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17CS36

Third Semester B.E. Degree Examination, June/July 2019
Discrete Mathematical Structures

Time: 3 hrs. **Max. Marks:** 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-I

1 a. Define tautology. Verify that [p --> (q -+ r)] -> [(p --> q) -> (p -> r)] is a tautology. **(06 Marks)**

b. If statement q has truth value 1, determine all truth value assignments for the primitive statements p, r, s for which the truth value of the statement :

(q [(ipvr) A is]) A [-+ (irAq)] is 1. **(04 Marks)**

c_ Establish the following logical equivalence :

i) pvqv(ipAlqAr)<=>pvqvr

ii) [(ip v lq) -> (p n q A r)] <7> p A q. **(10 Marks)**

OR

2 a. Establish the validity of following arguments :

i) **(lp v** ¯1(1) (r A s) ‘i) u r

r -> t (r A S) --> (p v t)

It ____ q --> (it A s)

.. p -It

q

.. p **(08 Marks)**

b. Let p(x), q(x) and r(x) be the following open statements :

p(x) : x^2 - 7x + 10 = 0 q(x) : x^2 - 2x - 3 = 0 r(x) < O.

Determine truth or falsity of following statements, where universe is all integers. If a statement is false, provide a counter example_

i)Vx[p(x) --> Ir(x)] ii) Vx[q(x) -> r(x)]

iii)]x[q(x) -> r(x)] iv) 3x [p(x) --> r(x)]. **(08 Marks)**

c. Prove that for all integers 'k' and `,C, if le and `C are both even, then k + iL is even and Id. is even by direct proof. **(04 Marks)**

Module-2

3 a. Define well ordering principle and prove the following by mathematical induction :

i) $^{+32}{}_{+B}{}^{E-2}$ + -- -- +$(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

ii) $1*3+2*4+3*5+----+n(n+2)= \frac{n(n+1)(2n+7)}{6}$. **(12 Marks)**

b. Find the coefficients of :

i. x^9y^3 in the expansion of $(2x - 3y)^{12}$

ii. $a^2b^3c^2d^5$ in the expansion of $(a + 2b\ 3c + 2d + \ ^{5)16}$. (08 Marks)

OR

4 a. A women has I I close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in following situations,

i. There is no restriction on the choice
ii. Two particular persons will not attend separately
iii. Two particular persons will not attend together. **(06 Marks)**

b. How many arrangements are there for all letters in word SOCIOLOGICAL? In how many of these arrangements all vowels are adjacent. **(06 Marks)**

c. For the Fibonacci sequence F_0, Fi, F2 ... prove that $k = \begin{vmatrix} 1 & \\ 2 & 2 \end{vmatrix}$ **(08 Marks)**

Module-3

5 a. Let A = {1, 2, 3, 41 and B { I, 2, 3, 4, 5,61.

i. How many functions are there from A to B?
ii. How many of these are one to one?
How many are onto?
iv. How many functions are there from B to A?
v. How many of these are onto?
vi. How many are one to one? **(06 Marks)**

h. A computer operator is given a magnetic tape that contains 500,000 words of four or fewer lowercase letters. Can it be that the 500,000 words are all distinct? **(06 Marks)**

c. Let f, g, h : R R where $f(x) = x^2$, $g(x) = x + 5$ and $h(x) = \sqrt{x^2 + 2}$. Show that (hog) of = ho(got). **(08 Marks)**

OR

6 a. Let A I, 2, 3, 6, 9, 181 and define R on A by xRy if "x divides y", Draw the Hasse diagram for the poset (A, R). Also write the matrix of relation. **(08 Marks)**

b. Consider Poset whose Hasse diagram is given below. Consider B = {3, 4, 51. Find upper and lower bounds of B, least upper bound and greatest lower bound of B. **(04 Marks)** (Ref Fig.Q6(b)).

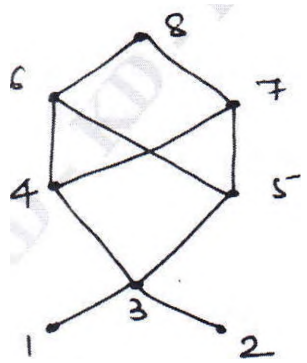

Fig.Q6(b)

c. Let A = I, 2, 3, 4, 51 x 11, 2, 3, 4, 51 and define R on A by (xi R (x2, y2) if + y, = x2 + y2.

i. Verify that R is an equivalence relation on A
ii. Determine equivalence classes [(1, 3)], [(2, 4)] and [(1, 1)]
iii. Determine partition of A induced by R. **(08 Marks)**

Module-4

7 a. In how many ways can the 26 letters of English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? **(08 Marks)**

b. There are eight letters to eight different people to be placed in eight different addressed envelops. Find the number of ways of doing this so that atleast one letter gets to right person. **(04 Marks)**

c. Four persons Pi, P2, P3, P4 who arrive late for a dinner party find that only one chair at each of five table T1, T2, T3, T4 and T_5 is vacant. P_1 will not sit at Ti or T2, P2 will not sit at T2, P3 will not sit at T3 or T4 and P4 will not sit at T4 or T5. Find the number of ways they can occupy the vacant chairs. **(08 Marks**

OR

8 a. Find the recurrence relation and the initial condition for the sequence 0, 2, 6, 12, 20, 30, 42, . . Hence find the general term of the sequence. **(10 Marks)**

b. If ao = 0, al = 1, a, = 4 and a; = 37 satisfy the recurrence relation a_{w}-, + ba_n+i + ca_n = 0 for n ?.. 0, determine the constants b and c and then solve the relation for a_n. (10 **Marks)**

Module-5

9 a. Merge sort the list -1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3. **(06 Marks)**

b. Determine whether the following graphs are isomorphic or not. **(06 Marks)**

Fig.Q9(b)

c. Define the following with an example to each.
i) Simple graph ii) Complete graph iii) Regular graph iv) Spanning sub graph v) Induced subgraph vi) Complete Bipartite graph vii) Tree viii) Complement of graph. **(08 Marks)**

OR

10 a. Define trail, circuit, path, cycle. In the graph shown below determine [Ref.Q10(a)]

i. a walk from b to d that is not a trail
ii. b-d trail that is not a path
a path from b to d
iv. a closed walk from b to b that is not a circuit
v. a circuit from b to b that is not cycle
vi. a cycle form b to b. (10 **Marks)**

Fig.Q10(a)

b. Define optimal tree and construct an optimal tree for a given set of weights {4, 15, 25, 5, 8, 161. Hence find the weight of optimal tree. **(06 Marks)**

c. Prove that in a graph. The sum of degrees of all vertices is an even number and is equal to twice the number of edges in the graph. **(04 Marks)**

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