# III B.TECH - I SEMESTER EXAMINATIONS - MAY, 2011 FINITE ELEMENT METHODS <br> (MECHANICAL ENGINEERING) <br> (MECHATRONICS) 

Time: 3hours
Max. Marks: $\mathbf{8 0}$

## Answer any FIVE questions <br> All Questions Carry Equal Marks

1.a) What are the characteristics of a stiffness matrix of an element? Explain different types of elements along with their applications.
b) State and explain the Galerkin's principle for solving the engineering problems. [8+8]
2. A stepped bar is subjected to an axial load of 300 kN as shown in figure 1. Find the nodal displacements, element stresses and strains and reactions. Take $E=2 \times 10^{5} \mathrm{~N} / \mathrm{mm}^{2}$. The cross sectional areas are $100 \mathrm{~mm}^{2}$ and $150 \mathrm{~mm}^{2}$.


Figure 1
3.a) Derive the stress strain equilibrium equations based on the theory of elasticity.
b) Calculate the deflection at the center and slopes at the ends of a simply supported beam of 2 m length subjected to a UDL of $50 \mathrm{kN} / \mathrm{m}$ throughout the length. Take EI $=$ $700 \mathrm{Nmm}^{2}$.
4.a) Show that the determinant of Jacobian matrix of a CST is twice than that of the area of a triangle.
b) Calculate the load vector for a triangular element subjected a variable traction load on the face $1-2$ of 5 MPa and on the face 2-3 a variable load of 2 MPa to 4 MPa . A point load of 400 kN acts at the node 3 with the inclination of $30^{\circ}$ with horizontal axis. [8+8]
5. An axi-symmetric body with a uniformly distributed load of 0.3 MPa on the conical surface shown in figure 2 . Calculate the equivalent loads at nodes 1,2 and 3 .
[16]


Figure 2
6.a) Derive the strain displacement relation matrix for the four nodded quadrilateral element.
b) Solve the following integral equation using one point, two point and three point Gaussian quadrature method and compare with exact solution $\int\left(1 / 1+x^{3}\right)+3 e^{x} d x$ with the limits from -1 to 1 .
7.a) Derive the stiffness matrix for the circular shaft subjected to a twisting moment.
b) Estimate the temperature profile in a pin fin of diameter 30 mm , whose length is 500 mm . The thermal conductivity of the fin material is $50 \mathrm{~W} / \mathrm{m} \mathrm{K}$ and heat transfer coefficient over the surface of the fin is $40 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$ at $30^{\circ} \mathrm{C}$. The tip is insulated and the base is exposed to heat flux of $400 \mathrm{~kW} / \mathrm{m}^{2}$.
8. Calculate the natural frequencies for the stepped bar shown in figure 3. Take $\mathrm{E}=2 \mathrm{E} 5$ $\mathrm{N} / \mathrm{mm}^{2}$; density $=1000 \mathrm{~kg} / \mathrm{m}^{3}$.

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1.a) Derive the equation for stiffness matrix in terms of strain displacement relation matrix and stress strain relation matrix.
b) State and explain the Minimum Potential energy principle.
2. Calculate displacement vector, stresses and reaction for the following figure 1. Take $\mathrm{E}=2 \times 10^{5} \mathrm{~N} / \mathrm{mm}^{2}$.


Figure 1
3.a) Derive the Hermite shape function for the two nodded beam element based on energy principle.
b) Calculate the load vector for the beam element subjected UDL, point loads and bending moments at ends.
4.a) Discuss the importance of iso parametric conditions in solving the CST problems.
b) Calculate the strain displacement matrix for the element with the coordinates $1(4,5)$, $2(9,2)$ and $3(6,8)$. And also calculate the strains the triangle whose nodal displacement values are $u_{1}=0.3 \mathrm{~mm}, \mathrm{v}_{1}=0.3 \mathrm{~mm}, \mathrm{u}_{2}=0.2 \mathrm{~mm}, \mathrm{v}_{2}=-0.4 \mathrm{~mm}, \mathrm{u}_{3}=0.3 \mathrm{~mm}, \mathrm{v}_{3}=0.5 \mathrm{~mm}$.
[8+8]
5. In an axi-symmetric problem, the element coordinates and displacements in r and z directions are given below

| Node No. | Coordinates | displacements |
| :---: | :---: | :---: |
| 1 | $(1,1)$ | $(0,0)$ |
| 2 | $(10,4)$ | $(-0.2,-0.1)$ |
| 3 | $(6,7)$ | $(0.6,0.8)$ |

Calculate the strains and stresses developed in the element.
6.a) Derive the shape functions for the four nodded quadrilateral element.
b) Evaluate $\int\left[3 \mathrm{e}^{\mathrm{x}}+\mathrm{x}^{2}+1 /(\mathrm{x}+2)\right] d \mathrm{x}$ over the limits -1 and +1 using one point, two point and three point Gauss quadrature formulae and compare with the exact solution. [8+8]
7.a) What is scalar field problems and how to solve them in finite element analysis.
b) Calculate the temperature values at the junction points of the composite slab made of two different materials with $25 \mathrm{~W} / \mathrm{m} \mathrm{K}$ of 0.25 m thick and $40 \mathrm{~W} / \mathrm{m} \mathrm{K}$ of 0.25 m thick. The inner wall is exposed to a convective heat transfer coefficient of $50 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}, 50^{\circ} \mathrm{C}$ and other wall is exposed to a heat flux of $50 \mathrm{~kW} / \mathrm{m}^{2}$. There is an internal heat generation of $500 \mathrm{~kW} / \mathrm{m}^{3}$ in the second layer of the composite slab.
8. Calculate natural frequency, displacements and mode shapes of the stepped bar shown in figure 2. Take $\mathrm{E}=2 \times 10^{5} \mathrm{~N} / \mathrm{mm}^{2}$ and $\rho=7200 \mathrm{~kg} / \mathrm{m}^{3}$.


Figure 2

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Time: 3hours
Max. Marks: 80

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1.a) Explain the historical development of finite element analysis and discuss the important applications.
b) A rod of 10 mm diameter, length 200 mm has nodal displacements due to axial loads as 1.2 mm and 2.8 mm . Calculate the displacements at the centre. Also find the stress, strain and strain energy for the rod. Take $\mathrm{E}=210 \mathrm{GPa}$.
2. Estimate the displacements, forces and stresses in the bar loaded shown in figure 1. The stiffness values of the bars are also shown in figure.


Figure 1
3. A beam is fixed at one end an supported by a roller at the other end, has a 20 kN concentrated load applied at the centre of the span of 10 m . Calculate the deflection and slope and also construct shear force and bending moment diagrams. Take $I=2500 \mathrm{~cm}^{4}$ and $\mathrm{E}=20 \times 10^{6} \mathrm{~N} / \mathrm{cm}^{2}$.
4.a) Derive the shape functions for the triangular element for the two dimensional structural problems.
b) Compute the strain displacement relation matrix and stiffness matrix of a triangular element with the co ordinates $1(3,4), 2(6,5)$ and $3(5,8)$ for the plane stress conditions. Take $\mathrm{E}=200 \mathrm{GPa}$, poison's ratio $=0.3$ and thickness 1 mm . All dimensions are in mm .
5. Compute the strain displacement matrix and also the strains of a axi-symmetric triangular element with the coordinates $\mathrm{r}_{1}=3 \mathrm{~cm}, \mathrm{z}_{1}=4 \mathrm{~cm}, \mathrm{r}_{2}=6 \mathrm{~cm}, \mathrm{z}_{2}=5 \mathrm{~cm}, \mathrm{r}_{3}=5 \mathrm{~cm}, \mathrm{z}_{3}=8$ cm . The nodal displacement values are $\mathrm{u}_{1}=0.01 \mathrm{~mm}, \mathrm{w}_{1}=0.01 \mathrm{~mm}, \mathrm{u}_{2}=0.01 \mathrm{~mm}, \mathrm{w}_{2}=$ $-0.04 \mathrm{~mm}, \mathrm{u}_{3}=-0.03 \mathrm{~mm}, \mathrm{w}_{3}=0.07 \mathrm{~mm}$.
6.a) Derive the load vector for the quadrilateral element subjected to a variable traction force on the side of the element.
b) Evaluate the integral $\iint\left(x^{2}+y^{2}\right) d x$ dy by Gaussian quadrature over the area of quadrilateral with the co-ordinates $1(1,1), 2(5,1), 3(6,6)$ and $4(1,4)$.
7. A large industrial furnace is supported on a long column of fire clay brick, which is $1 \times 1$ m on a side. During steady state operation, installation is such that three surfaces of the column are maintained at 600 K , while the remaining surface is exposed to an air stream for which $\mathrm{T}_{\alpha}=300 \mathrm{~K}$ and $\mathrm{h}=12 \mathrm{~W} / \mathrm{m}^{2} \mathrm{k}$. Determine the temperature distribution in the column and the heat rate to the air stream per unit length of column. Take $\mathrm{K}=1 \mathrm{~W} / \mathrm{mK}$
[16]
8.a) Define and distinguish between the lumped and consistent mass formulation.
b) Find the natural frequencies and modes of vibration for the following cases. a)one element cantilever beam b) two element cantilever beam by taking advantage of the symmetry about mid point.

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1.a) What is the need of finite element analysis for solving the engineering problems? Explain its significance in solving different solid mechanics problems
b) Describe different stresses and equilibrium conditions for the structural problems. [8+8]
2. A stepped bar is subjected an axial loaw of 200 kN at the place of change of cross section and material as shown in figure 1. Find (i) the nodal displacements (ii) the reaction forces (iii) the induced stress in each material. The materials are (1) Aluminimum bar with $\mathrm{E}=70 \times 10^{3} \mathrm{~N} / \mathrm{mm}^{2}$ and (2) Steel with $\mathrm{E}=200 \times 10^{3} \mathrm{~N} / \mathrm{mm}^{2}$.

3.a) Derive the load vector for beam element subjected to all three different loads i.e point loads, bending moments and uniformly distributed load.
b) A simply supported beam of span 2 m is subjected to a point load of 100 kN at the centre and a bending moment of $20 \mathrm{kN}-\mathrm{m}$ also at the centre. Calculate the maximum deflection and slope if the flexural rigidity is $800 \times 10^{3} \mathrm{~N}-\mathrm{m}^{2}$.
4.a) Evaluate the load vector for the triangular element subjected to a body force and a variable traction force on the side 1-2.
b) Estimate the shape function values at the point $\mathrm{P}(\mathrm{x}, \mathrm{y})$ in terms of x and y of a triangular element with the coordinates $1(0.0), 2(20,25)$ and $3(10,35)$.
[8+8]
5.a) The nodal coordinates of axi-symmetric triangular element are $\mathrm{r}_{1}=20 \mathrm{~mm}, \mathrm{z}_{1}=10 \mathrm{~mm}, \mathrm{r}_{2}$ $=40 \mathrm{~mm}, \mathrm{z}_{2}=10 \mathrm{~mm}, \mathrm{r}_{3}=30 \mathrm{~mm}, \mathrm{z}_{3}=50 \mathrm{~mm}$, Determine the strain displacement relation matrix.
b) Explain the methodology to simplify the given domain using symmetric boundary conditions.
6.a) Derive the strain displacement relation matrix for the quadrilateral element in terms of natural coordinates.
b) Explain the one-point and three point Gaussian quadrature method for the numerical integration with suitable example.
7.a) Derive the conductivity matrix for 3 noded triangular element with convection boundary condition at one of the faces of the element.
b) Estimate the temperature profile in a pin fin of diameter 30 mm , whose length is 750 mm . The thermal conductivity of the fin material is $50 \mathrm{~W} / \mathrm{m} \mathrm{K}$ and heat transfer coefficient over the surface of the fin is $40 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$ at $30^{\circ} \mathrm{C}$. The tip is also exposed to convection and the base temperature of the fin is $800^{\circ} \mathrm{C}$.
8.a) Compute the two longitudinal mode shapes and frequencies based on two element idealization for a free-free bar.
b) What do you understand by the terms equilibrium problems, Eigen value problems, and propagation problems.

