

Code No: 07A3BS02

**R07****Set No. 2**

II B.Tech I Semester Examinations, MAY 2011

MATHEMATICS - III

Common to ICE, E.COMP.E, ETM, E.CONT.E, EIE, ECE, EEE

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions  
All Questions carry equal marks

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- Find and plot the map of rectangular region  $0 \leq x \leq 1$ ;  $0 \leq y \leq 2$ , under the transformation  $w = \sqrt{2} e^{i\pi/4} z + (1-2i)$ .
  - Find the bilinear transformation that maps the points  $0, i, 1$  into the points  $-1, 0, 1$ . [8+8]
- Derive Cauchy Riemann equations in polar coordinates.
  - Prove that the function  $f(z) = \bar{z}$  is not analytic at any point.
  - Find the general and the principal values of (i)  $\log_e(1+\sqrt{3}i)$  (ii)  $\log_e(-1)$ . [5+5+6]
- Show that  $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$ .
  - Prove that  $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ .
  - Show that  $(n+1) P_{n+1}(x) - (2n+1)x P_n(x) + n P_{n-1}(x) = 0$ . [5+5+6]
- State and prove Taylor's theorem.
  - Find the Laurent series expansion of the function  $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$  in the region  $3 < |z+2| < 5$ . [8+8]
- Show that  $\beta(m, n) = 2 \int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta$  and deduce that  $\int_0^{\pi/2} \sin^n \theta d\theta = \int_0^{\pi/2} \cos^n \theta d\theta = \frac{\frac{1}{2} \Gamma(\frac{n+1}{2}) \Gamma(\frac{1}{2})}{\Gamma(\frac{n+2}{2})}$ .
  - Prove that  $\Gamma(n) \Gamma(1-n) = \frac{\pi}{\sin n\pi}$ .
  - Show that  $\int_0^{\infty} x^m e^{-a^2 x^2} dx = \frac{1}{2a^{m+1}} \Gamma(\frac{m+1}{2})$  and hence deduce that  $\int_0^{\infty} \cos(x^2) dx = \int_0^{\infty} \sin(x^2) dx = 1/2 \sqrt{\pi/2}$ . [5+5+6]
- Evaluate  $\int_C \frac{(e^z \sin 2z - 1) dz}{z^2(z+2)^2}$  where C is  $|z| = 1/2$  using Cauchy's integral formula.
  - Evaluate  $\int_C \frac{(e^{-2z}) z^2 dz}{(z-1)^3(z+2)}$  where C is  $|z+2| = 1$  using Cauchy's integral formula. [8+8]
- Use method of contour integration to prove that  $\int_0^{2\pi} \frac{d\theta}{1+a^2-2a\cos\theta} = \frac{2\pi}{1-a^2}$ ,  $0 < a < 1$ .

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(b) Evaluate  $\int_0^{\infty} \frac{dx}{(x^2+9)(x^2+4)^2}$  using residue theorem. [8+8]

8. (a) Find the poles and the residues at each pole of  $f(z) = \frac{1-e^z}{z^4}$ . Where  $z=0$  is a pole of order 4

(b) Evaluate  $\int_c \frac{(Z-3)}{Z^2+2Z+5} dz$  where  $c$  is the circle using residue theorem.

i.  $|z| = 1$

ii.  $|z+1-i| = 2$ .

[6+10]

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**R07****Set No. 4**

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MATHEMATICS - III**

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1. (a) Show that  $\beta(m,n) = 2 \int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta$  and deduce that
 
$$\int_0^{\pi/2} \sin^n \theta d\theta = \int_0^{\pi/2} \cos^n \theta d\theta = \frac{\frac{1}{2} \Gamma(\frac{n+1}{2}) \Gamma(\frac{1}{2})}{\Gamma(\frac{n+2}{2})}.$$
 (b) Prove that  $\Gamma(n) \Gamma(1-n) = \frac{\pi}{\sin n\pi}$ .  
 (c) Show that  $\int_0^{\infty} x^m e^{-a^2 x^2} dx = \frac{1}{2a^{m+1}} \Gamma(\frac{m+1}{2})$  and hence deduce that
 
$$\int_0^{\infty} \cos(x^2) dx = \int_0^{\infty} \sin(x^2) dx = 1/2 \sqrt{\pi/2}. \quad [5+5+6]$$
2. (a) Show that  $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$ .  
 (b) Prove that  $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ .  
 (c) Show that  $(n+1) P_{n+1}(x) - (2n+1)x P_n(x) + n P_{n-1}(x) = 0$ . [5+5+6]
3. (a) Use method of contour integration to prove that  $\int_0^{2\pi} \frac{d\theta}{1+a^2-2a\cos\theta} = \frac{2\pi}{1-a^2}$ ,  
 $0 < a < 1$ .  
 (b) Evaluate  $\int_0^{\infty} \frac{dx}{(x^2+9)(x^2+4)^2}$  using residue theorem. [8+8]
4. (a) Evaluate  $\int_C \frac{(e^z \sin 2z - 1) dz}{z^2(z+2)^2}$  where C is  $|z| = 1/2$  using Cauchy's integral formula.  
 (b) Evaluate  $\int_C \frac{(e^{-2z})z^2 dz}{(z-1)^3(z+2)}$  where C is  $|z+2| = 1$  using Cauchy's integral formula. [8+8]
5. (a) Derive Cauchy Riemann equations in polar coordinates.  
 (b) Prove that the function  $f(z) = \bar{z}$  is not analytic at any point.  
 (c) Find the general and the principal values of (i)  $\log_e(1+\sqrt{3}i)$  (ii)  $\log_e(-1)$ . [5+5+6]
6. (a) Find the poles and the residues at each pole of  $f(z) = \frac{1-e^z}{z^4}$ . Where  $z=0$  is a pole of order 4  
 (b) Evaluate  $\int_c \frac{(Z-3)}{Z^2+2Z+5} dz$  where c is the circle using residue theorem.  
 i.  $|z| = 1$

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ii.  $|z+1-i| = 2.$  [6+10]

7. (a) Find and plot the map of rectangular region  $0 \leq x \leq 1; 0 \leq y \leq 2$ , under the transformation  $w = \sqrt{2} e^{i\pi/4} z + (1-2i).$
- (b) Find the bilinear transformation that maps the points  $0, i, 1$  into the points  $-1, 0, 1.$  [8+8]
8. (a) State and prove Taylor's theorem.
- (b) Find the Laurent series expansion of the function  $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$  in the region  $3 < |z+2| < 5.$  [8+8]

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- Derive Cauchy Riemann equations in polar coordinates.
  - Prove that the function  $f(z) = \bar{z}$  is not analytic at any point.
  - Find the general and the principal values of (i)  $\log_e(1+\sqrt{3}i)$  (ii)  $\log_e(-1)$ . [5+5+6]
- Show that  $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$ .
  - Prove that  $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ .
  - Show that  $(n+1) P_{n+1}(x) - (2n+1)x P_n(x) + n P_{n-1}(x) = 0$ . [5+5+6]
- Find and plot the map of rectangular region  $0 \leq x \leq 1; 0 \leq y \leq 2$ , under the transformation  $w = \sqrt{2} e^{i\pi/4} z + (1-2i)$ .
  - Find the bilinear transformation that maps the points 0,i,1 into the points -1,0,1. [8+8]
- Find the poles and the residues at each pole of  $f(z) = \frac{1-e^z}{z^4}$ . Where  $z=0$  is a pole of order 4
  - Evaluate  $\int_c \frac{(z-3)}{z^2} (Z^2 + 2Z + 5) dz$  where  $c$  is the circle using residue theorem.
    - $|z| = 1$
    - $|z+1-i| = 2$ . [6+10]
- Show that  $\beta(m,n) = 2 \int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta$  and deduce that
 
$$\int_0^{\pi/2} \sin^n \theta d\theta = \int_0^{\pi/2} \cos^n \theta d\theta = \frac{\frac{1}{2} \Gamma(\frac{n+1}{2}) \Gamma(\frac{1}{2})}{\Gamma(\frac{n+2}{2})}$$
  - Prove that  $\Gamma(n) \Gamma(1-n) = \frac{\pi}{\sin n\pi}$ .
  - Show that  $\int_0^{\infty} x^m e^{-a^2 x^2} dx = \frac{1}{2a^{m+1}} \Gamma(\frac{m+1}{2})$  and hence deduce that
 
$$\int_0^{\infty} \cos(x^2) dx = \int_0^{\infty} \sin(x^2) dx = 1/2 \sqrt{\pi/2}$$
. [5+5+6]
- State and prove Taylor's theorem.
  - Find the Laurent series expansion of the function  $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$  in the region  $3 < |z+2| < 5$ . [8+8]

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7. (a) Evaluate  $\int_C \frac{(e^z \sin 2z - 1) dz}{z^2(z+2)^2}$  where C is  $|z| = 1/2$  using Cauchy's integral formula.
- (b) Evaluate  $\int_C \frac{(e^{-2z})z^2 dz}{(z-1)^3(z+2)}$  where C is  $|z+2| = 1$  using Cauchy's integral formula. [8+8]
8. (a) Use method of contour integration to prove that  $\int_0^{2\pi} \frac{d\theta}{1+a^2-2a\cos\theta} = \frac{2\pi}{1-a^2}$ ,  
 $0 < a < 1$ .
- (b) Evaluate  $\int_0^{\infty} \frac{dx}{(x^2+9)(x^2+4)^2}$  using residue theorem. [8+8]

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**R07****Set No. 3**

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1. (a) Find and plot the map of rectangular region  $0 \leq x \leq 1$ ;  $0 \leq y \leq 2$ , under the transformation  $w = \sqrt{2} e^{i\pi/4} z + (1-2i)$ .  
(b) Find the bilinear transformation that maps the points  $0, i, 1$  into the points  $-1, 0, 1$ . [8+8]
2. (a) State and prove Taylor's theorem.  
(b) Find the Laurent series expansion of the function  $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$  in the region  $3 < |z+2| < 5$ . [8+8]
3. (a) Evaluate  $\int_C \frac{(e^z \sin 2z-1) dz}{z^2(z+2)^2}$  where C is  $|z| = \frac{1}{2}$  using Cauchy's integral formula.  
(b) Evaluate  $\int_C \frac{(e^{-2z})z^2 dz}{(z-1)^3(z+2)}$  where C is  $|z+2| = 1$  using Cauchy's integral formula. [8+8]
4. (a) Find the poles and the residues at each pole of  $f(z) = \frac{1-e^z}{z^4}$ . Where  $z=0$  is a pole of order 4  
(b) Evaluate  $\int_c \frac{(Z-3)}{(Z^2+2Z+5)} dz$  where c is the circle using residue theorem.  
i.  $|z| = 1$   
ii.  $|z+1-i| = 2$ . [6+10]
5. (a) Derive Cauchy Riemann equations in polar coordinates.  
(b) Prove that the function  $f(z) = \bar{z}$  is not analytic at any point.  
(c) Find the general and the principal values of (i)  $\log_e(1+\sqrt{3}i)$  (ii)  $\log_e(-1)$ . [5+5+6]
6. (a) Show that  $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$ .  
(b) Prove that  $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ .  
(c) Show that  $(n+1) P_{n+1}(x) - (2n+1)x P_n(x) + n P_{n-1}(x) = 0$ . [5+5+6]
7. (a) Show that  $\beta(m,n) = 2 \int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta$  and deduce that  
$$\int_0^{\pi/2} \sin^n \theta d\theta = \int_0^{\pi/2} \cos^n \theta d\theta = \frac{\frac{1}{2} \Gamma(\frac{n+1}{2}) \Gamma(\frac{1}{2})}{\Gamma(\frac{n+2}{2})}$$
  
(b) Prove that  $\Gamma(n) \Gamma(1-n) = \frac{\pi}{\sin n\pi}$ .

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(c) Show that  $\int_0^{\infty} x^m e^{-a^2 x^2} dx = \frac{1}{2a^{m+1}} \Gamma\left(\frac{m+1}{2}\right)$  and hence deduce that

$$\int_0^{\infty} \cos(x^2) dx = \int_0^{\infty} \sin(x^2) dx = 1/2\sqrt{\pi/2}. \quad [5+5+6]$$

8. (a) Use method of contour integration to prove that  $\int_0^{2\pi} \frac{d\theta}{1+a^2-2a\cos\theta} = \frac{2\pi}{1-a^2}$ ,  
 $0 < a < 1$ .

(b) Evaluate  $\int_0^{\infty} \frac{dx}{(x^2+9)(x^2+4)^2}$  using residue theorem. [8+8]

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