(06 Marks)**

(b) Find by double integration, the centre of gravity of the area of the cardioid: $r = a(1+\cos\theta)$. **(07 Marks)**

(c) With usual notations, show that $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}} dx = \beta(m,n)$. **(07 Marks)**

Module-4

7. (a) A copper ball originally at 80^0C cools down to 60^0C in 20 minutes, if the temperature of the air being 40^0C. What will be the temperature of the ball after 40 minutes from the original? **(06 Marks)**

(b) Find the orthogonal trajectories of the family of curves $r^n \cos n\theta = a^n$, where a is a parameter. **(07 Marks)**

(c) Solve : $[3x^2y^4 + 2xy]dx + [2x^3y^3 - x^2]dy = 0$. **(07 Marks)**

OR

8. (a) Solve the differential equation $L[di/dt] + Ri = 200\sin 300t$, when $L = 0.05$ & $R = 100$ and find the value of the current i at any time t, if initially there is no current in the circuit. What value does i approach after a long time. **(06 Marks)**

(b) Solve : $[r\sin\theta - r^2]d\theta - [\cos\theta]dr = 0$. **(07 Marks)**

(c) Solve: $p^4 - [x+2y+1]p^3 + [x+2y+2xy]p^2 - 2xyp = 0$, where $p = dy/dx$. **(07 Marks)**

Module-5

9. (a) For what values λ and μ the system of equations $x+y+z=6;\ x+2y+3z=10;\ x+2y+\lambda z=\mu$, has (a) no solution (b) a unique solution and (iii) infinite number of solutions. **(06 Marks)**

(b) Using Rayleigh's power method, find largest eigen value and eigen vector of the matrix:

$\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by taking $X^{(0)} = [1,0,0]^T$ as initial eigen vector.(Perform 7 iterations) **(07 Marks)**

(c) Use Gauss-Jordan method solve the system of equations:
$83x + 11y - 4z = 95;\ 7x + 52y + 13z = 104; 3x + 8y + 29z = 71$ **(07 Marks)**

OR

10. (a) Find the rank of the matrix $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by applying elementary row operations. **(06 Marks)**

(b) Reduce the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ into the diagonal form. **(06 Marks)**

(c) Solve the system of equations $2x - 3y + 20z = 25;\ 20x + y - 2z = 17;\ 3x + 20y - z = -18$, using Gauss-Seidel method. (Carry out 4 iterations). **(07 Marks)**

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