Code No: 07A3BS02 **R07** **Set No. 2**

II B.Tech I Semester Examinations,MAY 2011
MATHEMATICS - III
Common to ICE, E.COMP.E, ETM, E.CONT.E, EIE, ECE, EEE

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Find and plot the map of rectangular region $0\leq x\leq 1$; $0\leq y\leq 2$, under the transformation $w = \sqrt{2}\, e^{i\pi/4} z + (1-2i)$.
   (b) Find the bilinear transformation that maps the points 0,i,1 into the points −1,0,1. [8+8]

2. (a) Derive Cauchy Riemann equations in polar coordinates.
   (b) Prove that the function $f(z) = \bar{z}$ is not analytic at any point.
   (c) Find the general and the principal values of (i) $\log_e(1+\sqrt{3}i)$ (ii) $\log_e(-1)$. [5+5+6]

3. (a) Show that $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$.
   (b) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.
   (c) Show that $(n+1) P_{n+1}(x) - (2n+1)\, x\, P_n(x) + n\, P_{n-1}(x) = 0$. [5+5+6]

4. (a) State and prove Taylor's theorem.
   (b) Find the Laurent series expansion of the function $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region $3< |z+2| <5$. [8+8]

5. (a) Show that $\beta(m,n)=2 \int_o^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta$ and deduce that $\int_o^{\pi/2} \sin^n\theta\, d\theta = \int_o^{\pi/2} \cos^n\theta\, d\theta = \frac{\frac{1}{2}\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right)}$.
   (b) Prove that $\Gamma(n)\, \Gamma((1-n) = \frac{\pi}{\sin n\pi}$.
   (c) Show that $\int_0^{\infty} x^m e^{-a^2x^2} dx = \frac{1}{2a^{m+1}} \Gamma\left(\frac{m+1}{2}\right)$ and hence deduce that $\int_o^{\infty} \cos(x^2) dx = \int_o^{\infty} \sin(x^2) dx = 1/2\sqrt{\pi/2}$. [5+5+6]

6. (a) Evaluate $\int_C \frac{(e^z \sin 2z-1)\quad dz}{z^2(z+2)^2}$ where C is $|\,z\,| = 1/2$ using Cauchy's integral formula.
   (b) Evaluate $\int_C \frac{(e^{-2z}) z^2\quad dz}{(z-1)^3(z+2)}$ where C is $|\,z+2\,| = 1$ using Cauchy's integral formula. [8+8]

7. (a) Use method of contour integration to prove that $\int_0^{2\pi} \frac{d\theta}{1+a^2-2acos\theta} = \frac{2\pi}{1-a^2}$, $0< a<1$.

(b) Evaluate $\int_0^\infty \frac{dx}{(x^2+9)(x^2+4)^2}$ using residue theorem. [8+8]

8. (a) Find the poles and the residues at each pole of $f(z) = \frac{1-e^z}{z^4}$. Where z=0 is a pole of order 4

   (b) Evaluate $\int_c \frac{(Z-3}{)}(Z^2+2Z+5)$ dz where c is the circle using residue theorem.

   i. | z | = 1
   ii. | z+1-i | = 2. [6+10]

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Code No: 07A3BS02 **R07** **Set No. 4**

II B.Tech I Semester Examinations,MAY 2011
MATHEMATICS - III
Common to ICE, E.COMP.E, ETM, E.CONT.E, EIE, ECE, EEE

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Show that $\beta(m,n)=2\int\limits_{o}^{\pi/2}\sin^{2m-1}\theta\cos^{2n-1}\theta d\theta$ and deduce that $\int\limits_{o}^{\pi/2}\sin^{n}\theta\, d\theta=\int\limits_{o}^{\pi/2}\cos^{n}\theta\, d\theta=\frac{\frac{1}{2}\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right)}$.

   (b) Prove that $\Gamma(n)\,\Gamma(1-n)=\frac{\pi}{\sin n\pi}$.

   (c) Show that $\int\limits_{0}^{\infty}x^{m}e^{-a^{2}x^{2}}dx=\frac{1}{2a^{m+1}}\Gamma\left(\frac{m+1}{2}\right)$ and hence deduce that $\int\limits_{o}^{\infty}\cos(x^{2})dx=\int\limits_{o}^{\infty}\sin(x^{2})dx=1/2\sqrt{\pi/2}$. [5+5+6]

2. (a) Show that $J_{n-1}(x)+J_{n+1}(x)=\frac{2n}{x}J_{n}(x)$.

   (b) Prove that $J_{-1/2}(x)=\sqrt{\frac{2}{\pi x}}\cos x$.

   (c) Show that $(n+1)P_{n+1}(x)-(2n+1)xP_{n}(x)+nP_{n-1}(x)=0$. [5+5+6]

3. (a) Use method of contour integration to prove that $\int\limits_{0}^{2\pi}\frac{d\theta}{1+a^{2}-2a\cos\theta}=\frac{2\pi}{1-a^{2}}$, $0<a<1$.

   (b) Evaluate $\int\limits_{0}^{\infty}\frac{dx}{(x^{2}+9)(x^{2}+4)^{2}}$ using residue theorem. [8+8]

4. (a) Evaluate $\int\limits_{C}\frac{(e^{z}\sin 2z-1)\ dz}{z^{2}(z+2)^{2}}$ where C is $|z|=1/2$ using Cauchy's integral formula.

   (b) Evaluate $\int\limits_{C}\frac{(e^{-2z})z^{2}\ dz}{(z-1)^{3}(z+2)}$ where C is $|z+2|=1$ using Cauchy's integral formula. [8+8]

5. (a) Derive Cauchy Riemann equations in polar coordinates.

   (b) Prove that the function $f(z)=\bar{z}$ is not analytic at any point.

   (c) Find the general and the principal values of (i) $\log_{e}(1+\sqrt{3}i)$ (ii) $\log_{e}(-1)$. [5+5+6]

6. (a) Find the poles and the residues at each pole of $f(z)=\frac{1-e^{z}}{z^{4}}$. Where z=0 is a pole of order 4

   (b) Evaluate $\int\limits_{c}\frac{(Z-3}{)}(Z^{2}+2Z+5)$ dz where c is the circle using residue theorem.

      i. $|z|=1$

ii. $| z+1-i | = 2$. [6+10]

7. (a) Find and plot the map of rectangular region $0 \leq x \leq 1$; $0 \leq y \leq 2$, under the transformation $w = \sqrt{2}\, e^{i\pi/4} z + (1-2i)$.

   (b) Find the bilinear transformation that maps the points 0,i,1 into the points –1,0,1. [8+8]

8. (a) State and prove Taylor's theorem.

   (b) Find the Laurent series expansion of the function $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z+2| < 5$. [8+8]

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Code No: 07A3BS02 **R07** **Set No. 1**

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MATHEMATICS - III
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Time: 3 hours **Max Marks: 80**
Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Derive Cauchy Riemann equations in polar coordinates.
   (b) Prove that the function f(z) = $\bar{z}$ is not analytic at any point.
   (c) Find the general and the principal values of (i) log $_e$(1+$\sqrt{3}$i) (ii) log $_e$(–1). [5+5+6]

2. (a) Show that J_{n-1} (x) + J_{n+1}(x) = $\frac{2n}{x} J_n(x)$.
   (b) Prove that J$_{-1/2}$(x) = $\sqrt{\frac{2}{\pi x}} \cos x$.
   (c) Show that (n+1) P_{n+1} (x) - (2n+1) x P_n(x) + n P_{n-1} (x) = 0. [5+5+6]

3. (a) Find and plot the map of rectangular region 0≤x≤1; 0≤y≤2, under the transformation w =$\sqrt{}$ 2 e$^{i\pi/4}$z + (1-2i).
   (b) Find the bilinear transformation that maps the points 0,i,1 into the points –1,0,1. [8+8]

4. (a) Find the poles and the residues at each pole of $f(z) = \frac{1-e^z}{z^4}$. Where z=0 is a pole of order 4
   (b) Evaluate $\int_c \frac{(Z-3}{)}(Z^2 + 2Z + 5)$ dz where c is the circle using residue theorem.
      i. | z | = 1
      ii. | z+1-i | = 2. [6+10]

5. (a) Show that β(m,n)=2 $\int_o^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta$ and deduce that
   $$\int_o^{\pi/2} \sin^n\theta\, d\theta = \int_o^{\pi/2} \cos^n\theta\, d\theta = \frac{\frac{1}{2}\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right)}.$$
   (b) Prove that Γ(n) Γ((1-n)= $\frac{\pi}{\sin n\pi}$.
   (c) Show that $\int_0^{\infty} x^m e^{-a^2x^2} dx = \frac{1}{2a^{m+1}}\Gamma\left(\frac{m+1}{2}\right)$ and hence deduce that
   $$\int_o^{\infty} \cos(x^2)dx = \int_o^{\infty} \sin(x^2)dx = 1/2\sqrt{\pi/2}.$$ [5+5+6]

6. (a) State and prove Taylor's theorem.
   (b) Find the Laurent series expansion of the function $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region 3< |z+2| <5. [8+8]

7. (a) Evaluate $\int_C \frac{(e^z \sin 2z - 1) \quad dz}{z^2(z+2)^2}$ where C is $|\, z \,| = 1/2$ using Cauchy's integral formula.

   (b) Evaluate $\int_C \frac{(e^{-2z}) z^2 \quad dz}{(z-1)^3(z+2)}$ where C is $|\, z+2 \,| = 1$ using Cauchy's integral formula. [8+8]

8. (a) Use method of contour integration to prove that $\int_0^{2\pi} \frac{d\theta}{1+a^2-2acos\theta} = \frac{2\pi}{1-a^2}$, $0 < a < 1$.

   (b) Evaluate $\int_0^{\infty} \frac{dx}{(x^2+9)(x^2+4)^2}$ using residue theorem. [8+8]

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Code No: 07A3BS02 **R07** **Set No. 3**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Find and plot the map of rectangular region $0 \leq x \leq 1$; $0 \leq y \leq 2$, under the transformation $w = \sqrt{2}\, e^{i\pi/4} z + (1-2i)$.
   (b) Find the bilinear transformation that maps the points 0,i,1 into the points –1,0,1. [8+8]

2. (a) State and prove Taylor's theorem.
   (b) Find the Laurent series expansion of the function $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z+2| < 5$. [8+8]

3. (a) Evaluate $\int\limits_C \frac{(e^z \sin 2z-1) \quad dz}{z^2(z+2)^2}$ where C is $|z| = 1/2$ using Cauchy's integral formula.
   (b) Evaluate $\int\limits_C \frac{(e^{-2z})z^2 \quad dz}{(z-1)^3(z+2)}$ where C is $|z+2| = 1$ using Cauchy's integral formula. [8+8]

4. (a) Find the poles and the residues at each pole of $f(z) = \frac{1-e^z}{z^4}$. Where z=0 is a pole of order 4
   (b) Evaluate $\int\limits_c \frac{(Z-3}{)}(Z^2 + 2Z + 5)$ dz where c is the circle using residue theorem.
      i. $|z| = 1$
      ii. $|z+1-i| = 2$. [6+10]

5. (a) Derive Cauchy Riemann equations in polar coordinates.
   (b) Prove that the function $f(z) = \bar{z}$ is not analytic at any point.
   (c) Find the general and the principal values of (i) $\log_e(1+\sqrt{3}i)$ (ii) $\log_e(-1)$. [5+5+6]

6. (a) Show that $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$.
   (b) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.
   (c) Show that $(n+1) P_{n+1}(x) - (2n+1)\, x\, P_n(x) + n\, P_{n-1}(x) = 0$. [5+5+6]

7. (a) Show that $\beta(m,n) = 2 \int\limits_o^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta$ and deduce that $\int\limits_o^{\pi/2} \sin^n\theta\, d\theta = \int\limits_o^{\pi/2} \cos^n\theta\, d\theta = \frac{\frac{1}{2}\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right)}$.
   (b) Prove that $\Gamma(n)\, \Gamma(1-n) = \frac{\pi}{\sin n\pi}$.

(c) Show that $\int\limits_0^\infty x^m e^{-a^2x^2}dx = \frac{1}{2a^{m+1}}\Gamma\left(\frac{m+1}{2}\right)$ and hence deduce that $\int\limits_o^\infty \cos(x^2)dx = \int\limits_o^\infty \sin(x^2)dx = 1/2\sqrt{\pi/2}$. [5+5+6]

8. (a) Use method of contour integration to prove that $\int\limits_0^{2\pi} \frac{d\theta}{1+a^2-2acos\theta} = \frac{2\pi}{1-a^2}$, 0< a<1.

(b) Evaluate $\int\limits_0^\infty \frac{dx}{(x^2+9)(x^2+4)^2}$ using residue theorem. [8+8]

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