I B.Tech II Semester Supplementary Examinations, February 2013 MATHEMATICAL METHODS
( Common to Mechanical Engineering, Electronics \& Communication Engineering, Chemical Engineering, Bio-Medical Engineering, Information Technology, Electronics \& Computer Engineering, Mining and Petroliem Technology)
Time: 3 hours
Max Marks: 75

## Answer any FIVE Questions

All Questions carry equal marks

1. (a) Find rank of $A=\left[\begin{array}{ccc}1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3\end{array}\right]$ using Echelon form
(b) Solve by Gauss Elimination method $2 x+y+z=10,3 x+2 y+3 z=18, x+4 y+9 z=16$
2. Find Eigen Vectors of $A=\left[\begin{array}{ccc}8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3\end{array}\right]$
3. Reduce the quadratic form $6 x^{2}+3 y^{2}+3 z^{2}-4 y z-4 z x-2 x y$ to canonical form by diagonalization. Also find its nature, rank, signature and index.
4. (a) Compute the real root of the equation $4 \sin x=e^{x}$ by Bisection method
(b) Find a real root of the equation $x \sin x+\cos x=0$ near $x=\pi$ using NewtonRaphson's method.
5. (a) Evaluate $\triangle^{2}\left[\frac{5 x+6}{x^{2}+5 x+6}\right]$, given that $\mathrm{h}=1$
(b) If $\mathrm{u}_{o}=5, \mathrm{u}_{1}=11, \mathrm{u}_{2}=40, \mathrm{u}_{3}=22, \mathrm{u}_{4}=140$, find $\mathrm{u}_{5}$ given that the general term is represented by a fourth degree polynomial. $[8+7]$
6. (a) The distance travelled by a vehicle at various time intervals during the initial running is given by the following table:

| Time, $\mathrm{t}(\mathrm{s})$ | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Distance travelled, $(\mathrm{km}) \mathrm{s}(\mathrm{t})$ | 10.0 | 14.5 | 19.5 | 25.5 | 32.0 |

Evaluate the velocity of the vehicle at time $\mathrm{t}=5$ and 9 seconds.
(b) Evaluate $\int_{0}^{1} \frac{d x}{1+x^{2}}$ by applying the Simpson's $3 / 8^{\text {th }}$ Rule by dividing the into six equal parts. Hence deduce an approximate value of $\log 2$.
$[8+7]$
7. (a) Solve $y^{1}=x^{2}+y^{2}$ subject to the condition $y(o)=0$ for $x=0.4$ by Taylor series method
(b) Solve $\mathrm{y}^{1}=1+\mathrm{xy}, \mathrm{y}(\mathrm{o})=1$ by Picard's method and hence find $\mathrm{y}(\mathrm{o} .1), \mathrm{y}(\mathrm{o} .2)[8+7]$
8. (a) Fit a power curve $\mathrm{y}=\mathrm{ax}^{b}$ to the following data

| X | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 7.1 | 27.8 | 62.1 | 110 | 161 |

(b) Fit a least square parabola $\mathrm{y}=\mathrm{a}+\mathrm{bx}+\mathrm{cx}^{2}$ to the following data

| x | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 1 | 5 | 10 | 22 | 38 |

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1. (a) Find rank of $A=\left[\begin{array}{cccc}-1 & 2 & 1 & 8 \\ 2 & 1 & -1 & 0 \\ 3 & 2 & 1 & 7\end{array}\right]$ by using Echelon form
(b) Find rank of $A=\left[\begin{array}{cccc}1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10\end{array}\right]$ using Normal Form
2. Verify Cayley - Hamilton theorem and find $A^{-1}$ if $A=\left[\begin{array}{lll}2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2\end{array}\right]$
3. Find the nature of the quadratic form $2 x^{2}+4 x y+y^{2}+3 y z+4 z^{2}$
4. (a) Find a real root of the equation $x^{3}+3 x^{2}-12 x-11=0$ near -1 using NewtonRaphson's method.
(b) Compute the real root of the equation $x \sin x=1$ by Iteration method. [ $8+7]$
5. The following table gives the population of a town during the last six censuses. Estimate, using Newton's interpolation formula, the increase in the population during the period 1986 to 1988.

| year | 1911 | 1921 | 2931 | 1941 | 1951 | 1961 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Population <br> (in thousands) | 12 | 15 | 20 | 27 | 39 | 52 |

6. (a) From the following data find $f$ ' (0)

| x | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{f}(\mathrm{x})$ | 43 | 40 | 38 | 42 | 45 | 50 |

(b) By considering 4 strips, find the value of $\int_{3}^{7} x^{2} \log x d x$
7. (a) Solve $\mathrm{y}^{1}=\mathrm{x}^{2} \mathrm{y}-1, \mathrm{y}(0)=1$ by Taylor series method and hence find y at $\mathrm{x}=0.1$
(b) Solve $\mathrm{y}^{1}=\mathrm{y}, \mathrm{y}(\mathrm{o})=1$ by Picard's method and compare the solution with exact solution.
8. (a) Determine the constants a and b such that $\mathrm{y}=\mathrm{ab}^{x}$ to the following data by the method of least squares

| x | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 4 | 3 | 4.243 | 5.196 | 6 | 6.708 |

(b) Fit a least square parabola $\mathrm{y}=\mathrm{a}+\mathrm{bx}+\mathrm{cx}^{2}$ to the following data

| x | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 1.1 | 1.3 | 1.6 | 2 | 2.7 | 3.4 | 4.1 |

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1. (a) Find rank using Normal Form $A=\left[\begin{array}{llll}1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5\end{array}\right]$
(b) Solve Homogeneous equations $\mathrm{x}_{1}+2 \mathrm{x}_{2}+3 \mathrm{x}_{3}=0,2 \mathrm{x}_{1}+3 \mathrm{x}_{2}+\mathrm{x}_{3}=0,4 \mathrm{x}_{1}+5 \mathrm{x}_{2}+4 \mathrm{x}_{3}=0$ $\mathrm{X}_{1}+\mathrm{x}_{2}-2 \mathrm{x}_{3}=0$
2. Find Eigen vectors of $A=\left[\begin{array}{lll}2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2\end{array}\right]$
3. Reduce the quadratic form $3 x^{2}-2 y^{2}-z^{2}-4 x y+12 y z-+8 z x$ to canonical form by orthogonal transformation .Also find its nature, rank index signature and the transformation which transforms quadratic form to canonical from.
4. (a) Compute the real root of the equation $x \sin x=1$ by iteration method.
(b) Find the real root of the equation $\cos x=x^{2}$ by Newton's method. [8+7]
5. (a) If the interval of differencing is unity, prove the following:
(i) $\Delta\left(\frac{2^{x}}{x!}\right)=\frac{2^{x}(1-x)}{(x+1)!}$
(ii) $\Delta\left\{\tan ^{-1}\left(\frac{n-1}{n}\right)\right\}=\tan ^{-1}\left(\frac{1}{2 n^{2}}\right)$
(b) Using the Newton's forward differences formula, find the interpolating polynomial for the function $\mathrm{y}=\mathrm{f}(\mathrm{x})$ given by $\mathrm{f}(0)=1, \mathrm{f}(1)=2, \mathrm{f}(2)=1, \mathrm{f}(3)=$ 10 . Hence evaluate $f(0.75)$ and $f(-0.5)$.
$[8+7]$
6. (a) Find the value of $\cos (1.74)$ from the following table.

| X | 1.7 | 1.74 | 1.78 | 1.82 | 1.86 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Sin x | 0.9857 | 0.9916 | 0.9781 | 0.9691 | 0.9584 |

(b) Evaluate $\int_{4}^{5.2} \log x d x$ using Simpon's $3 / 8^{\text {th }}$ Rule with the aid of the following Table.

| x | 4.0 | 4.2 | 4.4 | 4.6 | 4.8 | 5.0 | 5.2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\log \mathrm{x}$ | 1.3863 | 1.4351 | 1.4816 | 1.5261 | 1.5686 | 1.6094 | 1.6487 |
| $[8+7]$ |  |  |  |  |  |  |  |

7. (a) Given $\frac{d y}{d x}=\frac{x^{2}}{x^{2}+1}$ with $y(0)=0$ use Picard's method second approximation to Obtain y and find $\mathrm{y}(1)$
(b) Solve $y^{1}=x y+y^{2}, y(o)=1$ by R-K method fourth order and hence find $y(o .1)$, $\mathrm{y}(\mathrm{o} .2)$
[8+7]
8. (a) Fit a second degree polynomial to the following data by the method of least squares

| x | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 1 | 1.8 | 1.3 | 2.5 | 6.3 |

(b) Fit a straight line of the form $\mathrm{y}=\mathrm{a}+\mathrm{bx}$ to the following data

| x | 0 | 5 | 10 | 15 | 20 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 12 | 15 | 17 | 22 | 24 | 30 |

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1. (a) Using Echelon form, find rank of $A=\left[\begin{array}{cccc}1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & 8\end{array}\right]$
(b) Solve system of equations $\mathrm{x}+\mathrm{y}+\mathrm{z}=3,2 \mathrm{x}+3 \mathrm{y}+2 \mathrm{z}=7,4 \mathrm{x}+2 \mathrm{y}+3 \mathrm{z}=9 \quad[7+8]$
2. Verify Cayley - Hamilton theorem and find $A^{-1}$ if $A=\left[\begin{array}{ccc}1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1\end{array}\right]$
3. Find the nature of the quadratic form $2 x^{2}+4 x y+y^{2}+3 y z+4 z^{2}$
4. (a) Find a real root the equation $1+\tan ^{-1}(x)-x=0$ near $\mathrm{x}=1$ correct up to 4 decimal places using iteration method
(b) By using bisection method find an approximate root of the equation $\sin x=$ $\frac{1}{x}$ that lies between $\mathrm{x}=1$ and $\mathrm{x}=1.5$ (measured in radians). Carryout computation upto $7^{\text {th }}$ stage.
5. (a) Interpolate by means of Gauss's backward formula the sales of a concern for the year 1976 for the given data

| Year: | 1940 | 1950 | 1960 | 1970 | 1980 | 1990 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Sales (in lakhs of Rs.) | 17 | 20 | 27 | 32 | 36 | 38 |

(b) Calculate f (1.30) from the following table.

| $\mathrm{X}:$ | 0.0 | 1.2 | 2.4 | 3.7 |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{~F}(\mathrm{x}):$ | 3.41 | 2.68 | 1.37 | -1.18 |

6. (a) The velocity v of a particle moving in a straight line covers a distance at time $t$. They are related as shown in the following Table. Find $v(x)$ at $x=10$ and $\mathrm{x}=15$.

| x | 0 | 10 | 20 | 30 | 40 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| v | 45 | 60 | 65 | 54 | 42 |

(b) Find the area bounded by the cure $y=x^{3}-x+1, x-$ axis between $x=0$ and $\mathrm{x}=1.2$ by using
(i) Trapezoidal Rule (ii) Simpson' $1 / 3$ rule.
7. (a) Solve $y^{1}=x^{2}+y, y(o)=1$ by modified Euler's method and find $y(0.02), y(0.04)$
(b) Solve $\mathrm{y}^{1}+\mathrm{y}=0, \mathrm{y}(\mathrm{o})=1$ by R-K method and hence find $\mathrm{y}(0.1), \mathrm{y}(0.2) \quad[8+7]$
8. (a) Fit a curve of the type $y=\mathrm{ae}^{b x}$ to the data by the method of least squares

| x | 77 | 100 | 185 | 239 | 285 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 2.4 | 3.4 | 7 | 11.1 | 19.6 |

(b) Fit a curve of the type $\mathrm{y}=\mathrm{ab}^{x}$ to the following data by the method of least squares

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | $[7+8]$ |  |  |  |  |  |  |  |

