USN 18MAT11

0.046

First Semester B.E. Degree Examination, Deg.2018/Jan.2019
Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Show that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ are intersect orthogonally. (06 Marks)

b. Find the radius of curvature of the curve $y = a \log \sec(\frac{x}{a})$ at any point (x, y). (06 Marks)

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$. (08 Marks)

OR

2 a. With usual notation, prove that $\tan\phi = r\frac{d\theta}{dr}$. (06 Marks)

b. Find the pedal equation of the curve $r = ae^{\theta\cot\alpha}$ (06 Marks)

c. Find the radius of curvature for the curve $r = a(1 + \cos\theta)$. (08 Marks)

Module-2

3 a. Using Maclaurin's expansion. Prove that $\sqrt{1 + \sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} \ldots$ (06 Marks)

b. Evaluate $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x + d^x}{4}\right)^{\frac{1}{x}}$ (07 Marks)

c. Find the dimensions of the rectangular box open at the top of maximum capacity whose surface is 432 sq.cm. (07 Marks)

OR

4 a. If $u = f(y - z, z - x, x - y)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (06 Marks)

b. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$. Find Jacobian $= \frac{\partial(u,v,w)}{\partial(x,y,z)}$. (07 Marks)

c. Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$. (07 Marks)

Important Note

Module-3

5 a. Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)} dxdy$, by changing into polar coordinates. (06 Marks)

b. Find the volume of the tetrahedron bounded by the planes :

$x = 0,\ y = 0,\ z = 0,\ \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.$ (07 Marks)

c. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ (07 Marks)

OR

6 a. Evaluate $\int_0^{1}\int_x^{\sqrt{x}} xy\, dy\, dx$ by change of order of integration. (06 Marks)

b. Evaluate $\int_{-1}^{1}\int_0^{z}\int_{x-z}^{x+z} (x + y + z)dy\, dx\, dz$. (07 Marks)

c. Prove that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}}\, d\theta = \pi$. (07 Marks)

Module-4

7 a. A body in air at 25°C cools from 100°C to 75°C in 1 minute, find the temperature of the body at the end of 3 minutes. (06 Mark

b. Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$ (07 Marks)

c. Solve $xyp^2 - (x^2 + y^2)p + xy = 0$. (07 Marks)

OR

8 a. Solve $\frac{dy}{dx} + y\tan x = y^3 \sec x$. (06 Marks)

b. Show that the family of parabolas $y^2 = 4a(x + a)$ is self orthogonal. (07 Marks)

c. Find the general solution of the equation $(px - y)(py + x) = 0$ by reducing into Clairaut's from, taking the substitution $X = x^2$, $Y = y^2$. (07 Marks)

Module-5

9 a. Find the rank of the matrix :

$$A = \begin{vmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{vmatrix}$$

(07 Marks)

b. Solve the system of equations

$12x + y + z = 31$
$2x + 8y\ z = 24$
$3x + 4y + 10z = 58$

By Gauss —Siedal method. **(07 Marks)**

c. Diagonalize the matrix :

$$A = \begin{vmatrix} -1 & 3 \\ -2 & 4 \end{vmatrix}$$

(06 Marks)

OR

10 a. For what values of X, and M the system of equations :

$x + 2y + 3z = 6$
$x + 3y + 5z = 9$
$2x + 5y + \ = M$

has i) no solution ii) a unique solution iii) infinite number of solution. **(07 Marks)**

b. Find the largest eigen value and the corresponding eigen vector of :

$$A = \begin{vmatrix} 6 & -2 & \\ -2 & 3 & - \\ 2 & -1 & 3 \end{vmatrix}$$

by Rayleigh's power method, use [1 1 as the initial eigen vector (carry out 6 iterations). **(07 Marks)**

Solve the system of`equations :

$x + y + z = 9$
$2x + y - z = 0$
$2x + 5y + 7z = 52$

By Gauss elimination method. **(06 Marks)**