Code No: R21053 **R10** SET - 1

II B. Tech I Semester, Regular Examinations, Nov – 2012
MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE AND ENGINEERING
(Com. to CSE, IT, ECC)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
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1. a) Construct the truth tables of the following formulas

   i) $(Q \wedge (P \rightarrow Q)) \rightarrow P$

   ii) $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$

   b) Show the following implication without constructing the truth table.

   $(P \rightarrow Q) \rightarrow Q \Rightarrow P \vee Q$

   c) Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x) P(x) \wedge (\exists x) Q(x)$ (7M+4M+4M)

2. a) What is the property of GCD(a, b) used by Euclid's algorithm? Write and explain the algorithm with example.

   b) What is modular arithmetic? How the basic operations are defined? (8M+7M)

3. a) Define the following and give examples:

   i) inverse function

   ii) One-to-one function

   iii) Onto function

   iv) bijective function

   b) Let P(A) be the power set of any non empty set A, then prove that the relation $\subseteq$ of set inclusion is not an equivalence relation. (8M+7M)

4. a) Find the number of edges in a 4-regular graph of order 3.

   b) Is the degree sequence (1, 2, 3, 3, 3, 5, 5) a graphic? If so, draw the graph for the same

   c) How many vertices will the graph have if it contains 21 edges, 3 vertices of degree 4, and the other vertices of degree 3. (5M+5M+5M)
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5. a) Explain Kruskal's algorithm with example.
   b) Discuss the following with suitable example
   i) Graph coloring
   ii) planar graph (8M+7M)

6. a) Prove that if a, b $\in$ R where $\langle R, +, \cdot \rangle$ is a ring, then $(a + b)^2 = a^2 + a \cdot b + b \cdot a + b^2$ where $a^2 = a \cdot a$.
   b) If H = { 1, -1 } and G = { 1, -1, i, - i } , then find the right cosets of H in G. (8M+7M)

7. a) Find the coefficient of x^{10} in
   i) $(1 + x + x^2 + \ldots)^2$
   ii) $\dfrac{1}{(1-x)^3}$
   b) How many ways are there to seat 10 boys and 10 girls around a circular table, if boys and girls seat alternatively.
   c) Find the number of arrangements of the letters of TENNESSEE. (7M+4M+4M)

8. a) Solve the Fibonacci relation $a_n = a_{n-1} + a_{n-2}$ with $a_0 = 0$ and $a_1 = 1$ as initial conditions.
   b) Solve the recurrence relation $a_n = a_{n-1} + n^2$ where $a_0 = 7$ by substitution method. (8M+7M)

Code No: R21053 **R10** SET - 2

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Answer any **FIVE** Questions
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1. a) Show that the truth values of the following are independent of their components.
   $(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$
   b) Write down the symbolic form of the following statement and write its negation.
   If k, m, n are any integers where k - m and m – n are odd, then k – n is even.
   c) Verify the validity of the following argument.
   All boys are players. Sachin is a boy. Therefore Sachin is a player.
   d) Determine the validity of the following argument.
   If two sides of a triangle are equal, then two opposite angles are equal. Two sides of a triangle are not equal. Therefore, the opposite angles are not equal. (3M+4M+4M+4M)

2. a) Compute the inverse of each element in Z_7 using Fermat's theorem.
   b) State and explain the division theorem. (8M+7M)

3. a) Let $f: A \rightarrow R$ be defined by $f(x) = (x-2) / (x-3)$, where $A = R - \{3\}$. Is the function f bijective? Find f^{-1}.
   b) (i) What is relation? Give properties of binary relation.
   (ii) Describe about permutation functions and recursive function with examples.
   (8M+7M)

4. a) Define the following terms and give an example of each.
   i) Complement graph ii) Complete graph
   iii) Isomorphic graphs iv) Hamiltonian graph
   b) Find the number of vertices in a graph containing 3 vertices of degree 4, 2 vertices of degree 3 and remaining vertices of degree 2. Given that number of edges in G is 11.
   c) Is the degree sequence (2, 2, 3, 3, 4) a graphic? If so, draw the graph for the same.
   (7M+4M+4M)
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5. a) Explain Prim's algorithm with example.
   b) Use Euler's formula to show that the graph $K_{3,3}$ is non-planar. (8M+7M)

6. a) If for a group G, f: G → G is given by $f(x) = x^2$ for all $x \in G$ is a homomorphism, prove that G is Abelian.
   b) Let G be a multiplicative group and f: G → G, such that for $a \in G$, $f(a) = a^{-1}$. Prove that f is one-to-one, onto. (8M+7M)

7. a) Consider a set of integers from 1 to 250. Find how many of these numbers are divisible by 3 or 5 or 7. Also indicate how many are divisible by 3 or 7 but not by 5.
   b) An identifier in a programming language consists of a letter followed by alphanumeric characters. Find the number of legal identifiers of length at most 10. (8M+7M)

8. a) Solve the recurrence relation $a_n - 4\, a_{n-1} + 3\, a_{n-2} = 0$ for n >= 2 with initial conditions $a_0 = 2$ and $a_1 = 4$ by using generating functions.
   b) What does the recurrence relation: $T(0) = 1, T(n) = T(n-1) + 3^n$ evaluates to? (8M+7M)

Code No: R21053 **R10** SET - 3

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1. a) Are $(P \rightarrow Q) \rightarrow R$ and $P \rightarrow (Q \rightarrow R)$ logically equivalent? Justify your answer by using the rules of logic to simplify both expressions and also by using truth tables.

   b) Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$, $\neg R \vee P$ and Q, using rule CP. (8M+7M)

2. a) Compute the inverse of each element of Z_{12}, if it exists, using Euler's theorem.

   b) State and prove Fermat's theorem. (8M+7M)

3. a) If f: R → R, g: R → R, where R is the set of real numbers. Find f **o** g, g **o** f where $f(x) = x^2 - 2$, $g(x) = x + 4$. State whether these functions are injective, subjective or bijective.

   b) Let $(L, \leq)$ be a lattice and a, b, c $\in$ L, Show that the following are equivalent

   i) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

   ii) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ (8M+7M)

4. a) List and explain different representations of graphs. Illustrate with an example for each.

   b) Suppose G is a connected planar graph with 14 regions each of degree 4. Find the number of vertices in G.

   c) Is there a simple graph with degree sequence (1, 1, 3, 3, 3, 4, 6, 7)? (7M+4M+4M)

5. a) What are the steps involved in graph traversal using Breadth-First Search algorithm? Illustrate with an example.

   b) What is the chromatic number of a cycle graph and a complete graph of n vertices?

   (8M+7M)
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6. a) Show that the set N of natural numbers is a semi group under the operation x * y = max {x, y}. Is it a monoid?

   b) Show that the set of all positive rational numbers forms an abelian group under the composition defined by a * b = ab/4. (8M+7M)

7. a) In a survey of students at Florida University, the following information was obtained. 260 were taking a statistics course, 208 were taking a Mathematics course, 160 were taking a Computer programming course, 76 were taking statistics and Mathematics, 48 were taking Statistics and Computer programming, 62 were taking Mathematics and Computer programming, 32 were taking all 3 kinds of courses and 150 none of the 3 courses.

   i) How many students were surveyed?

   ii) How many were taking Statistics and Mathematics but not Computer programming?

   iii) How many were taking Computer programming but not taking Mathematics or Statistics?

   iv) How many were taking a Mathematics but not taking Statistics or Computer programming?

   b) Find the coefficient of x^{20} in $(x^3 + x^4 + x^5 + \ldots)^5$ (8M+7M)

8. a) Solve $a_n + 7a_{n-1} + 10 a_{n-2} = 0$, $n >= 2$ with $a_0 = 10$, $a_1 = 41$.

   b) Solve the Recurrence relation by using substitution method
   $a_n = a_{n-1} + n$ where $a_0 = 2$ (8M+7M)

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1. a) Show that the truth values of the following formula is independent of their components.
   $(P \leftrightarrow Q) \leftrightarrow ((P \wedge Q) \vee (\neg P \wedge \neg Q))$
   b) Show the following formula is a tautology without constructing truth table.
   $(P \wedge (P \rightarrow Q)) \rightarrow Q$
   c) Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$
   d) Test whether the following is a valid argument.
   If Sachin hits a century, then he gets a free car. Sachin does not get a free car. Therefore, Sachin has not hit a century. (3M+4M+4M+4M)

2. a) What is the principle of mathematical induction? Explain with example.
   b) State the following theorems
      i) The fundamental theorem of arithmetic
      ii) Division theorem
      iii) Fermat's theorem
      iv) Euler's theorem. (8M+7M)

3. a) Let $(L, \leq)$ be a lattice and $a, b, c \in L$. Then prove the following
      i) $a \vee b = b$ iff $a \leq b$
      ii) $a \wedge b = a$ iff $a \leq b$
   b) Draw a poset diagram representing the positive divisors of 36 and determine all maximal, minimal, elements and greatest, least elements if they exist.
   c) Let $X = \{ 1, 2, 3, 4 \}$ and $R = \{ (1, 2), (2, 3), (3, 4) \}$ be a relation on X. Find R^+.
   (7M+4M+4M)

4. a) Define the following terms. Give one suitable example for each.
      i) Euler path ii) Euler circuit
      iii) Hamiltonian graph iv) Isomorphic graphs
   b) When it can be said that two graphs G_1 and G_2 are isomorphic? How can it be discovered? Explain with example. (8M+7M)

5. a) Define tree. Explain in detail about properties of spanning tree.
   b) Find whether K_5 is planar or not. (8M+7M)
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6. a) List and explain the properties of an algebraic system.
   b) Let Z be the set of integers. **o** is an operation in Z defined by a **o** b = a + b +1. Prove that (Z, **o**) is a semi-group. (8M+7M)

7. a) In a language survey of students, it is found that 80 students know English, 60 know French, 50 know German, 30 know English and French, 20 know French and German, 15 know English and German and 10 students know all the three languages. How many students know
      i) at least one language
      ii) English only
      iii) French and one but not both out of English and German
      iv) at least two languages.
   b) Prove that C(n+3, r) – 3C(n+2, r) + 3C(n+1, r) – C(n, r) = C(n, r-3) (8M+7M)

8. a) For $n \geq 2$ suppose that there are n people at a party and that each of those people shakes hands exactly one time with all the other people there and no one shakes hands with himself or herself. If a_n counts the total number of handshakes, frame a recurrence relation per a_n and solve it.
   b) Solve the recurrence relation $a_r = 2\, a_{r-1} + 1$ with $a_1 = 7$ for $r > 1$ by substitution method. (8M+7M)