# II B.Tech I Semester Examinations,MAY 2011 MATHEMATICS-II <br> Common to CE, CHEM, AE, BT, MMT 

Time: 3 hours
Max Marks: 75
Answer any FIVE Questions
All Questions carry equal marks

1. (a) i. Show that the product of the eigen values of a matrix $A$ is equal to its determinant.
ii. Show that the sum of the eigen values of a matrix is the trace of the matrix.
(b) Find the eigen values and eigen vectors of $A=\left[\begin{array}{ccc}-1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0\end{array}\right]$
2. (a) Find the skew-Hermitian form for $\mathrm{A}=\left[\begin{array}{cc}i & 0 \\ 0 & -i\end{array}\right]$ with $\mathrm{X}=\left[\begin{array}{l}1 \\ i\end{array}\right]$.
(b) Find the Hermitian form of $\mathrm{A}=\left[\begin{array}{cc}3 & 2-i \\ 2+i & 4\end{array}\right]$ with $\mathrm{X}=\left[\begin{array}{c}1+i \\ 2 i\end{array}\right]$.
3. (a) Find the Fourier Transform of $\left\{\begin{array}{cc}\cos x & 0<x<a \\ 0 & x \geq a\end{array}\right.$
(b) Find $\mathrm{f}(\mathrm{x})$ if $\mathrm{F}_{C}[\mathrm{f}(\mathrm{x})]=16 \frac{(-1)^{n-1}}{n^{3}}$ if $0<x<8$
4. (a) Form the partial differential equation from
$\mathrm{F}\left(\mathrm{x}-y-\mathrm{z}, x^{2}-z^{2}\right)=0$
(b) Solve the partial differential equation

$$
\begin{equation*}
\left(\frac{p}{2}+x\right)^{2}+\left(\frac{q}{2}+y\right)^{2}=1 \tag{15}
\end{equation*}
$$

5. Reduce the quadratic form $5 x^{2}+26 y^{2}+10 z^{2}+4 y z+6 x y+14 x z$ to canonical form by orthogonalization.
6. (a) Find non-singular matrices P and Q so that PAQ is in the normal form,

$$
\text { where } \mathrm{A}=\left[\begin{array}{cccc}
3 & 2 & -1 & 5 \\
5 & 1 & 4 & -2 \\
1 & -4 & 11 & -19
\end{array}\right]
$$

(b) Find the rank of the matrix A by reducing it to the normal form,

$$
\text { where } \mathrm{A}=\left[\begin{array}{cccc}
1 & 1 & 1 & 1  \tag{15}\\
1 & 2 & 3 & -4 \\
2 & 3 & 5 & -5 \\
3 & -4 & -5 & 8
\end{array}\right]
$$

7. Solve the boundary value problem $\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial y^{2}}=0$, with $u(0, y)=0$

$$
\begin{align*}
& \mathrm{u}(\mathrm{x}, \mathrm{~b})=0 \text { and } \mathrm{u}(\mathrm{x}, 0)=0 \\
& \mathrm{u}(\mathrm{a}, \mathrm{y})=\mathrm{Ky}(\mathrm{~b}-\mathrm{y}), 0<y<b \tag{15}
\end{align*}
$$

8. (a) Obtain the Fourier series for the function $f(x)=x \operatorname{Sin} x$ in $[0,2 \pi]$
(b) Find the half range cosine series for $\begin{array}{r}f(x)=1 \text { in }[0,1] \\ =x \text { in }[1,2]\end{array}$


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1. Determine diagonal matrix orthogonally similar to the real symmetric matrix

$$
A=\left[\begin{array}{lll}
1 & 2 & 0  \tag{15}\\
2 & 2 & 2 \\
0 & 2 & 3
\end{array}\right]
$$

3. (a) Find the rank of the matrix by reducing it to Echelon form from the matrix $\left[\begin{array}{cccc}1 & 2 & -4 & 5 \\ 2 & -1 & 3 & 6 \\ 8 & 1 & 9 & 7\end{array}\right]$.
(b) Reduce the matrix $\left[\begin{array}{cccc}5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0\end{array}\right]$ into Echelon form and hence find its
4. A string is stretched and fastened to two points L apart. Motion is started by displacing the string in the form $\mathrm{y}=\mathrm{K} \mathrm{x}(\mathrm{L}-\mathrm{x})$ from which it is released at time $\mathrm{t}=0$. Find the displacement of the string at any point x at any time t .
[15]
5. (a) Obtain the Fourier series for the function

$$
\begin{gathered}
f(x)=\pi+x \text { in }-\pi<x<0 \\
=\pi-x \text { in } 0<x<\pi
\end{gathered}
$$

(b) Find the half range Sine series for

$$
\begin{align*}
f(x) & =\frac{1}{4}-x \text { if } 0<x<\frac{1}{2}  \tag{15}\\
& =x-\frac{3}{4} \text { if } \frac{1}{2}<x<1 .
\end{align*}
$$

6. Diagonalize the matrix $A=\left[\begin{array}{ccc}3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3\end{array}\right]$ and hence find $A^{4}$.
7. Reduce the quadratic form $3 x_{1}^{2}-2 x_{2}^{2}-x_{3}^{2}-4 x_{1} x_{2}+12 x_{2} x_{3}+8 x_{1} x_{3}$ by orthogonal transforms and hence find rank, index and signature.
8. (a) Find the Fourier cosine Transform of

$$
\begin{aligned}
\mathrm{F}(\mathrm{x}) & =\mathrm{x} .0<x<\frac{1}{2} \\
& =1-\mathrm{x}, \frac{1}{2}<x<1 \\
& =0, x>1
\end{aligned}
$$

(b) Find $f(x)$ if its finite Fourier Sine transform is $\frac{(1-\operatorname{Cosn\pi })}{n^{2} \pi^{2}}$


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1. Diagonalize the matrix $A=\left[\begin{array}{ccc}2 & 2 & -2 \\ 2 & 1 & 2 \\ 0 & 1 & -3\end{array}\right]$ and hence find $A^{4}$.
2. (a) Using Fourier integral show that $\int_{0}^{\infty} \frac{1-\cos \pi \lambda}{\lambda} \sin x \lambda d y=\frac{\pi}{2}, 0<x<\pi$
(b) Find the finite Fourier sine and cosine transforms of

$$
\begin{aligned}
\mathrm{f}(\mathrm{x}) & =1 \text { in } 0<x<\frac{\pi}{2} \\
& =-1 \text { in } \frac{\pi}{2}<x<\pi
\end{aligned}
$$

3. (a) Solve: $x+y+z=6 ; x-y+2 z=5 ; 2 x-2 y+3 z=7$
(b) Test for consistency and solve $2 \mathrm{x}+3 \mathrm{y}+7 \mathrm{z}=5 ; 3 \mathrm{x}+\mathrm{y}-3 \mathrm{z}=12 ; 2 \mathrm{x}+19 \mathrm{y}-47 \mathrm{z}=32$.
4. (a) Prove that eigen values of a real symmetric matrix are always real.
(b) Express the matrix A as the sum of a symmetric and a skew symmetric ma-

$$
\text { trices, where } \mathrm{A}=\left[\begin{array}{ccc}
4 & 2 & -3  \tag{15}\\
1 & 3 & -6 \\
-5 & 0 & -7
\end{array}\right]
$$

5. Solve the partial differential equation $\frac{\partial u}{\partial t}=a^{2} \frac{\partial^{2} u}{\partial x^{2}}$ with the conditions
(a) $u \rightarrow 0$, ast $\rightarrow \infty$
(b) $\frac{\partial u}{\partial x}=0$ when $\mathrm{x}= \pm \mathrm{a}, \mathrm{t}>0$
(c) $\mathrm{u}=\mathrm{x}$, when $\mathrm{t}=0$ and $-a<x<a$
6. (a) Obtain the Fourier series for the function $f(x)=\cos x$ in $(-\pi, \pi)$
(b) Find the half range cosine series for $f(x)=x \operatorname{Sin} x$ in $[0, \pi]$
7. Reduce the quadratic form $4 x^{2}+3 y^{2}+z^{2}-8 x y-6 y z+4 x z$ to canonical form by orthogonal transformation and hence find the nature of the quadratic form.
8. (a) Form the partial differential equation from

$$
\mathrm{z}=\mathrm{e}^{n y} \mathrm{f}(\mathrm{x}-\mathrm{y})[\mathrm{n} \text { is a constant }]
$$

(b) Solve the partial differential equation

$$
\begin{equation*}
\left(x^{2}-y z\right) p+\left(y^{2}-z x\right) q=z^{2}-x y \tag{15}
\end{equation*}
$$

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1. (a) Find the Fourier Transform of $\mathrm{F}(\mathrm{x})=\mathrm{e}^{i k x} \quad a<x<b$

$$
=0, x<a, x>b
$$

(b) Find the finite Fourier sine transforms of $\mathrm{f}(\mathrm{x})=\mathrm{I}$ if $0<x<\frac{\pi}{2}$ $=-1$ if $\frac{\pi}{2}<$
2. (a) Obtain the Fourier series for the function
$\mathrm{F}(\mathrm{x})=\mathrm{x} \operatorname{Sin} \mathrm{x}$ in $[-\pi, \pi]$
Deduce that $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5-7}-\frac{1}{7.9}+\ldots . .=\frac{1}{4}(\pi-2)$
(b) Find the half range Sine series for $f(x)=\pi x-x^{2}$ in (0, $\pi$ )
3. (a) i. Prove that the transpose of unitary matrix is unitary.
ii. Prove that the inverse of unitary matrix is unitary.
(b) Show that the matrix $\left[\begin{array}{ccc}3 & 7-4 i & -2 \\ 7+4 i & -2 & 3 \\ -2 & 5 i & 3-i\end{array}\right.$
is a Hermitian matrix.
5. Reduce the quadratic form $x^{2}+y^{2}+2 z^{2}-2 x y+4 x z+4 y z$ to the canonical form.
[15]
6. Solve the boundary value problem $\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial y^{2}}=0$ with $\frac{\partial u}{\partial x}(0, \mathrm{y})=\frac{\partial u}{\partial x}(\pi, \mathrm{y})=u(\mathrm{x}$, $\pi)=0$ and $u(x, 0)=x^{2}, 0<x<\pi$
[15]
7. (a) Show that the system of equations $2 x_{1}-2 x_{2}+x_{3}=\lambda x_{1} ; 2 x_{1}-3 x_{2}+2 x_{3}=$ $-\lambda x_{2} ;-x_{1}+2 x_{2}=\lambda x_{3}$ can possess a non-trivial solutions only if $\lambda=1, \lambda=$ -3 . Obtain the general solution in each case.
(b) Solve completely the system of equations:

$$
2 x-2 y+5 z+3 w=0 ; 4 x-y+z+w=0
$$

$$
\begin{equation*}
3 x-2 y+3 z+4 w=0 ; x-3 y+7 z+6 w=0 \tag{15}
\end{equation*}
$$

8. (a) Form the partial differential equation from $\mathrm{f}\left(\mathrm{xyz}, x^{2}+y^{2}+z^{2}\right)=0$
(b) Solve the partial differential equation

$$
\begin{equation*}
\mathrm{z}\left(p^{2}-q^{2}\right)=\mathrm{x}-\mathrm{y} \tag{15}
\end{equation*}
$$

