

Code No: 07A3BS02

R07**Set No. 2**

II B.Tech I Semester Examinations, MAY 2011

MATHEMATICS - III

Common to ICE, E.COMP.E, ETM, E.CONT.E, EIE, ECE, EEE

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Find and plot the map of rectangular region $0 \leq x \leq 1; 0 \leq y \leq 2$, under the transformation $w = \sqrt{2} e^{i\pi/4} z + (1-2i)$.
(b) Find the bilinear transformation that maps the points $0, i, 1$ into the points $-1, 0, 1$. [8+8]
2. (a) Derive Cauchy Riemann equations in polar coordinates.
(b) Prove that the function $f(z) = \bar{z}$ is not analytic at any point.
(c) Find the general and the principal values of (i) $\log_e(1+\sqrt{3}i)$ (ii) $\log_e(-1)$. [5+5+6]
3. (a) Show that $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$.
(b) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.
(c) Show that $(n+1) P_{n+1}(x) - (2n+1)x P_n(x) + n P_{n-1}(x) = 0$. [5+5+6]
4. (a) State and prove Taylor's theorem.
(b) Find the Laurent series expansion of the function $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z+2| < 5$. [8+8]
5. (a) Show that $\beta(m, n) = 2 \int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta$ and deduce that $\int_0^{\pi/2} \sin^n \theta d\theta = \int_0^{\pi/2} \cos^n \theta d\theta = \frac{\frac{1}{2} \Gamma(\frac{n+1}{2}) \Gamma(\frac{1}{2})}{\Gamma(\frac{n+2}{2})}$.
(b) Prove that $\Gamma(n) \Gamma(1-n) = \frac{\pi}{\sin n\pi}$.
(c) Show that $\int_0^{\infty} x^m e^{-a^2 x^2} dx = \frac{1}{2a^{m+1}} \Gamma(\frac{m+1}{2})$ and hence deduce that $\int_0^{\infty} \cos(x^2) dx = \int_0^{\infty} \sin(x^2) dx = 1/2 \sqrt{\pi/2}$. [5+5+6]
6. (a) Evaluate $\int_C \frac{(e^z \sin 2z - 1) dz}{z^2(z+2)^2}$ where C is $|z| = 1/2$ using Cauchy's integral formula.
(b) Evaluate $\int_C \frac{(e^{-2z}) z^2 dz}{(z-1)^3(z+2)}$ where C is $|z+2| = 1$ using Cauchy's integral formula. [8+8]
7. (a) Use method of contour integration to prove that $\int_0^{2\pi} \frac{d\theta}{1+a^2-2a\cos\theta} = \frac{2\pi}{1-a^2}$, $0 < a < 1$.

Code No: 07A3BS02

R07

Set No. 2

(b) Evaluate $\int_0^{\infty} \frac{dx}{(x^2+9)(x^2+4)^2}$ using residue theorem. [8+8]

8. (a) Find the poles and the residues at each pole of $f(z) = \frac{1-e^z}{z^4}$. Where $z=0$ is a pole of order 4

(b) Evaluate $\int_c \frac{(z-3)}{z} (z^2 + 2z + 5) dz$ where c is the circle using residue theorem.

i. $|z| = 1$

ii. $|z+1-i| = 2$.

[6+10]

FIRSTRANKER

Code No: 07A3BS02

R07**Set No. 4**

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1. (a) Show that $\beta(m,n) = 2 \int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta$ and deduce that
- $$\int_0^{\pi/2} \sin^n \theta d\theta = \int_0^{\pi/2} \cos^n \theta d\theta = \frac{\frac{1}{2} \Gamma(\frac{n+1}{2}) \Gamma(\frac{1}{2})}{\Gamma(\frac{n+2}{2})}.$$
- (b) Prove that $\Gamma(n) \Gamma(1-n) = \frac{\pi}{\sin n\pi}$.
- (c) Show that $\int_0^{\infty} x^m e^{-a^2 x^2} dx = \frac{1}{2a^{m+1}} \Gamma(\frac{m+1}{2})$ and hence deduce that
- $$\int_0^{\infty} \cos(x^2) dx = \int_0^{\infty} \sin(x^2) dx = 1/2 \sqrt{\pi/2}. \quad [5+5+6]$$
2. (a) Show that $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$.
- (b) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.
- (c) Show that $(n+1) P_{n+1}(x) - (2n+1)x P_n(x) + n P_{n-1}(x) = 0$. [5+5+6]
3. (a) Use method of contour integration to prove that $\int_0^{2\pi} \frac{d\theta}{1+a^2-2a\cos\theta} = \frac{2\pi}{1-a^2}$, $0 < a < 1$.
- (b) Evaluate $\int_0^{\infty} \frac{dx}{(x^2+9)(x^2+4)^2}$ using residue theorem. [8+8]
4. (a) Evaluate $\int_C \frac{(e^z \sin 2z - 1) dz}{z^2(z+2)^2}$ where C is $|z| = 1/2$ using Cauchy's integral formula.
- (b) Evaluate $\int_C \frac{(e^{-2z})z^2 dz}{(z-1)^3(z+2)}$ where C is $|z+2| = 1$ using Cauchy's integral formula. [8+8]
5. (a) Derive Cauchy Riemann equations in polar coordinates.
- (b) Prove that the function $f(z) = \bar{z}$ is not analytic at any point.
- (c) Find the general and the principal values of (i) $\log_e(1+\sqrt{3}i)$ (ii) $\log_e(-1)$. [5+5+6]
6. (a) Find the poles and the residues at each pole of $f(z) = \frac{1-e^z}{z^4}$. Where $z=0$ is a pole of order 4
- (b) Evaluate $\int_c \frac{(z-3)}{z} (Z^2 + 2Z + 5) dz$ where c is the circle using residue theorem.
- i. $|z| = 1$

Code No: 07A3BS02

R07

Set No. 4

ii. $|z+1-i| = 2$. [6+10]

7. (a) Find and plot the map of rectangular region $0 \leq x \leq 1$; $0 \leq y \leq 2$, under the transformation $w = \sqrt{2} e^{i\pi/4} z + (1-2i)$.
- (b) Find the bilinear transformation that maps the points $0, i, 1$ into the points $-1, 0, 1$. [8+8]
8. (a) State and prove Taylor's theorem.
- (b) Find the Laurent series expansion of the function $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z+2| < 5$. [8+8]

FIRSTRANKER

Code No: 07A3BS02

R07**Set No. 1**

II B.Tech I Semester Examinations, MAY 2011

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2. (a) Show that $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$.
(b) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.
(c) Show that $(n+1) P_{n+1}(x) - (2n+1)x P_n(x) + n P_{n-1}(x) = 0$. [5+5+6]
3. (a) Find and plot the map of rectangular region $0 \leq x \leq 1; 0 \leq y \leq 2$, under the transformation $w = \sqrt{2} e^{i\pi/4} z + (1-2i)$.
(b) Find the bilinear transformation that maps the points 0,i,1 into the points -1,0,1. [8+8]
4. (a) Find the poles and the residues at each pole of $f(z) = \frac{1-e^z}{z^4}$. Where $z=0$ is a pole of order 4
(b) Evaluate $\int_c \frac{(z-3)}{z^2} (Z^2 + 2Z + 5) dz$ where c is the circle using residue theorem.
i. $|z| = 1$
ii. $|z+1-i| = 2$. [6+10]
5. (a) Show that $\beta(m,n) = 2 \int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta$ and deduce that
$$\int_0^{\pi/2} \sin^n\theta d\theta = \int_0^{\pi/2} \cos^n\theta d\theta = \frac{\frac{1}{2} \Gamma(\frac{n+1}{2}) \Gamma(\frac{1}{2})}{\Gamma(\frac{n+2}{2})}$$

(b) Prove that $\Gamma(n) \Gamma(1-n) = \frac{\pi}{\sin n\pi}$.
(c) Show that $\int_0^{\infty} x^m e^{-a^2 x^2} dx = \frac{1}{2a^{m+1}} \Gamma(\frac{m+1}{2})$ and hence deduce that
$$\int_0^{\infty} \cos(x^2) dx = \int_0^{\infty} \sin(x^2) dx = 1/2 \sqrt{\pi/2}$$
. [5+5+6]
6. (a) State and prove Taylor's theorem.
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Code No: 07A3BS02

R07

Set No. 1

7. (a) Evaluate $\int_C \frac{(e^z \sin 2z - 1) dz}{z^2(z+2)^2}$ where C is $|z| = 1/2$ using Cauchy's integral formula.

(b) Evaluate $\int_C \frac{(e^{-2z})z^2 dz}{(z-1)^3(z+2)}$ where C is $|z+2| = 1$ using Cauchy's integral formula.

[8+8]

8. (a) Use method of contour integration to prove that $\int_0^{2\pi} \frac{d\theta}{1+a^2-2a\cos\theta} = \frac{2\pi}{1-a^2}$,
 $0 < a < 1$.

(b) Evaluate $\int_0^\infty \frac{dx}{(x^2+9)(x^2+4)^2}$ using residue theorem. [8+8]

FIRSTRANKER

Code No: 07A3BS02

R07**Set No. 3**

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- (b) Find the bilinear transformation that maps the points $0, i, 1$ into the points $-1, 0, 1$. [8+8]
2. (a) State and prove Taylor's theorem.
- (b) Find the Laurent series expansion of the function $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z+2| < 5$. [8+8]
3. (a) Evaluate $\int_C \frac{(e^z \sin 2z-1) dz}{z^2(z+2)^2}$ where C is $|z| = 1/2$ using Cauchy's integral formula.
- (b) Evaluate $\int_C \frac{(e^{-2z})z^2 dz}{(z-1)^3(z+2)}$ where C is $|z+2| = 1$ using Cauchy's integral formula. [8+8]
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- (b) Evaluate $\int_c \frac{(Z-3)}{Z^2} (Z^2 + 2Z + 5) dz$ where c is the circle using residue theorem.
 - i. $|z| = 1$
 - ii. $|z+1-i| = 2$. [6+10]
5. (a) Derive Cauchy Riemann equations in polar coordinates.
- (b) Prove that the function $f(z) = \bar{z}$ is not analytic at any point.
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- (b) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.
- (c) Show that $(n+1) P_{n+1}(x) - (2n+1)x P_n(x) + n P_{n-1}(x) = 0$. [5+5+6]
7. (a) Show that $\beta(m,n) = 2 \int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta$ and deduce that

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- (b) Prove that $\Gamma(n) \Gamma(1-n) = \frac{\pi}{\sin n\pi}$.

Code No: 07A3BS02

R07

Set No. 3

(c) Show that $\int_0^{\infty} x^m e^{-a^2 x^2} dx = \frac{1}{2a^{m+1}} \Gamma\left(\frac{m+1}{2}\right)$ and hence deduce that

$$\int_0^{\infty} \cos(x^2) dx = \int_0^{\infty} \sin(x^2) dx = 1/2\sqrt{\pi/2}. \quad [5+5+6]$$

8. (a) Use method of contour integration to prove that $\int_0^{2\pi} \frac{d\theta}{1+a^2-2a\cos\theta} = \frac{2\pi}{1-a^2}$,
 $0 < a < 1$.

(b) Evaluate $\int_0^{\infty} \frac{dx}{(x^2+9)(x^2+4)^2}$ using residue theorem. [8+8]

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