Code No: R31031 **R10** **Set No: 1**

III B.Tech. I Semester Regular Examinations, November/December - 2012

FINITE ELEMENT METHODS

(Common to Mechanical Engineering & Auto Mobile Engineering)

Time: 3 Hours **Max Marks: 75**

Answer any FIVE Questions

All Questions carry equal marks

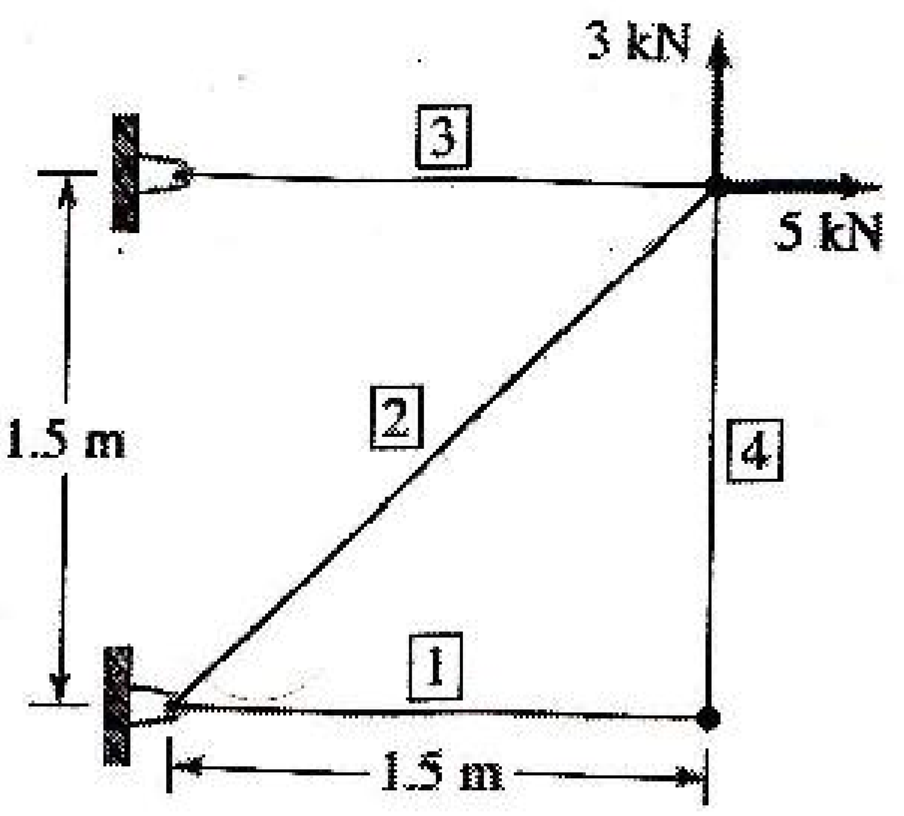
*****

1. (a) Explain the difference between the plane stress and plane strain condition.
   (b) Derive the element stiffness matrix for one dimensional element.

2. (a) Explain node numbering scheme. What is its significance?
   (b) Explain the discretisation process with an example.

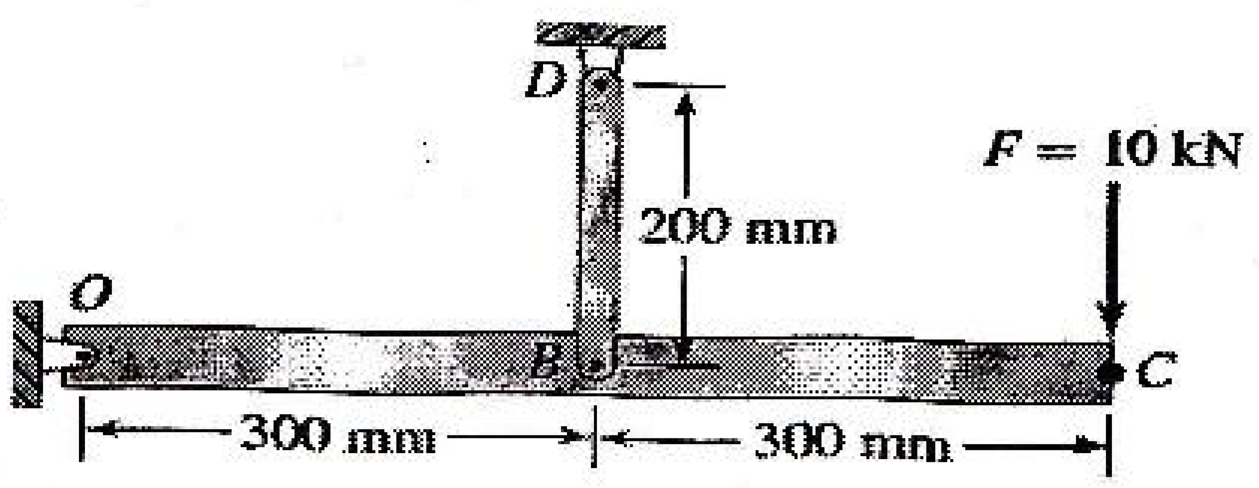
3. The plane truss shown in figure 1 is composed of members having a square 15 mm × 15 mm cross section and modulus of elasticity $E = 69$ GPa.
   (a) Assemble the global stiffness matrix.
   (b) Compute the nodal displacements in the global coordinate system for the loads shown.

figure 1



4. Determine the deflection of point C in figure 2. The modulus of elasticity of the beam OBC is 207 GPa and the dimensions of the cross section are 40 mm X 40 mm. For the elastic rod BD, the modulus of elasticity is 60 GPa and the cross sectional area is 78.54 mm^2. Also find the support reactions and stress within the each element.

Figure 2



5. (a) Derive the expression for consistent load vector due to self-weight in a Constant Strain Triangle element.
   (b) The vertices of a Constant Strain Triangle element are given by (3, 2), (7, 9) and (12, 5). Determine the Shape functions and strain - nodal displacement matrix, B.

6. (a) Use Gaussian quadrature to obtain an exact value for the integral $I = \int_{-1}^{1}\int_{-1}^{1}(r^3 - 1)(s-1)^2\,drds$
   (b) The vertices of a four noded quadrilateral element is given by A (0, 0), B (20, 0), C (20, 10) and D (0, 10). All the dimensions are in mm. Determine the Jacobian and strain displacement relation at a point P (15, 8).

7. (a) Explain the different boundary conditions that can be applied to a one dimensional heat transfer problem
   (b) A metallic fin which is 1mm thick and 600 mm long extends from plane wall whose temperature is 300 ^{0}C. Determine the temperature distribution from the fin to the air at 20 ^{0}C with h = 9 W/m^2 ^{0}C. Take the thermal conductivity of the fin, k= 20 W/m ^{0}C. Width of the fin is 100 mm.

8. (a) Differentiate between the consistent mass matrix and lumped mass matrix.
   (b) A uniform cantilever beam of length L, Young's modulus E and density ρ is modelled by a single element. Calculate frequencies and modes by consistent mass matrix.

*****

Code No: R31031 | R10 | Set No: 2

III B.Tech. I Semester Regular Examinations, November/December - 2012

FINITE ELEMENT METHODS

(Common to Mechanical Engineering & Auto Mobile Engineering)

Time: 3 Hours **Max Marks: 75**

Answer any FIVE Questions

All Questions carry equal marks

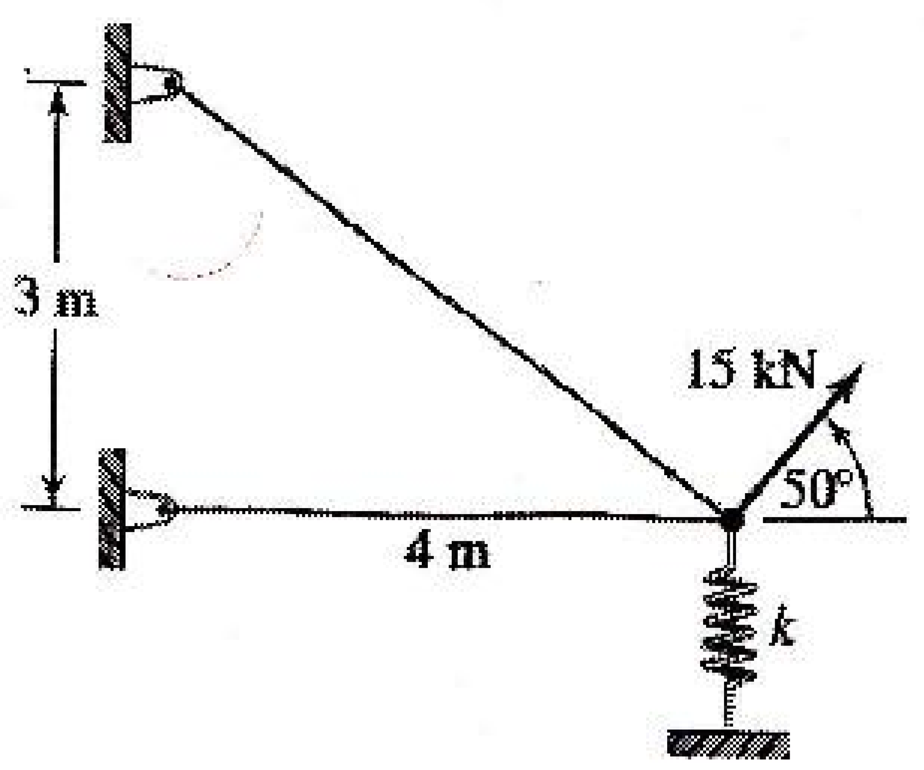
*****

1. Find the approximate solution to the following boundary value problem by using Galerkin method. Compare the solution with the exact solution. $\frac{d^2u}{dx^2} = x \quad 0 < x < 1; \quad u(0) = 0$ and $u(1) = 0$

2. (a) Define the shape function. What are the properties of a shape functions?
   (b) Explain elimination method for boundary conditions.

3. Figure 1 shows a two-member plane truss supported by a linearly elastic spring. The truss members are of a solid circular cross section having d = 20 mm and E = 80 GPa. The linear spring has stiffness constant 50 N/mm.
   Assemble the system global stiffness matrix and calculate the global displacements of the unconstrained node.

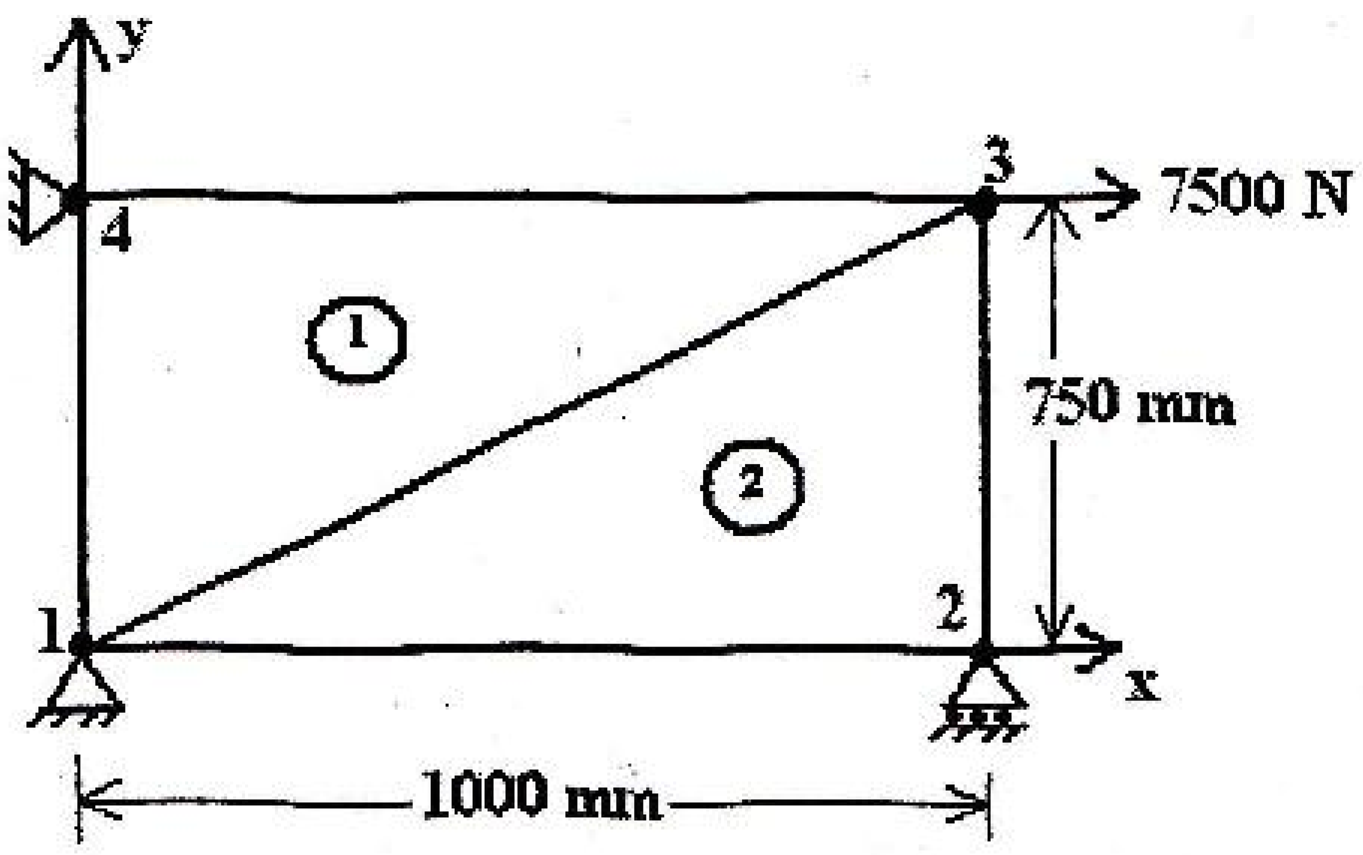
Figure 1



4. (a) What are the nodal variables associated with a 2 noded flexural element?
   (b) Derive the interpolation functions that describe the distribution of displacement in terms of the nodal variables for a 2 noded flexural element. Present these interpolation functions graphically.
   (c) Develop the expression for the normal stress using the above interpolation functions.

5. Determine the nodal displacement and element stresses for the thin plate shown in figure 2. Body force is neglected. Take, Thickness (t) = 10mm, Young's modulus (E) = 2×10^5 N/mm^2, Poisson's ratio (ν) = 0.25. Assume plane stress condition.

Figure 2



6. A four noded quadrilateral element is defined by the vertices A (0, 0), B (2, 0), C (2, 1) and D (0, 1) in counter clockwise. Evaluate the Jacobian matrix and its determinant for all the 2 X 2 Gauss quadrature points.

7. Using Galerkin approach, derive the stiffness matrix for a torsional triangular element. Explain the procedure of evaluating shear stress component.

8. A simply supported beam of span length 800 mm is having a rectangular cross section 75 mm X 25 mm. Young's modulus E and density ρ of the material are given by 200 GPa and 7850 kg/m^3. Evaluate eigenvalues and eigenvectors of the beam using two element model.

*****

Code No: R31031 **R10** **Set No: 3**

III B.Tech. I Semester Regular Examinations, November/December - 2012

FINITE ELEMENT METHODS

(Common to Mechanical Engineering & Auto Mobile Engineering)

Time: 3 Hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks
*****

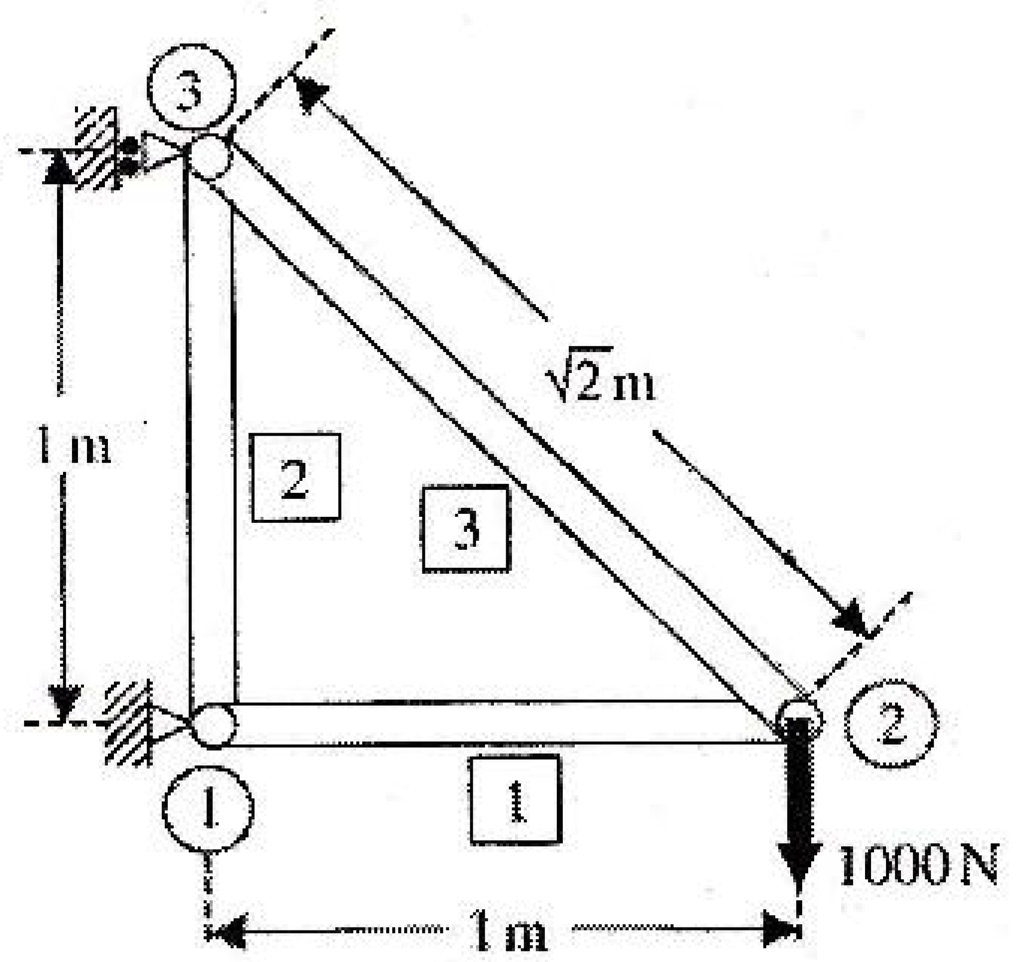
1. The functional form of a bar clamped at one end and left free at the other end and subjected to uniform axial load q is given by $I = \int_0^L \left[\frac{1}{2} AE \left(\frac{du}{dx} \right)^2 - qu \right] dx$

   The essential boundary is $u(0) = 0$, obtain the approximate solution to the problem by using Rayleigh-Ritz method.

2. (a) Derive the element stiffness matrix, force vector for a two noded axial element using potential energy approach and the natural coordinate system for shape functions. Assume constant body force and traction.
   (b) Compare the elimination and the penalty approach in imposing the essential boundary conditions.

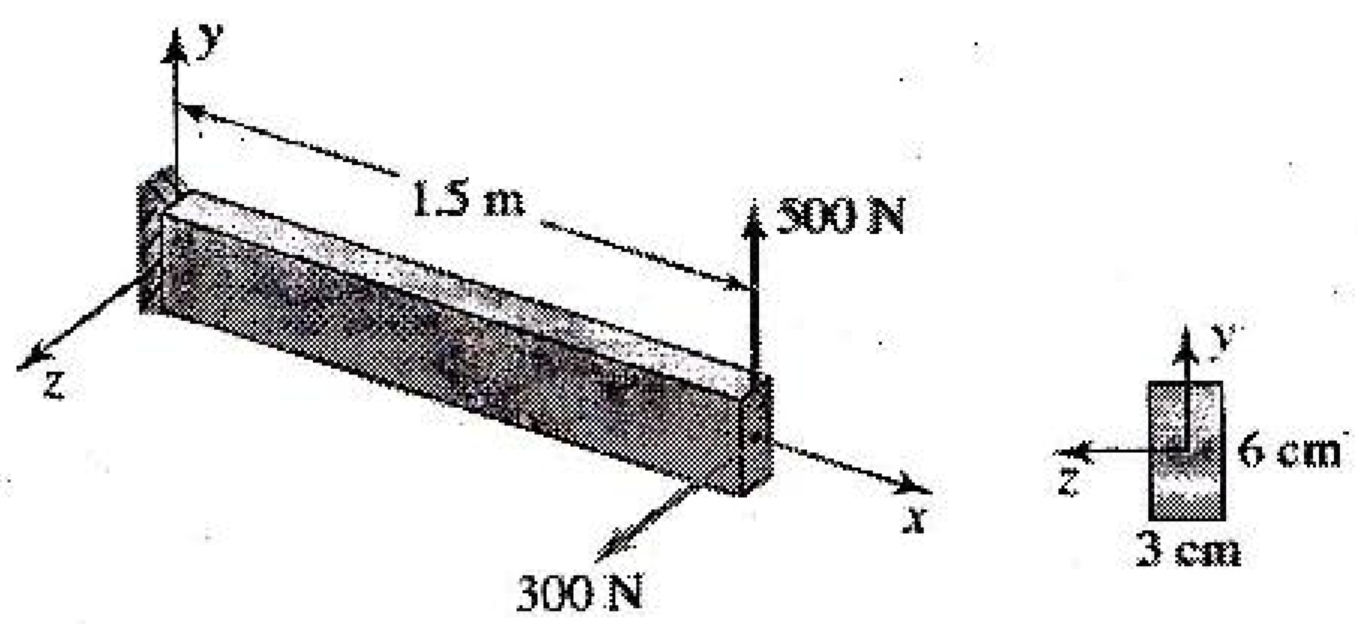
3. The plane truss shown in Figure 1 is composed of members having 0.1 m^2 cross section area and modulus of elasticity $E = 70$ GPa. (a) Assemble the global stiffness matrix.(b) Compute the nodal displacements in the global coordinate system.

Figure 1



4. The cantilevered beam depicted in Figure 2 is subjected to two-plane bending. The loads are applied such that the planes of bending correspond to the principal moments of inertia. Model the beam as a single element and compute the deflections of the free end, node 2. Determine the exact location and magnitude of the maximum bending stress. (Use $E = 207$ GPa.)

Figure 2



|""|""||""||"""|

5. Derive the shape functions for a CST element using natural coordinate system. Verify the properties of the shape functions. Using these shape functions establish strain-displacement relationship for CST element.

6. (a) Use Gaussian quadrature to obtain exact values for the following integrals. Verify exactness by analytical integration. $\int_1^6 (y^3 + 2y)dy$

   (b) For a four noded quadrilateral element derive the relationship between the gradients of the field variable in the global coordinate system to the natural coordinate system.

7. (a) A three noded element is defined by the vertices A (1, 1), B (10, 4) and C (6, 7) in counter clockwise. The temperatures at these nodes are given by 120 ^{0}C, 140 ^{0}C and 80 ^{0}C respectively. Assuming a linear distribution, determine the temperature at point P (7, 4).

8. (a) Derive the consistent mass matrix for a linear triangle element.
   (b) What are the properties of the eigenvectors? Explain the evaluation of the eigen vectors by any one method.

*****

Code No: R31031 **R10** **Set No: 4**

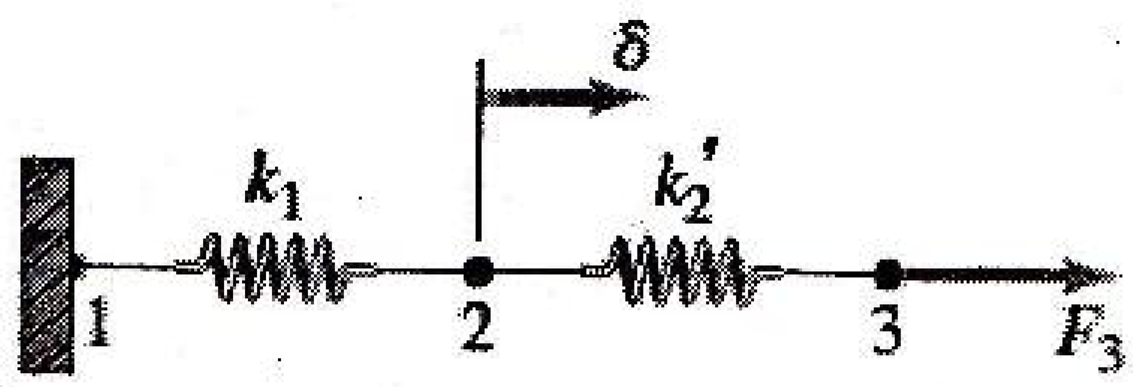
III B.Tech. I Semester Regular Examinations, November/December - 2012
FINITE ELEMENT METHODS
(Common to Mechanical Engineering & Auto Mobile Engineering)

Time: 3 Hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks
*****

1. Explain the principle of minimum potential energy. Use this principle to determine the force F_3 required for the displacement of δ for the spring system in figure 1

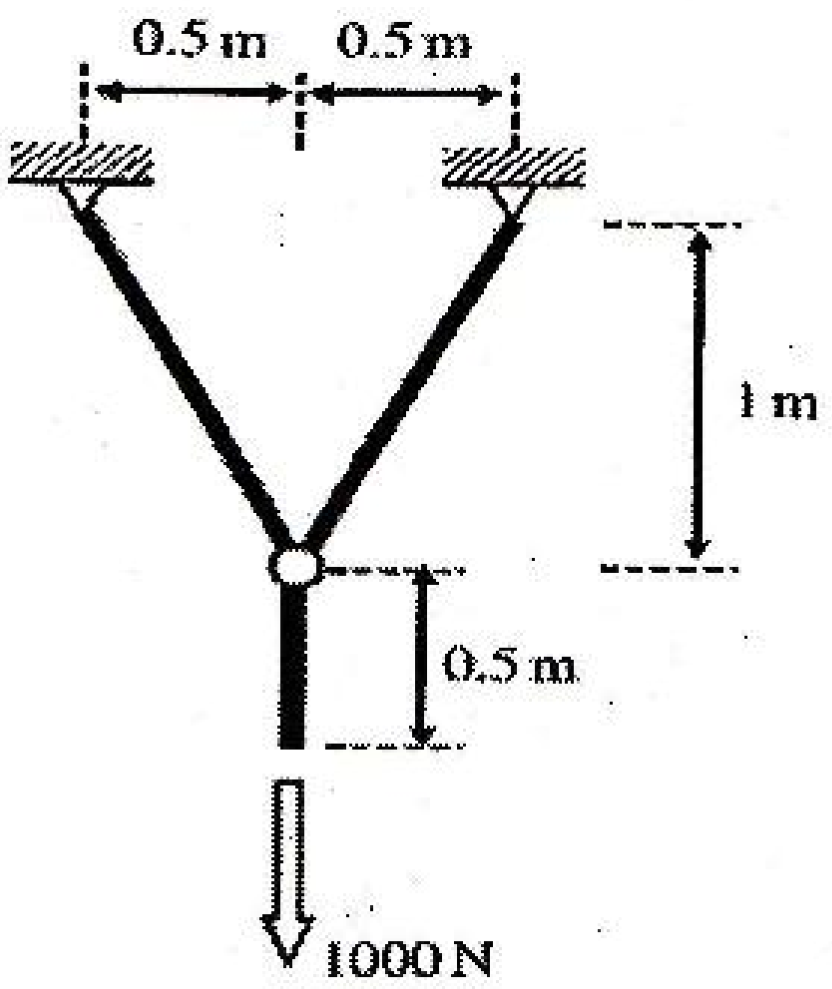
Figure 1



2. (a) What are the convergence requirements that an interpolation polynomial must satisfy?
(b) Explain different mesh generation techniques.

3. The plane truss shown in Figure 2 is composed of members having 0.01 m^2 cross section area and modulus of elasticity E = 70 GPa.
(a) Assemble the global stiffness matrix.
(b) Compute the nodal displacements in the global coordinate system
(c) Compute the axial stress in each element

Figure 2

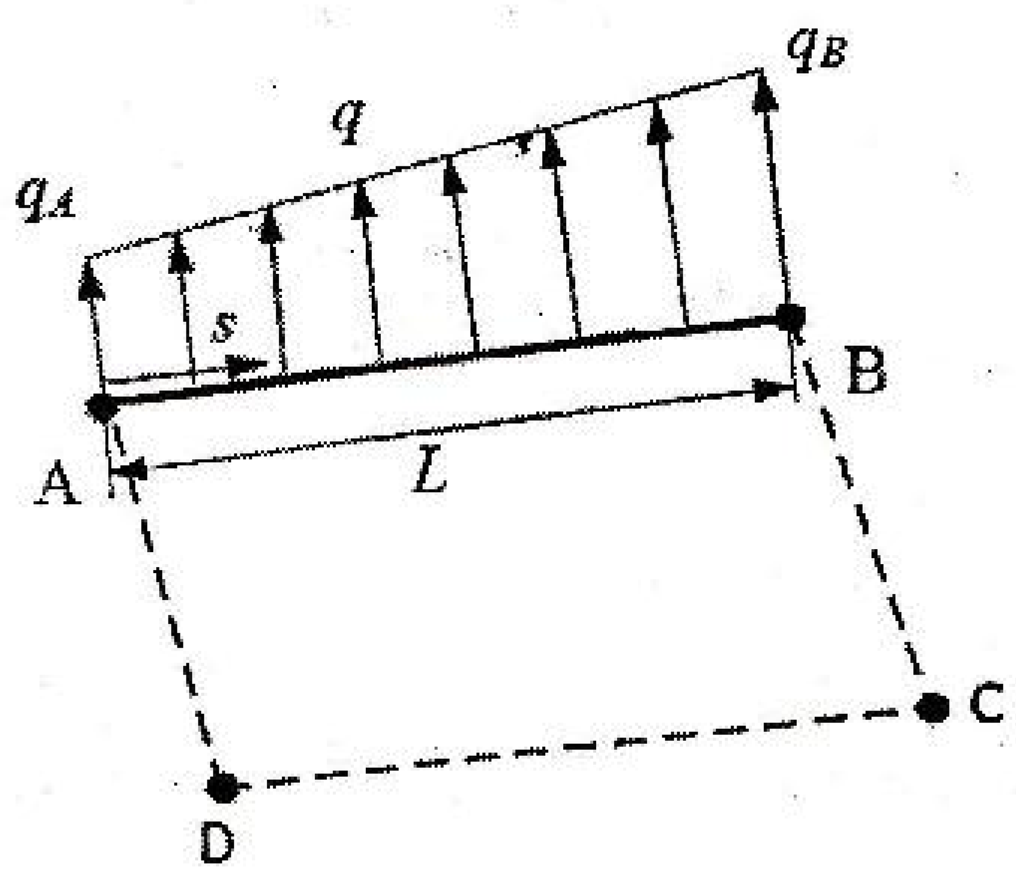


4. (a) Derive the stiffness matrix for 2-noded flexural element using minimum potential approach. Use Hermite shape functions in dimensionless length coordinate.
(b) Present the work equivalent nodal force and moments for uniform distributed load on 2-noded flexural element.

5. A long hollow cylinder of inside diameter 100 mm and outside diameter 140 mm is subjected to an internal pressure of $4 N/mm^2$. By using two elements on the 15mm length, calculate the displacements at the inner radius. Young's Modulus is 200 GPa and Poisson's Ratio =0.3.

6. (a) Differentiate between Sub-parametric, Iso-parametric and Super-parametric elements with examples.
(b) The traction on a four noded element ABCD is as shown in Figure 3. Derive the equivalent nodal force vector.

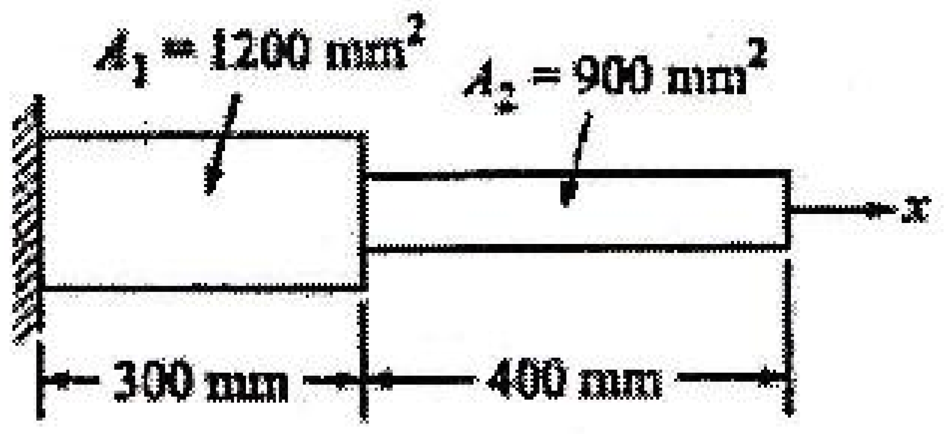
Figure 3



7. (a) Determine the temperature distribution in a plane wall of thickness 60 mm, which has an internal heat source of $0.3 MW/m^3$ and the thermal conductivity of the material is 21 W/m °C. Assume that the surface temperature of the wall is 40 °C. Let the cross-sectional area for heat flow, $A = 1 m^2$.
(b) What are the forced and natural boundary conditions in the Heat Transfer analysis?

8. Consider axial vibration of the steel bar as shown in Figure 4. Develop the global stiffness and mass matrix. Determine the natural frequency and mode shapes.

Figure 4



*****