Code No: 07A4BS02 **R07** **Set No. 2**

II B.Tech II Semester Examinations,APRIL 2011
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. Find the singular points of the function.
   (a) $\frac{z^2}{z^4+1}$
   (b) $\frac{z^2}{1-z^4}$ [16]

2. (a) If tan(x+iy) = A+iB show that $A^2+B^2+2A\cot 2x = 1$.
   (b) Find the principal value of $(2i)^{2i}$ [8+8]

3. Show by the method of residues, $\int_0^{\pi} \frac{d\theta}{a+b\cos\theta} = \frac{\pi}{\sqrt{a^2-b^2}}$ (a > b > 0). [16]

4. (a) Evaluate $\int_C \frac{z+2}{z} dz$ where C is
      i. the semi circle $z = 2e^{i\theta}, \pi \leq \theta \leq 2\pi$
      ii. the circle $z = 2e^{i\theta}, -\pi \leq \theta \leq \pi$
   (b) Use Cauchy's integral formula to evaluate $\oint_c \frac{(e^z+z\sinh z)}{(z-\pi i)^2} dz$ where c is the circle $|z| = 4$ [10+6]

5. (a) Find the bilinear transformation which maps vertices (1 + i, -i, 2-i) of the triangle T of the z-plane in to the points (0, 1, i) of the w-plane.
   (b) Find the image of the semi-infinite strip $x \geq 0, 0 \leq y \leq \pi$ under the mapping w = cosh z. [8+8]

6. (a) State and prove Fundamental theorem of Algebra.
   (b) State and prove Liouville's theorem. [8+8]

7. Using $\beta - \Gamma$ functions
   (a) Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\, d\theta = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \frac{1}{2}\pi\sqrt{2}$.
   (b) Prove that $\int_0^1 (1-x^n)^{\frac{1}{n}}\, dx = \frac{\left[\Gamma\left(\frac{1}{n}\right)\right]^2}{n^2\,\Gamma\left(\frac{2}{n}\right)}$.
   (c) Prove that $\beta(m, n) = \beta(m+1, n) + \beta(m, n+1)$ [5+5+6]

8. The necessary and sufficient conditions for the function f(z) = u(x, y) + i v(x, y) to be analytic in the region R, are

(a) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ are continuous functions of x and y in R.

(b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ [16]

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Code No: 07A4BS02 **R07** **Set No. 4**

II B.Tech II Semester Examinations,APRIL 2011
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) If tan(x+iy) = A+iB show that $A^2+B^2+2A\cot 2x = 1$.
   (b) Find the principal value of $(2i)^{2i}$ [8+8]

2. The necessary and sufficient conditions for the function f(z) = u(x, y) + i v(x, y) to be analytic in the region R, are
   (a) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ are continuous functions of x and y in R.
   (b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ [16]

3. Using $\beta - \Gamma$ functions
   (a) Prove that $\int_0^{\frac{\pi}{2}} \sqrt{tan\theta}\, d\theta = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \frac{1}{2}\pi\sqrt{2}$.
   (b) Prove that $\int_0^1 (1-x^n)^{\frac{1}{n}}\, dx = \frac{\left[\Gamma\left(\frac{1}{n}\right)\right]^2}{n^2\,\Gamma\left(\frac{2}{n}\right)}$.
   (c) Prove that β(m, n) = β(m+1, n) + β(m, n+1) [5+5+6]

4. Show by the method of residues, $\int_0^{\pi} \frac{d\theta}{a+b\cos\theta} = \frac{\pi}{\sqrt{a^2-b^2}}$ (a > b > 0). [16]

5. Find the singular points of the function.
   (a) $\frac{z^2}{z^4+1}$
   (b) $\frac{z^2}{1-z^4}$ [16]

6. (a) Evaluate $\int_C \frac{z+2}{z} dz$ where C is
      i. the semi circle $z = 2e^{i\theta}$, $\pi \leq \theta \leq 2\pi$
      ii. the circle $z = 2e^{i\theta}$, $-\pi \leq \theta \leq \pi$
   (b) Use Cauchy's integral formula to evaluate $\oint_c \frac{(e^z+z\sinh z)}{(z-\pi i)^2} dz$ where c is the circle |z| = 4 [10+6]

7. (a) State and prove Fundamental theorem of Algebra.
   (b) State and prove Liouville's theorem. [8+8]

8. (a) Find the bilinear transformation which maps vertices (1 + i, -i, 2-i) of the triangle T of the z-plane in to the points (0, 1, i) of the w-plane.

(b) Find the image of the semi-infinite strip $x \geq 0, 0 \leq y \leq \pi$ under the mapping $w = \cosh z$. [8+8]

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Code No: 07A4BS02 **R07** **Set No. 1**

II B.Tech II Semester Examinations,APRIL 2011
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. Show by the method of residues, $\int\limits_0^{\pi} \frac{d\theta}{a+b\cos\theta} = \frac{\pi}{\sqrt{a^2-b^2}}$ (a > b > 0). [16]

2. (a) Evaluate $\int\limits_C \frac{z+2}{z}dz$ where C is
   i. the semi circle z = $2e^{i\theta}$, $\pi \leq \theta \leq 2\pi$
   ii. the circle z = $2e^{i\theta}$, $-\pi \leq \theta \leq \pi$

   (b) Use Cauchy's integral formula to evaluate $\oint\limits_c \frac{(e^z+z\sinh z)}{(z-\pi i)^2}dz$ where c is the circle |z| = 4 [10+6]

3. (a) Find the bilinear transformation which maps vertices (1 + i, -i, 2-i) of the triangle T of the z-plane in to the points (0, 1, i) of the w-plane.

   (b) Find the image of the semi-infinite strip $x \geq 0, 0 \leq y \leq \pi$ under the mapping w = cosh z. [8+8]

4. The necessary and sufficient conditions for the function f(z) = u(x, y) + i v(x, y) to be analytic in the region R, are

   (a) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ are continuous functions of x and y in R.

   (b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ [16]

5. Find the singular points of the function.

   (a) $\frac{z^2}{z^4+1}$

   (b) $\frac{z^2}{1-z^4}$ [16]

6. (a) State and prove Fundamental theorem of Algebra.

   (b) State and prove Liouville's theorem. [8+8]

7. (a) If tan(x+iy) = A+iB show that $A^2+B^2+2A\cot 2x = 1$.

   (b) Find the principal value of $(2i)^{2i}$ [8+8]

8. Using $\beta - \Gamma$ functions

   (a) Prove that $\int\limits_0^{\frac{\pi}{2}} \sqrt{tan\theta}\ d\theta = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \frac{1}{2}\pi\sqrt{2}$.

   (b) Prove that $\int\limits_0^1 (1-x^n)^{\frac{1}{n}}\ dx = \frac{\left[\Gamma\left(\frac{1}{n}\right)\right]^2}{n^2\,\Gamma\left(\frac{2}{n}\right)}$.

(c) Prove that β(m, n) = β(m+1, n) + β(m, n+1) [5+5+6]

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Code No: 07A4BS02 **R07** **Set No. 3**

II B.Tech II Semester Examinations,APRIL 2011
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Find the bilinear transformation which maps vertices (1 + i, -i, 2-i) of the triangle T of the z-plane in to the points (0, 1, i) of the w-plane.
   (b) Find the image of the semi-infinite strip $x \geq 0, 0 \leq y \leq \pi$ under the mapping w = cosh z. [8+8]

2. (a) State and prove Fundamental theorem of Algebra.
   (b) State and prove Liouville's theorem. [8+8]

3. The necessary and sufficient conditions for the function f(z) = u(x, y) + i v(x, y) to be analytic in the region R, are
   (a) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ are continuous functions of x and y in R.
   (b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ [16]

4. Find the singular points of the function.
   (a) $\frac{z^2}{z^4+1}$
   (b) $\frac{z^2}{1-z^4}$ [16]

5. Using $\beta - \Gamma$ functions
   (a) Prove that $\int_0^{\frac{\pi}{2}} \sqrt{tan\theta}\, d\theta = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \frac{1}{2}\pi\sqrt{2}$.
   (b) Prove that $\int_0^1 (1-x^n)^{\frac{1}{n}}\, dx = \frac{\left[\Gamma\left(\frac{1}{n}\right)\right]^2}{n^2\,\Gamma\left(\frac{2}{n}\right)}$.
   (c) Prove that β(m, n) = β(m+1, n) + β(m, n+1) [5+5+6]

6. Show by the method of residues, $\int_0^{\pi} \frac{d\theta}{a+b\cos\theta} = \frac{\pi}{\sqrt{a^2-b^2}}$ (a > b > 0). [16]

7. (a) If tan(x+iy) = A+iB show that $A^2+B^2+2A\cot 2x = 1$.
   (b) Find the principal value of $(2i)^{2i}$ [8+8]

8. (a) Evaluate $\int_C \frac{z+2}{z} dz$ where C is
      i. the semi circle $z = 2e^{i\theta}$, $\pi \leq \theta \leq 2\pi$
      ii. the circle $z = 2e^{i\theta}$, $-\pi \leq \theta \leq \pi$

(b) Use Cauchy's integral formula to evaluate $\oint\limits_{c} \frac{(e^z + z\sinh z)}{(z-\pi i)^2} dz$ where c is the circle $|z| = 4$ [10+6]

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