Code No: A109210201   **R09**   **Set No. 2**

II B.Tech I Semester Examinations,MAY 2011
MATHEMATICS-III
Common to ICE, E.COMP.E, ETM, EIE, ECE, EEE

Time: 3 hours   Max Marks: 75

Answer any FIVE Questions
All Questions carry equal marks
⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Express x^3+3x^2+4x+3 interms of Legendre polynomial.

   (b) Evaluate $\int_0^1 x(1-x^2)^{-\frac{1}{2}}U_4(x)dx$.   [8+7]

2. Find the Taylor's and Laurent's series which represents the function $\frac{(z^2-1)}{(z+3)(z+2)}$ when

   (a) $|z| \leq 2$

   (b) $2 < |z| < 3$

   (c) $|z| \geq 3$ [15]

3. (a) Find the analytic function f(z) = u(r, θ)+ i v(r, θ) such that $u(r,\theta) = r^2\cos 2\theta - r\cos\theta + 2$.

   (b) S.T. The function u= 1/2 log (x^2+y^2) is harmonic & find its conjugate. [15]

4. (a) Evaluate $\int_0^\infty \frac{\log x}{1+x^2}dx$

   (b) Find the Residues of f (z) = $\frac{1}{z(e^z-1)}$ [8+7]

5. (a) Evaluate $\int_c (z^2 + 3z + 2)dz$ where C is the arc of the cycloid x=a($\theta + \sin\theta$), y = a(1 - cosθ) between the points (0,0) & (aπ, 2a)

   (b) Evaluate $\int_c (z^2 + 3z)dz$ along the straight line from (2,0) to (2,2) and then from (2,2) to (0,2)   [15]

6. (a) Draw the undirected graph represented by the adjacency matrix A given below.
   $$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

   (b) Convert the following tree into binary tree (figure 1).   [7+8]

7. Using Jacobi Series, P.T. $J_0^2 + 2\{J_1^2 + J_2^2 + --\} = 1$ [15]

8. (a) Find the points at which w = cosh z is not conformal.

   (b) Find the image of the strip bounded by x = 0 and x = $\frac{\pi}{4}$ under the transformation w = cos z [7+8]
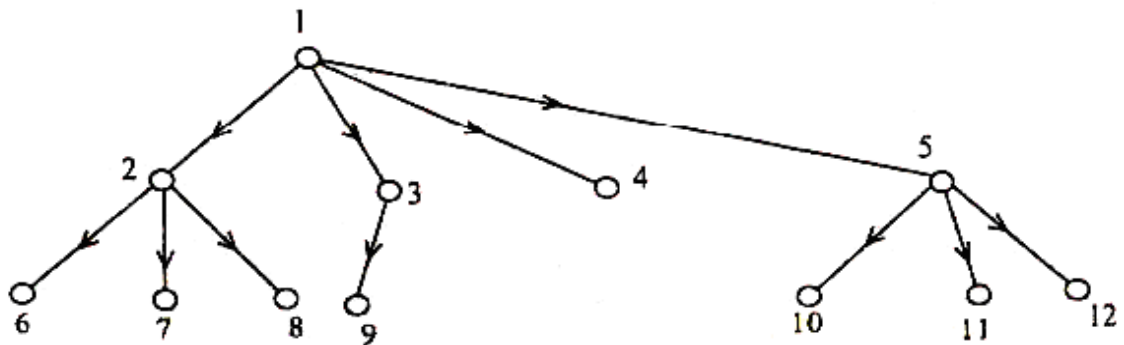
⋆ ⋆ ⋆ ⋆

1

Figure – 1

****

Code No: A109210201 $\boxed{\textbf{R09}}$ $\boxed{\textbf{Set No. 4}}$

II B.Tech I Semester Examinations,MAY 2011
MATHEMATICS-III
Common to ICE, E.COMP.E, ETM, EIE, ECE, EEE
Time: 3 hours Max Marks: 75
Answer any FIVE Questions
All Questions carry equal marks
⋆ ⋆ ⋆ ⋆ ⋆

1. Find the bilinear transformation which maps $z_1 = 1, z_2 = i, z_3 = -1$ in to the points $w_1 = i, w_2 = 0, w_3 = -i$ respectively. Find the fixed and critical points of this transformation and find the image of $|z| < 1$ [15]

2. (a) Show that when $|z + 1| < 1$, $z^{-2} = 1 + \sum\limits_{n=1}^{\infty} (n + 1)z^n$

 (b) Find the Laurent series expansion of f (z) = $\frac{z^2 - 6z - 1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z + 2| < 5$. [7+8]

3. (a) S.T. $J_3(x)$ is an even function when 'n' is even & odd function when 'on' is odd.

 (b) Evaluate $\int_0^\alpha x^{-3/2}(1 - e^{-x})\mathbf{dx}$ or $\int_0^\alpha t^{-3/2}(1 - e^{-t})dt$ [15]

4. (a) Evaluate contour integral of the real integral $\int\limits_0^{2\pi} \frac{Cos3\theta \ d\theta}{5 - 4\cos\theta}$

 (b) The only singularities of a single valued function f (z) are poles of order 1 and 2 at z = -1 and z = 2 with residues at these poles i and 2 respectively. If f (0) = $\frac{7}{4}$, f(1) = $\frac{5}{4}$, determine the function f (z). [8+7]

5. (a) Verify whether the graph G given below Figure 2 contain an Eulerian circuit.
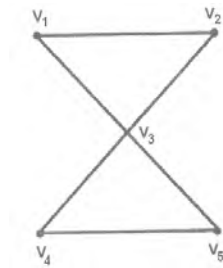
2

Figure 2:

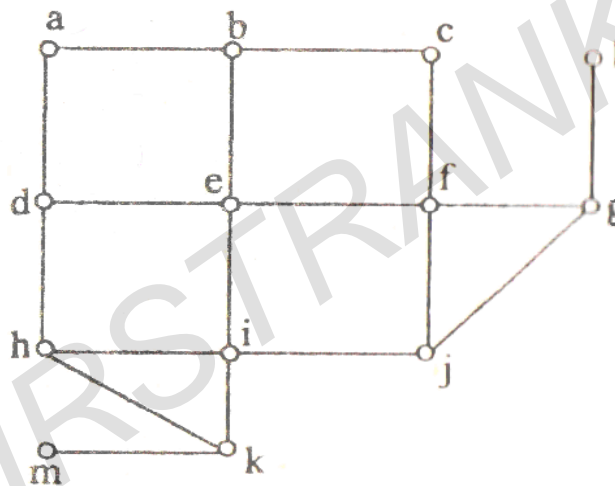(b) Using D F S (Depth first search) to produce a spanning tree for the simple graph Figure 3.

[7+8]

Figure 3:

6. (a) S.T. the real & imaginary parts of the function w = log z satisfy the C-R equations when z is not zero.

   (b) S.T. f(z) = z+2$\bar{z}$ is not analytic anywhere in the complex plane. [15]

7. Let 'C' denotes the boundary of the square whose sides lie along the lenes x = ±2, y = ±2 where 'C' is described in the positive sense evaluate the following integrals

   (a) $\int_C \frac{\tan(z/2)}{(z-x_0)^2}dx\,(|x_0|<2)$

   (b) $\int_C \frac{\cosh z}{z^4}dz$ [15]

8. P.T. $\frac{1}{\sqrt{1-2xt+t^2}}= P_0(x) + P_1(x)t + P_2(x)t^2 + -----$ [15]

⋆ ⋆ ⋆ ⋆ ⋆

Code No: **A109210201** | **R09** | | **Set No. 1** |

II B.Tech I Semester Examinations,MAY 2011
MATHEMATICS-III
Common to ICE, E.COMP.E, ETM, EIE, ECE, EEE
Time: 3 hours   Max Marks: 75
Answer any FIVE Questions
All Questions carry equal marks
⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Find the image of the triangle with vertices at i,1+i,1-i in the z-plane, under the transformation $e^{\frac{5\Pi i}{3}}.(z-2+4i)$

   (b) Find the image of the infinite strip, $0 < y < \frac{1}{2}$ under the mapping function w $= \frac{1}{z}$.   [7+8]

2. (a) Find the residue of $\frac{cos(z-i)}{(z+2i)^3}$.

   (b) Evaluate $\int \frac{sinz\,dz}{z\,cosz}$ where c is $|z| = \pi$.   [8+7]

3. (a) Evaluate $\int_c (y^2 + 2xy)dx + (x^2 - 2xy)dy$ where 'C' is the boundary of the region by y = x² & x = y²

   (b) Evaluate $\int_0^{1+i} z^2 dz$ along y = x²   [15]

4. (a) S.T. an analytic function of constant absolute value is constant.

   (b) S.T. both the real & imaginary parts of an analytic function are harmonic.
                                                                                     [15]

5. (a) P.T. $\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x)$

   (b) S.T. $4J_n^{11}(x) = J_{n-2}(x) - 2 J_n(x) + J_{n+2}(x)$ [15]

6. (a) S.T. $\int_0^1 x^2 P_{n+1}(x)P_{n-1}(x)dx = \frac{2n(n+1)}{(4n^2-1)(2n+3)}$

   (b) S.T. $2P_3(x)+3P_1(x) = 5x^3$ [15]

7. Find the minimal spanning tree for the following Graph (Figure 4).   [15]

8. Expand f(z)= $\frac{z+3}{z(z^2-z-2)}$ in powers of z.

   (a) With in the unit circle about the origin

   (b) With in the annular region between the concentric circles about the origin having radii 1 and 2 respectively.

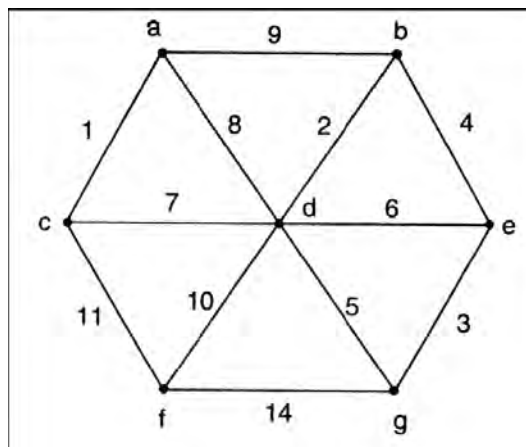   (c) The exterior to the circle of radius 2.   [15]

⋆ ⋆ ⋆ ⋆ ⋆

4

Figure – 4

******

Code No: A109210201　　　　　**R09**　　　　　**Set No. 3**

II B.Tech I Semester Examinations,MAY 2011
MATHEMATICS-III
Common to ICE, E.COMP.E, ETM, EIE, ECE, EEE

Time: 3 hours　　　　　　　　　　　　　　　　　　Max Marks: 75

Answer any FIVE Questions
All Questions carry equal marks
⋆ ⋆ ⋆ ⋆ ⋆

1. Expand the function f $(z) = \frac{4z+4}{z(z-3)(z+2)}$ in powers of z, when

   (a) $|z| \leq 1$
   (b) $1 \leq |z| \leq 2$
   (c) $|z| > 2$　　　　　　　　　　　　　　　　　　　　　　　　　[15]

2. (a) Show that the transformation $w = \frac{3-z}{z-2}$ transforms the circle $|z - \frac{5}{2}| = \frac{1}{2}$ in the z-plane in to the imaginary axis in the w-plane.

   (b) For the mapping w = 1/z, find the image of the family of circles $x^2 + y^2 = ax$,where a is real.　　　　　　　　　　　　　　　　　　[8+7]

3. (a) Determine the value of J$_{1/2}$ (x)

   (b) P.T. $\int_0^{\pi/2} \sin^7 \theta \cos^7 \theta d\theta = \frac{1}{280}$　　　　　　　　　　　　　[15]

4. (a) Find 'k' such that f(x,y) = $x^3 + 3kxy^2$ may be harmonic & find its conjugate.

   (b) Find the conjugate harmonic of u = $e^{x^2-y^2}$ cos2xy. Hence find f(z) in terms of 'z'.　　　　　　　　　　　　　　　　　　　　　　　　　[15]

5. (a) Find whether the following (figure 5) is a binary tree.

   (b) Suppose all vertices in a graph have odd degree 'K' Show that total number of edges in G is multiple of K.　　　　　　　　　　　　　[8+7]

6. If P$_6$(2)= a & P$_7$(2)=b, then P.T.　　　　　　　　　　　　　　[15]

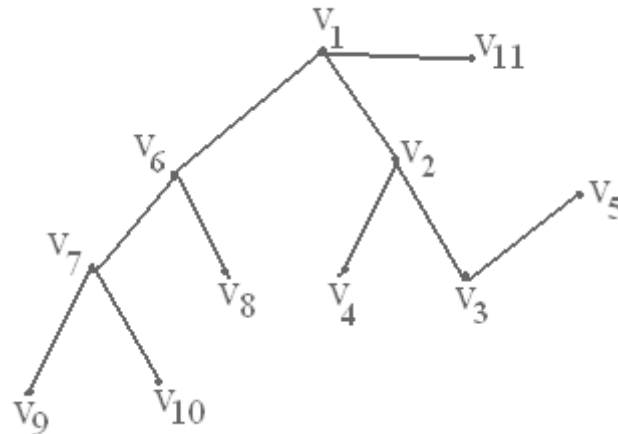   (a) $P_6^1(2) = \frac{7}{3}(b - 2a)$　　　　(b)　$P_8(2)=\frac{1}{8}(30b-7a)$

5

Figure 5:

7. (a) Using complex variable techniques evaluate $\int_0^{2\pi} \frac{\sin^2\theta\, d\theta}{5-4\cos\theta}$.

   (b) The only singularities of a single valued function f(z) are poles of order 2 and 1 at z =1 & z = 2 with residues of these poles as 1 and 3 respectively. If f (0) = $\frac{3}{2}$, f (-1) = 1, determine the function. [8+7]

8. (a) From the integral $\int_0 \frac{dz}{z+4}$ S.T $\int_0^\pi \frac{1+4\cos\theta}{17+8\cos\theta} = 0$ where C: $|z| = 1$

   (b) If C is a closed curve described in + ve sense and f(z_0)= $\int_c \frac{z^4+z}{(z-z_0)^4} dz$ show that f(z_0) = $8\pi i z_0$ is where z_0 is a point inside 'C' and f(z_0) = 0 if z_0 lies outside 'C'. [15]

$\star\star\star\star\star$