Model Question Paper-1 with effect from 2018-19
(CBCS Scheme)

USN | | | | | | | | | | | **18MAT11**

First Semester B.E. Degree Examination
Calculus and Linear Algebra
(Common to all Branches)

Time: 3 Hrs Max.Marks: 100

Note: Answer any FIVE full questions, choosing at least ONE question from each module.

Module-1

1. (a) With usual notation, prove that $1/p^2 = 1/r^2 + 1/r^4 [dr/d\theta]^2$. **(06 Marks)**

   (b) For the cardiod : $r = a(1-\cos\theta)$, show that ρ^2/r is constant. **(06 Marks)**

   (c) Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x-2a)^3$. **(08 Marks)**

OR

2. (a) Find the pedal equation of the curve : $r^m = a^m(\cos m\theta + \sin m\theta)$. **(06 Marks)**

   (b) Show that the radius of curvature for the catenary $y = c\cosh(x/c)$ at any point (x, y) varies as square of the ordinate at that point. **(06 Marks)**

   (c) Show that the angle between the pair of curves: $r = a\log\theta$ & $r = a/\log\theta$ is $2\tan^{-1} e$. **(08 Marks)**

Module-2

3. (a) Using Maclaurin's series, prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{24} + \ldots$ **(06 Marks)**

   (b) Evaluate (i) $\lim_{x\to 0}\left[\left(a^x + b^x + c^x\right)/3\right]^{1/x}$ (ii) $\lim_{x\to\pi/2}[\cos x]^{(\pi/2)-x}$. **(07 Marks)**

   (c) Examine the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ for its extreme values. **(07 Marks)**

OR

4. (a) Find du/dt at $t = 0$, if $u = e^{x^2+y^2+z^2}$ and $x = t^2 + 1$, $y = t\cos t$, $z = \sin t$. **(06 Marks)**

   (b) If $u = yz/x, v = zx/y, w = xy/z$, then show that $\partial(u,v,w)/\partial(x,y,z) = 4$. **(07 Marks)**

   (c) Find the maximum and minimum distances of the point $(1,2,3)$ from the sphere $x^2 + y^2 + z^2 = 56$. **(07 Marks)**

Module-3

5. (a) Evaluate : $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dydxdz$

   (b) Find by double integration the area lying between the circle $x^2 + y^2 = a^2$ and the line $x + y = a$ in the first quadrant. **(06 Marks)**

   (c) Show that $\int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta = \pi$ **(07 Marks)**

OR

6. (a) Change the order of integration and hence evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dxdy$. **(06 Marks)**

(b) A pyramid is bounded by three coordinate planes and the plane $x + 2y + 3z = 6$. Compute the volume by double integration. **(07 Marks)**

(c) Evaluate: $\int_0^1 x^{3/2}(1-x)^{1/2}\, dx$, by expressing in terms beta & gamma functions. **(07 Marks)**

Module-4

7. (a) If the temperature of the air is $30^0 C$ and a metal ball cools from $100^0 C$ to $70^0 C$ in 15 minutes, find how long will it take for the metal ball to reach a temperature of $40^0 C$. **(06 Marks)**

(b) Find the orthogonal trajectories of the family of curves $[x^2/a^2]dx + [y^2/(b^2+\lambda^2)]dy = 1$, where λ is a parameter. **(07 Marks)**

(c) Solve : $[y^4 + 2y]dx + [xy^3 + 2y^4 - 4x]dy = 0.$ **(07 Marks)**

OR

8. (a) The current i in an electrical circuit containing an inductance L anda resistance R in series and, acted upon an e.m.f. $E \sin \omega t$ satisfies the differential equation $L[di/dt] + Ri = E \sin \omega t$. Find the value of the current at any time t, if initially there is no current in the circuit. **(06 Marks)**

(b) Solve: $dy + [x \sin 2y - x^3 \cos^2 y]dx = 0$ **(07 Marks)**

(c) Find the general and singular solution of $[px - y][x - py] = 2p$, by using the substitution $x^2 = u$ & $y^2 = v$ **(07 Marks)**

Module-5

9. (a) Find the rank of the matrix $\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$ by applying elementary row operations. **(06 Marks)**

(b) Using Rayleigh's power method, find largest eigen value and eigen vector of the matrix: $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ by taking $X^{(0)} = [1,0,0]^T$ as initial eigen vector.(Perform 7 iterations) **(07 Marks)**

(c) Use Gauss-Jordan method solve the system of equations: $x + 4y - z = -5;\ x + y - 6z = -12; 3x - y - z = 4$ **(07 Marks)**

OR

10. (a) For what values λ and μ the system of equations $x + 2y + 3z = 6;\ x + 3y + 5z = 9;\ 2x + 5y + \lambda z = \mu$, has (a) no solution (b) a unique solution and (iii) infinite number of solutions. **(06 Marks)**

(b) Reduce the matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ into the diagonal form. **(07 Marks)**

(c) Solve the system of equations $7x + 52y + 13z = 104;\ 3x + 8y + 29z = 71;\ 83x + 11y - 4z = 95$, using Gauss-Seidel method. (Carry out 4 iterations). **(07 Marks)**

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