

USN

15MAT11

**First Semester B.E. Degree Examination, June/July 2019**  
**Engineering Mathematics - I**

Time: 3 hrs.

Max. Marks: 80

**Note: Answer any FIVE full questions, choosing ONE full question from each module.**

**Module-1**

- 1 a. Find the  $n^{\text{th}}$  derivative of  $7x + 6$  (05 Marks)  
 $2x^2 + 7x + 6$   
 b. Find the angle between the radius vector and the tangent for the curve  $r'' = a'(\cos\theta + \sin\theta)$ . (05 Marks)  
 c. Show that the radius of curvature at any point  $\theta$  on the cycloid  $x = a(\theta - \sin\theta)$ ,  $y = a(1 - \cos\theta)$  is  $4a \cos(\theta/2)$  (06 Marks)

**OR**

- 2 a. If  $x = \sin t$  and  $y = \cos t$ , prove that  $(1 - x^2)y'' = 2x$  (05 Marks)  
 b. Find the pedal equation of the curve  $r = a \sec 2\theta$ . (05 Marks)  
 c. Prove with usual notation  $\tan \theta = \frac{r}{dr/d\theta}$  (06 Marks)

**Module-2**

- a. Expand  $e^x$  using Maclaurin's series upto third degree term. (05 Marks)  
 b. Evaluate  $\lim_{x \rightarrow 0} \frac{1 - \sin^2 x}{x^2}$  (05 Marks)  
 c. If  $u = e^{(x^2 + y^2)}$  (flax - by), prove that  $b \frac{\partial u}{\partial x} + a \frac{\partial u}{\partial y} = 2abu$  (06 Marks)

**OR**

- 4 a. Expand  $\sin x$  in ascending power of  $t/2$  upto the term containing  $x^4$ . (05 Marks)  
 b. If  $u = \tan^{-1} \frac{y}{x}$ , show that  $x u_x + y u_y = \sin 2u$ . (05 Marks)  
 c. If  $u = yz$ ,  $v = \frac{zx}{y}$ ,  $w = \frac{xy}{z}$ . Find  $\frac{a(u, v, w)}{a(x, y, z)}$  (06 Marks)

**Module-3**

- 5 a. Find the angle between the surfaces  $x^2 + y^2 + z^2 = 9$  and  $x^2 + y^2 - z = 3$  at the point  $(2, -1, 2)$ . (05 Marks)  
 b. Show that  $\mathbf{F} = (y + z)\mathbf{i} + (x + z)\mathbf{j} + (x + y)\mathbf{k}$  is irrotational. Also find a scalar function  $\phi$  such that  $\mathbf{F} = \nabla\phi$ . (05 Marks)  
 c. Prove that  $\nabla \cdot (\phi \mathbf{A}) = \phi(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot \nabla \phi$ . (06 Marks)

**OR**

- 6 a. Prove that  $\text{Curl}(\nabla \times \mathbf{A}) = \nabla(\text{Curl } \mathbf{A}) + \text{grad}(\nabla \cdot \mathbf{A})$  (05 Marks)  
 b. A particle moves along the curve  $C$ ;  $x = t^3 - 4t$ ,  $y = t^2 + 4t$ ,  $z = 8t^2 - 3t^3$  where  $t$  denotes the time. Find the component of acceleration at  $t = 2$  along the tangent. (05 Marks)

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- c. Show that  $F = (2xy^2 + yz)i + (2x^2y + xz + 2yz^2)j + (2y^2z + xy)k$  is a conservative force field. Find its scalar potential. (06 Marks)

**Module-4**

- 7 a. Obtain the reduction formula for  $\int \sin^n x \, dx$  (05 Marks)
- b. Solve  $(y^2 e^{x'''} + 4x^3)dx + (2xy e^{''''} - 3y^2)dy = 0$ . (05 Marks)
- c. Find the orthogonal trajectories of  $r = a(1 + \sin \theta)$ . (06 Marks)

**OR**

- 8 a. Evaluate  $\int_0^2 x^2 \sin 2x - x^2 \, dx$  (05 Marks)
- b. Solve  $(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$ . (05 Marks)
- c. A bottle of mineral water at a room temperature of  $72^\circ\text{F}$  is kept in a refrigerator where the temperature is  $44^\circ\text{F}$ . After half an hour, water cooled to  $61^\circ\text{F}$
- i) What is the temperature of the mineral water in another half an hour?
- ii) How long will it take to cool to  $50^\circ\text{F}$ ? (06 Marks)

**Module-5**

- 9 a. Find the rank of the matrix  $A = \begin{bmatrix} -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$  (05 Marks)
- b. Find the largest eigen value and corresponding eigenvector of the matrix  $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$  by power method taking  $X^{(0)} = [1, 1, 1]$  (05 Marks)
- c. Reduce the matrix  $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$  to the diagonal form. (06 Marks)
- 10 a. Use Gauss elimination method to solve
- $$\begin{aligned} 2x + y + 4z &= 12 \\ 4x + 11y - z &= 33 \\ 8x - 3y + 2z &= 20 \end{aligned}$$
- (05 Marks)
- b. Find the inverse transformation of the following linear transformation.
- $$\begin{aligned} y_1 &= x_1 + 2x_2 + 5x_3 \\ y_2 &= 2x_1 + 4x_2 + 11x_3 \\ y_3 &= -x_1 + 2x_2 \end{aligned}$$
- (05 Marks)
- c. Reduce the quadratic form  $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_2 + 2x_1x_3 + 2x_2x_3$  to the Canonical form. (06 Marks)