Code No: X0522 **R07** SET - 1

II B. Tech I Semester, Supplementary Examinations, Nov – 2012
MATHEMATICAL FOUNDATIONS OF COMPTER SCIENCE
(Com. to CSE, IT)

Time: 3 hours Max. Marks: 80

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
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1. a) Obtain the principle conjunctive normal form of the A given by $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$ and hence find disjunctive normal form of A
   b) Show that $(P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology. (8M+8M)

2. a) Using CP or otherwise obtain the following implication
   $(\forall x)(P(x) \rightarrow Q(x)),\ (\forall x)(R(x) \rightarrow \neg Q(x)) \Rightarrow (\forall x)(R(x) \rightarrow \neg P(x))$
   b) Show that $R \rightarrow S$ can be derived from the premises
   $P \rightarrow (Q \rightarrow S), \neg R \vee P, Q$ (Use Rule CP) (8M+8M)

3. a) Show that the relation R is defined N x N by (a,b) R (c,d) if and only if $a+d = b+c$ is an equivalence relation.
   b) What is Lattice? Explain its properties. (8M+8M)

4. a) Let Z be the set of integers. o is an operation in Z defined by $a \circ b = a + b + 1$. Prove that (Z, o) is a semi group.
   b) Show that the set N of natural numbers is a semi group under the operation $x * y = \max\{x, y\}$. Is it a monoid? (8M+8M)

5. a) A sample of 80 people revealed that 25 like cinema and 60 like television programmes. Find the number of people who like both cinema and television programmes.
   b) Find the number of arrangements of the letters of TENNESSEE.
   c) A cricket 11 is to be selected out of 14 players of whom 5 are bowlers. Find the number of ways in which this can be done so as to include at least 3 bowlers.
   d) How many 3-digit numbers can be formed if 3 and 4 are not adjacent to each other.
   (4M+4M+4M+4M)

6. a) Solve the recurrence relation $a_n = a_{n-1} + n^2$ where $a_0 = 7$ by substitution method.
   b) Solve the recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = 0$ for $n >= 2$ with initial conditions $a_0 = 10$ and $a_1 = 41$ by using generating functions. (8M+8M)

7. a) What are the steps involved in graph traversal using Breadth-First Search algorithm? Illustrate with an example.
   b) State and prove Euler's formula for a connected planar graph. (8M+8M)

8. a) State and prove the first theorem of graph theory.
   b) What is Hamiltonian cycle? How to determine whether Hamiltonian cycle exists in a given graph or not? (8M+8M)
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Code No: X0522 **R07** SET - 2

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Answer any **FIVE** Questions
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1. a) Obtain PDNF and PCNF for the following $P \rightarrow (P \wedge (Q \rightarrow P)$
   b) Show that $\neg (P \vee (\neg P \wedge Q)) \Leftrightarrow \neg P \wedge Q$
   c) Prove that $(P \rightarrow Q) \Rightarrow (\neg Q \rightarrow \neg P)$ (8M+4M+4M)

2. a) Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$, $(\neg R \vee P)$ and Q.
   b) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises
   $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$, and $\neg M$ (8M+8M)

3. a) Let R denote a relation on the set of all ordered pairs of positive integers by
   (x,y) R (u,v) iff xv = yu. Show that R is an equivalence relation.
   b) Let X = { 1, 2, 3 } and f, g, h and s are functions from X to X given by
   f = { (1,2), (2,3), (3,1) }, g = { (1,2), (2,1), (3,3) }, h = { (1,1), (2,2), (3,1) }
   s = {(1, 1), (2, 2), (3, 3)}.
   Find f o g, g o f, f o h o g, s o g, g o s, s o s, f o s, f o f. (8M+8M)

4. a) Show that the set of all positive rational numbers forms an abelian group under the composition defined by a * b = ab/4.
   b) If for a group G, f: G → G is given by $f(x) = x^2$ for all $x \in G$ is a homomorphism, prove that G is abelian. (8M+8M)

5. a) Consider a set of integers from 1 to 250. Find how many of these numbers are divisible by 3 or 5 or 7. Also indicate how many are divisible by 3 or 7 but not by 5 and divisible by 3 or 5.
   b) In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, PUN, DOG, or BYTE occurs? (8M+8M)

6. a) Solve the Fibonacci relation $a_n = a_{n-1} + a_{n-2}$ with $a_0 = 0$ and $a_1 = 1$ as initial conditions.
   b) Solve the recurrence relation $a_n - 4\, a_{n-1} + 3\, a_{n-2} = 0$ for n >= 2 with initial conditions $a_0 = 2$ and $a_1 = 4$ by using generating functions. (8M+8M)

7. a) List and explain the representations of graphs with example.
   b) Explain the algorithm for Depth-First Search traversal of a graph. (8M+8M)

8. a) Prove that in any non-directed graph there is an even number of vertices of odd degree.
   b) Define isomorphism. What are the steps followed in discovering the isomorphism. (8M+8M)
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Code No: X0522 **R07** SET - 3

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Time: 3 hours Max. Marks: 80

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1. a) Obtain the product of sums canonical form of the following formulae.

   i) $(\neg P \rightarrow R) \wedge (Q \rightarrow P)$

   ii) $\neg (P \vee Q)$

   b) Write the symbolic statement of the following.

   i) If Rita and Sita go to I.T. camp and Jim and John go to P.C. camp then the college gets the good name.

   ii) It is not true that Ramu reads Times or DC but not The Hindu.

   c) Let P: Ramu is smart, Q: Shamu is clever. R: Dholu is intelligent be the prepositions. Write The following prepositions into statement form.

   i) $(P \rightarrow R) \wedge (Q \rightarrow P)$

   ii) $Q \leftrightarrow P$ (8M+4M+4M)

2. a) Show that the following premises are inconsistent

   i) If the contract is valid, then John is liable for penalty.

   ii) If John is liable for penalty, he will go bankrupt.

   iii) If the bank will loan him money, he will not go bankrupt.

   iv) As a matter of fact, the contract is valid and the bank will loan him money.

   b) Show the following using the automatic theorem

   i) $P \wedge \neg P \wedge Q \Rightarrow R$ ii) $R \Rightarrow P \vee \neg P \vee Q$ (8M+8M)

3. a) Consider the relation R = { (1,3), (1,4), (3,2), (3,3), (3,4) } on A = {1, 2, 3, 4}.

   i) Find the matrix representation of R.

   ii) Find R^{-1}

   iii) Draw the directed graph of R

   iv) Find the composition relation of R o R

   b) Let f: R→ R be defined by f(x) = 2x +5. Show that f is bijection and find f^{-1}. (8M+8M)
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4. a) Show that the set of all elements a of an Abelian group G which satisfy $a^2 = e$ forms a subgroup of G.

   b) Let G be a multiplicative group and f: G $\rightarrow$ G, such that for $a \in G$, $f(a) = a^{-1}$. Prove that f is one-to-one, onto. (8M+8M)

5. a) How many ways are there to seat 10 boys and 10 girls around a circular table, if boys and girls seat alternatively.

   b) Find the number of different ways in which 4 boys and 6 girls may be arranged in a row so that no two boys shall be together.

   c) Prove that $C(n+3, r) - 3C(n+2, r) + 3C(n+1, r) - C(n, r) = C(n, r-3)$ (4M+4M+8M)

6. a) Solve $a_n + a_{n-1} - 5\, a_{n-2} + 3a_{n-3} = 0$

   b) Solve $a_n + 7a_{n-1} + 10\, a_{n-2} = 0$, $n >= 2$ with $a_0 = 10$, $a_1 = 41$. (8M+8M)

7. a) Prove that a complete bipartite graph $K_{m,n}$ is planar if $m <= 2$ or $n <= 2$.

   b) Suppose that G is a connected planar graph. Determine $| V |$ if G has 35 regions each of degree 6. (8M+8M)

8. a) Distinguish between cycle and circuit. Give suitable example for each.

   b) Determine the number of edges in

      i) Complete graph K_n

      ii) Complete bipartite graph $K_{m,n}$

      iii) Cycle graph C_n

      iv) Path graph P_n (8M+8M)

Code No: X0522 **R07** SET - 4

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1. a) Write converse and contra positive of the following.
   i) $P \rightarrow (Q \rightarrow R)$ ii) $(P \wedge (P \rightarrow Q)) \rightarrow Q$
   b) Show the following implications without constructing the truth tables.
   i) $((P \vee \neg P) \rightarrow Q) \rightarrow ((P \vee \neg P) \rightarrow R) \Rightarrow (Q \rightarrow R)$ ii) $\neg Q \wedge (P \rightarrow Q) \Rightarrow \neg P$ (8M+8M)

2. a) Determine the validity of argument: My father praises me if I can be proud of myself. Either I do well in sports or I can't be proud of myself. If I study hard, then I can't do well in sports. Therefore if father praises me, then I do not study well.
   b) Verify the validity of the following inference: If one person is more successful than other, then he has worked harder to deserve success. Naveen has not worked harder than Anil. Therefore, Naveen is not more successful than Anil. (8M+8M)

3. a) What is Hasse diagram? What is the procedure for drawing it for a poset? Draw Hasse diagram of Poset $(D_{12}, |)$.
   b) Show that x! is primitive recursive, where 0! = 1 and n! = n*(n-1)! (8M+8M)

4. a) Let (S, *) be a semi group where S = { a, b }, a * a = b. Show that
   i) a * b = b * a ii) b * b = b
   b) The operation o defined on Z such that a o b = a + b – ab for a, b $\in$ Z. Show that (Z, o) is a monoid. (8M+8M)

5. a) In a language survey of students it is found that 80 students know English, 60 know French, 50 know German, 30 know English and French, 20 know French and German, 15 know English and German and 10 students know all the three languages. How many students know
   i) at least one language ii) English only
   iii) French and one but not both out of English and German iv) at least two languages.
   b) Show that
   i) C(n, r) = C(n-1, r-1) + C(n-1, r) ii) P(n, r) = n P(n-1, r-1) (8M+8M)

6. a) Solve the recurrence relation $a_r = 2\, a_{r-1} + 1$ with $a_1 = 7$ for r > 1 by substitution method.
   b) Find the coefficient of x^{10} in i) $(1 + x + x^2 + \ldots)^2$ ii) $1 / (1 – x)^3$ (8M+8M)

7. a) Show that a complete graph k_n is planar iff n<=4.
   b) Explain the algorithm for Breadth-First search traversal of a graph. (8M+8M)

8. a) Define the following graphs with a suitable example for each graph
   i) Complement graph ii) Euler path iii) Hamiltonian graph iv) Multi graph.
   b) Prove that a non-directed multigraph has an Euler circuit if it is connected and all its vertices are of even degree. (8M+8M)
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