Code No: A109210201 **R09** **Set No. 2**

II B.Tech I Semester Examinations,MAY 2011
MATHEMATICS-III
Common to ICE, E.COMP.E, ETM, EIE, ECE, EEE

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Express x^3+3x^2+4x+3 interms of Legendre polynomial.
   (b) Evaluate $\int_0^1 x(1-x^2)^{-\frac{1}{2}}U_4(x)dx$. [8+7]

2. Find the Taylor's and Laurent's series which represents the function $\frac{(z^2-1)}{(z+3)(z+2)}$ when
   (a) $|z| \leq 2$
   (b) $2 < |z| < 3$
   (c) $|z| \geq 3$ [15]

3. (a) Find the analytic function f(z) = u(r, θ)+ i v(r, θ) such that $u(r,\theta) = r^2\cos 2\theta - r\cos\theta + 2$.
   (b) S.T. The function u= 1/2 log (x^2+y^2) is harmonic & find its conjugate. [15]

4. (a) Evaluate $\int_0^\infty \frac{\log x}{1+x^2}\,dx$
   (b) Find the Residues of f (z) = $\frac{1}{z(e^z-1)}$ [8+7]

5. (a) Evaluate $\int_c (z^2+3z+2)dz$ where C is the arc of the cycloid x=a(θ + sinθ), y = a(1 - cosθ) between the points (0,0) & (aπ, 2a)
   (b) Evaluate $\int_c (z^2+3z)dz$ along the straight line from (2,0) to (2,2) and then from (2,2) to (0,2) [15]

6. (a) Draw the undirected graph represented by the adjacency matrix A given below.
   $$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$
   (b) Convert the following tree into binary tree (figure 1). [7+8]

7. Using Jacobi Series, P.T. $J_0^2 + 2\{J_1^2 + J_2^2 + --\} = 1$ [15]

8. (a) Find the points at which w = cosh z is not conformal.
   (b) Find the image of the strip bounded by x = 0 and x = $\frac{\pi}{4}$ under the transformation w = cos z [7+8]

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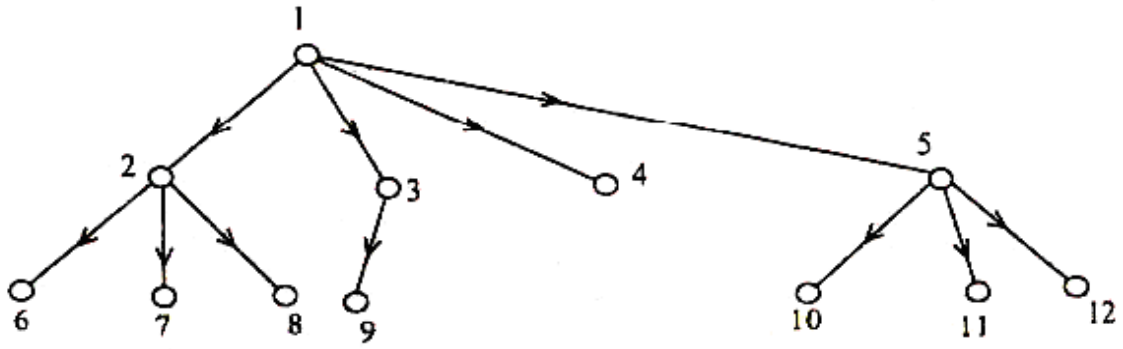

Figure – 1

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Code No: A109210201 **R09** **Set No. 4**

II B.Tech I Semester Examinations,MAY 2011
MATHEMATICS-III
Common to ICE, E.COMP.E, ETM, EIE, ECE, EEE

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks
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1. Find the bilinear transformation which maps $z_1 = 1, z_2 = i, z_3 = -1$ in to the points $w_1 = i, w_2 = 0, w_3 = -i$ respectively. Find the fixed and critical points of this transformation and find the image of $|z| < 1$ [15]

2. (a) Show that when $|z+1| < 1$, $z^{-2} = 1 + \sum_{n=1}^{\infty} (n+1)z^n$

   (b) Find the Laurent series expansion of f (z) = $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z+2| < 5$. [7+8]

3. (a) S.T. J_3(x) is an even function when 'n' is even & odd function when 'on' is odd.

   (b) Evaluate $\int_0^\alpha x^{-3/2}(1-e^{-x})$dx or$\int_0^\alpha t^{-3/2}(1-e^{-t})dt$ [15]

4. (a) Evaluate contour integral of the real integral $\int_0^{2\pi} \frac{Cos3\theta \ \ d\theta}{5-4\cos\theta}$

   (b) The only singularities of a single valued function f (z) are poles of order 1 and 2 at z = -1 and z = 2 with residues at these poles i and 2 respectively. If f (0) = $\frac{7}{4}$, f(1) = $\frac{5}{4}$, determine the function f (z). [8+7]

5. (a) Verify whether the graph G given below Figure 2 contain an Eulerian circuit.

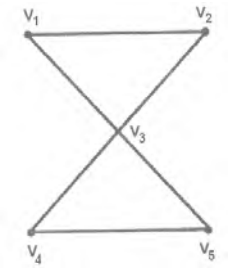

Figure 2:

(b) Using D F S (Depth first search) to produce a spanning tree for the simple graph Figure 3. [7+8]

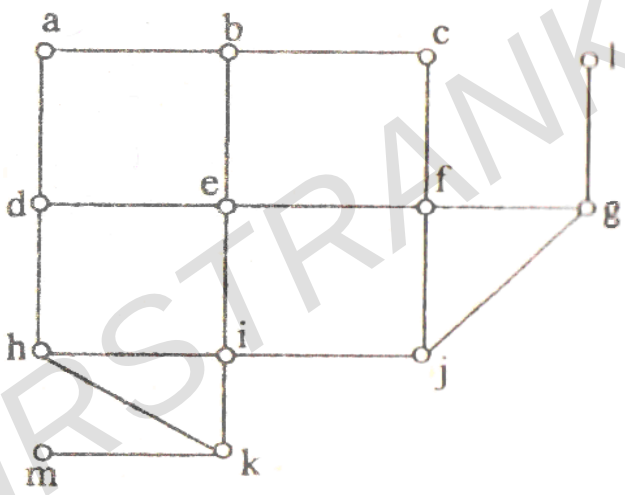

Figure 3:

6. (a) S.T. the real & imaginary parts of the function w = log z satisfy the C-R equations when z is not zero.

   (b) S.T. f(z) = z+2$\overline{z}$ is not analytic anywhere in the complex plane. [15]

7. Let 'C' denotes the boundary of the square whose sides lie along the lenes x = ±2, y = ±2 where 'C' is described in the positive sense evaluate the following integrals

   (a) $\int\limits_C \frac{\tan(z/2)}{(z-x_0)^2} dx \; (|x_0| < 2)$

   (b) $\int\limits_C \frac{\cosh z}{z^4} dz$ [15]

8. P.T. $\frac{1}{\sqrt{1-2xt+t^2}}$= $P_0(x) + P_1(x)t + P_2(x)t^2 + - - - - -$ [15]

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Code No: A109210201 **R09** **Set No. 1**

II B.Tech I Semester Examinations,MAY 2011
MATHEMATICS-III
Common to ICE, E.COMP.E, ETM, EIE, ECE, EEE

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Find the image of the triangle with vertices at i,1+i,1-i in the z-plane, under the transformation $e^{\frac{5\Pi i}{3}}.(z-2+4i)$
   (b) Find the image of the infinite strip, $0 < y < \frac{1}{2}$ under the mapping function w $= \frac{1}{z}$. [7+8]
2. (a) Find the residue of $\frac{cos(z-i)}{(z+2i)^3}$.
   (b) Evaluate $\int \frac{sinz\,dz}{z\,cosz}$ where c is $|z| = \pi$. [8+7]
3. (a) Evaluate $\int\limits_c (y^2+2xy)dx + (x^2-2xy)dy$ where 'C' is the boundary of the region by y = x^2 & x = y^2
   (b) Evaluate $\int\limits_0^{1+i} z^2 dz$ along y = x^2 [15]
4. (a) S.T. an analytic function of constant absolute value is constant.
   (b) S.T. both the real & imaginary parts of an analytic function are harmonic. [15]
5. (a) P.T. $\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x)$
   (b) S.T. $4J_n^{11}(x)$ = J_{n-2}(x) - 2 J_n(x) + J_{n+2} (x) [15]
6. (a) S.T. $\int_0^1 x^2 P_{n+1}(x) P_{n-1}(x) dx = \frac{2n(n+1)}{(4n^2-1)(2n+3)}$
   (b) S.T. $2P_3(x)+3P_1(x) = 5x^3$ [15]
7. Find the minimal spanning tree for the following Graph (Figure 4). [15]
8. Expand f(z)= $\frac{z+3}{z(z^2-z-2)}$ in powers of z.
   (a) With in the unit circle about the origin
   (b) With in the annular region between the concentric circles about the origin having radii 1 and 2 respectively.
   (c) The exterior to the circle of radius 2. [15]

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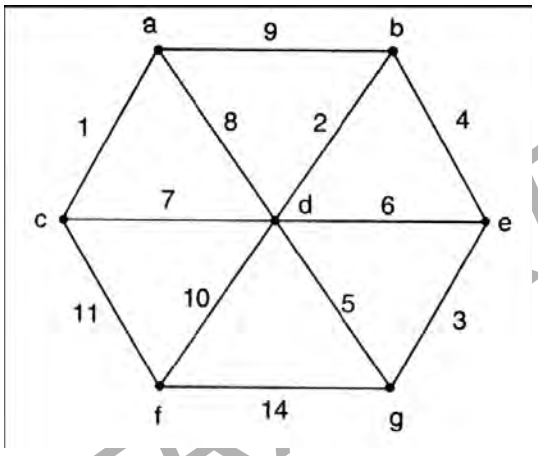

Figure – 4

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Code No: A109210201 **R09** **Set No. 3**

II B.Tech I Semester Examinations,MAY 2011
MATHEMATICS-III
Common to ICE, E.COMP.E, ETM, EIE, ECE, EEE

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks

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1. Expand the function f (z) = $\frac{4z+4}{z(z-3)(z+2)}$ in powers of z, when
   (a) $|z| \leq 1$
   (b) $1 \leq |z| \leq 2$
   (c) $|z| > 2$ [15]
2. (a) Show that the transformation $w = \frac{3-z}{z-2}$ transforms the circle $|z - \frac{5}{2}| = \frac{1}{2}$ in the z-plane in to the imaginary axis in the w-plane.
   (b) For the mapping w = 1/z, find the image of the family of circles $x^2 + y^2 = ax$,where a is real. [8+7]
3. (a) Determine the value of $J_{1/2}$ (x)
   (b) P.T. $\int_0^{\pi/2} \sin^7\theta \cos^7\theta d\theta = \frac{1}{280}$ [15]
4. (a) Find 'k' such that f(x,y) = $x^3 + 3kxy^2$ may be harmonic & find its conjugate.
   (b) Find the conjugate harmonic of u = $e^{x^2-y^2}$ cos2xy. Hence find f(z) in terms of 'z'. [15]
5. (a) Find whether the following (figure 5) is a binary tree.
   (b) Suppose all vertices in a graph have odd degree 'K' Show that total number of edges in G is multiple of K. [8+7]
6. If $P_6(2)$= a & $P_7(2)$=b, then P.T. [15]
   (a) $P_6^1(2) = \frac{7}{3}(b - 2a)$ (b) $P_8(2) = \frac{1}{8}(30b - 7a)$

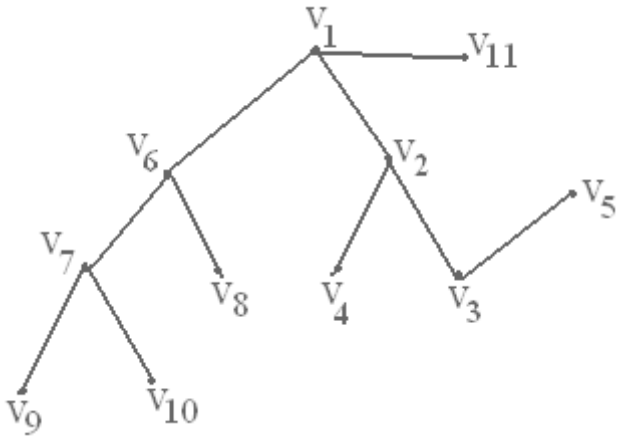

Figure 5:

7. (a) Using complex variable techniques evaluate $\int_0^{2\pi} \frac{\sin^2\theta \cdot d\theta}{5-4\cos\theta}$.

   (b) The only singularities of a single valued function f(z) are poles of order 2 and 1 at z =1 & z = 2 with residues of these poles as 1 and 3 respectively. If f (0) = $\frac{3}{2}$, f (-1) = 1, determine the function. [8+7]

8. (a) From the integral $\int_0 \frac{dz}{z+4}$ S.T $\int_0^{\pi} \frac{1+4\cos\theta}{17+8\cos\theta} = 0$ where C: $|z| = 1$

   (b) If C is a closed curve described in + ve sense and f(z_0)= $\int_c \frac{z^4+z}{(z-z_0)^4} dz$ show that f(z_0) = $8\pi i z_0$ is where z_0 is a point inside 'C' and f(z_0) = 0 if z_0 lies outside 'C'. [15]

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