Subject Code-: R10107/R10 **Set No - 1**

I B.Tech I Semester Supplementary Examinations June - 2012

MATHEMATICAL METHODS

(Common to Computer Science & Engineering, Electrical & Electronic Engineering, Civil Engineering, Electronics & Instrumentation Engineering, Aeronautical Engineering, Bio-Technology, & Automobile Engineering)

Time: 3 hours **Max. Marks : 75**

Answer any FIVE Questions
All Questions carry equal marks
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1.(a) Find the rank of the matrix $\begin{bmatrix} 10 & -2 & 3 & 0 \\ 2 & 10 & 2 & 4 \\ -1 & -2 & 10 & 1 \\ 2 & 3 & 4 & 9 \end{bmatrix}$.

(b) Solve the system if equations using Gauss – Seidel method
x + 4y + 15z = 24
x + 12y +z = 26
10x + y – 2z = 10

[8M + 7M]

2.(a) Verify Cayley - Hamilton theorem for A and deduce A^{-1} if A = $\frac{1}{4}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

(b) Is the matrix A = $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ diagonalizable? If so, find the modal matrix.

[8M + 7M]

3.(a) Find the eigen values of the matrix $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and hence reduce the Q.F 2xy + 2yz +2zx to canonical form. What is the diagonal equivalent matrix?

(b) Show that the matrix $\begin{bmatrix} cos\phi & 0 & sin\phi \\ sin\theta sin\phi & cos\theta & -sin\theta cos\phi \\ -cos\theta sin\phi & sin\theta & cos\theta cos\phi \end{bmatrix}$ is an orthogonal matrix.

[8M + 7M]

4.(a) Establish the formula $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{n}\right)$ and hence compute the value of $\sqrt{11}$ correct to four decimal places.

(b) If $[a, b]$ is the initial guess interval and if f (a) and f (b) are the function values at x = a and x = b, then derive that the approximated root is given by x =$\frac{a\, f(b) - b\, f(a)}{f(b) - f(a)}$.

[8M + 7M]

Subject Code-: R10107/R10 **Set No - 1**

5.(a) Find the Lagrange's polynomial for the following data and using it find the value of i(t) when t = 1.6.

t	1.2	2.	2.5	3.
i(t)	1.36	0.58	0.34	0.2

(b) The approximate value of sin (1.5) is computed from the three terms of the series Sin(x) = $x\frac{x^3}{3!}+\frac{x^3}{5!}+\cdots$ is found to be y_3 = 1.00078, while the exact value is y_e = 0.997495. Find the absolute error, relative error and percentage errors of y_3.

[8M + 7M]

6.(a) The population of a certain town is shown in the following table:

Year	1931	1941	1951	1961	1971
Population y(x)	40.62	60.80	79.95	103.56	132.65

Find the growth rate of the population in the year 1931.

(b) The Velocity υ of a particle given at various times are recorded in the following table:

t (seconds)	0	2	4	6	8	10	12
υ(mps)	4	6	16	34	60	94	136

Find

(i) The distance moved by the particle in 12 seconds and

(ii) The acceleration at t = 2 seconds.

[7M + 8M]

7.(a) Using Runge – Kutta fourth order method. Find y when x = 1.2 in steps of 0.1 given that $\frac{dy}{dx} = x^2 + y^2$ and y (1) = 1.5.

(b) By using Taylor series method, solve$\frac{dy}{dx} = dy, y(0) = 2$, to find y (0.2) and compare it with that obtained by the exact solution.

[8M + 7M]

8.(a) Fit an exponential curve of the from $y(x) = ae^{bx}$ to the following data.

x	1	2	3	4	5
y	2.6	3.3	4.2	5.4	6.9

(b) Fit an exponential model y(x) = a e^{bx} to the following data

x	0	1	2	3	4	5
y	0.500	0.485	0.471	0.457	0.443	0.430

[8M + 7M]

Subject Code-: R10107/R10 **Set No - 2**

I B.Tech I Semester Supplementary Examinations June - 2012

MATHEMATICAL METHODS

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Time: 3 hours **Max. Marks : 75**

Answer any FIVE Questions
All Questions carry equal marks
*** * * * ***

1.(a) Find the inverse of the matrix A = $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ operations.

(b) Solve the following equations
$x_1 + 2x_2 + 3x_3 = 0$
$2x_1 + 3x_2 + x_3 = 0$
$4x_1 + 5x_2 + 4x_3 = 0$
$x_1 + x_2 - 2x_3 = 0$

[8M + 7M]

2.(a) Show that the matrix A = $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ has less than three Linearly independent eigen vectors. Is it possible to obtain a similarity transformation that wills diagenalize this matrix?

(b) Show that the matrix A = $\frac{1}{2}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$ is orthogonal.

[8M + 7M]

3.(a) By Lagrange's reduction transform the quadratic form $X^T AX$ to sum of squares form for A = $\begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & -2 \\ 4 & -2 & 18 \end{bmatrix}$.

(b) Show that the matrix A = $\begin{bmatrix} a+ic & -b+id \\ b+id & a-ic \end{bmatrix}$ is unitary if and only if $a^2 + b^2 + c^2 + d^2 = 1$

[8M + 7M]

4.(a) Using Newton – Raphson method find a positive real root of the equation $x^3 - x - 10 = 0$, with $x_0 = 1.0$.

(b) Find a real root of f(x) = x sin x – 1 correct up to three decimal places starting with x = 1 by Newton Raphson method.

[8M + 7M]

Subject Code-: R10107/R10 **Set No - 2**

5.(a) The values of an elliptic integral

$K(m) = \int_0^{\frac{\pi}{2}} (1 - m\sin^2\theta)^{-\frac{1}{2}} d\theta$

For certain equidistant values of m are given below. Determine K (0.25) from Newton Backward Difference Formula.

m	0.20	0.22	0.24	0.26	0.28	0.30
K(m)	1.659624	1.669850	1.680373	1.691208	1.702374	1.713889

(b) Prove the following

(i) $\Delta\nabla = \Delta - \nabla$

(ii) $\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta} = \Delta + \nabla$

(iii) $\nabla E = E\Delta = \nabla$

[8M + 7M]

6.(a) Find the area bounded by the curve $y = e^{-\frac{x^2}{2}}$, x – axis between x = 0 and x = 3 by using Simpson's $\frac{3}{8}$ formula.

(b) Deduce Simpson's $\frac{1}{3}$ rule from the Newton Forward Difference Formula and hence use it find $\int_0^1 \sin x \, dx$.

[8M + 7M]

7.(a) Find y (0.1), z (0.1), y (0.2) and z (0.2) from the system of equation $y^1 = x + z, z^1 = x - y^2$, given that y (0) = 2 and z (0) = 1 by RK fourth order.

(b) Solve by using first order Runge – Kutta method to find y (0.1) the differential equation $\frac{dy}{dx} = (1 + x^2)y$ and y (0) = 1.

[8M + 7M]

8.(a) Fit exponential curve of the from $y(x) = ae^{bx}$ to the following data.

x	2	3	4	5	6
y	144.0	172.8	207.4	248.8	298.5

(b) Fit the least square line $y = a_0 + a_1 x$ for the data prints (-1,10), (0,9), (1,7), (2,5), (3,4), (4,3),(5,0) and (6,-1).

[8M + 7M]

Subject Code-: R10107/R10 **Set No - 3**

I B.Tech I Semester Supplementary Examinations June - 2012

MATHEMATICAL METHODS

(Common to Computer Science & Engineering, Electrical & Electronic Engineering, Civil Engineering, Electronics & Instrumentation Engineering, Aeronautical Engineering, Bio-Technology, & Automobile Engineering)

Time: 3 hours **Max. Marks : 75**

Answer any FIVE Questions
All Questions carry equal marks
*** * * * ***

1.(a) Determine the rank of the matrix A = $\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$.

(b) Solve the system of equations
$x + y + w = 0$
$y + z = 0$
$x + y + z + w = 0$
$x + y + 2z = 0$

[8M + 7M]

2.(a) Find the eigen values and vectors of $\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$.

(b) Show that the matrix A=$\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalizable. Also find the transforming matrix, and diagonal matrix.

[8M + 7M]

3.(a) Find the orthogonal matrix P that will Diagonalize the symmetric matrix
A =$\begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{bmatrix}$.

(b) Prove that A =$\begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$ is skew – Hermitian and also unitary. Find the eigen values and eigen vectors.
Diagonalize the following matrices by Orthogonal Transformation.

[8M + 7M]

4.(a) Find a positive real root of f(x) = cos x + 1 – 3x = 0 correct to two decimal places by bisection method.

(b) Using Regula Falsi method, compute the real root of the following equations correct up to three decimal places:
(i) $x e^x = 1$
(ii) $e^x \sin x = 1$
(iii) $x = 3 e^{-x}$

[8M + 7M]

Subject Code-: R10107/R10 **Set No - 3**

5.(a) The population of a certain village in thousands is given in the following table. By using Central Forward Difference Formula estimate the village population in 1936.

Year	1901	1911	1921	1931	1941	1951
Population	12	15	20	27	39	52

(b) Evaluate
(i) $\Delta \sin(ax+b)$,
(ii) $\Delta^2(3e^x)$

[7M + 8M]

6.(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ from the data near x = 1.5 using Central Backward Difference formula.

x	1	1.2	1.4	1.6	1.8	2
y	3.00	6.26	11.07	17.84	26.99	39.00

(b) A curve is observed to pass through the points given in the following table:

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	2	2.4	2.7	2.8	3	2.6	2.1

By using Simpson's rule find the
(i) The area bounded by the curve and x axis between x = 1 and x = 4
(ii) The volume of revolution of the area about the x – axis.

[7M + 8M]

7.(a) Solve by Taylor series expansion the initial value problem $\acute{y} = x + y^2$ for x = 0.2(0.2)0.6 given that y (0) = 0.

(b) Apply Euler method to find the solution of $\frac{dy}{dx} = \frac{y-x}{y+x}$, with y(0) = 1 for $0 \le x \le 0.1$ with h = 0.025.

[8M + 7M]

8.(a) Fit a straight line of form $y(x) = a_0 + a_1 x$ to the data

x	1	2	3	4	6	8
y	2.4	3.1	3.5	4.2	5.0	6.0

(b) Find a weighted least square parabola for the following data by choosing the weights 1,4,2,4 and 1 respectively

x	0	2	3	4	5
y	-1	1	7	17	31

[8M + 7M]

Subject Code-: R10107/R10 **Set No - 4**

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MATHEMATICAL METHODS

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Time: 3 hours **Max. Marks : 75**

Answer any FIVE Questions
All Questions carry equal marks
*** * * * ***

1.(a) Find the rank of $\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$.

(b) Find the values of x for which the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & x & 3 & 1 \\ 0 & 0 & 1 & x \\ 0 & 0 & 1 & 1 \end{bmatrix}$

[8M + 7M]

2.(a) Find the eigen values and eigen vector of A = $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 4 & 3 \\ 0 & 2 & 0 \end{bmatrix}$

(b) Find the modal matrix P to A = $\begin{bmatrix} -2 & 2 & 3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ verify that P^{-1} AP is a diagonal matrix by similarity transformation.

[8M + 7M]

3.(a) Show that the matrix $\begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$ is a Hermitian matrix.

(b) Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_1x_3$ to canonical form. Identify the nature, rank, index and signature.

[8M + 7M]

4.(a) Show that the iteration scheme f(x) = $\frac{-1}{x^2-3}$ converges and hence find a real root of f(x) = $x^3 - 3x + 1 = 0$ near x = 3.

(b) Find a root of the following equations using the bisection method correct to three decimal places:

(i) $x^2 - 4x - 9 = 0$

(ii) $x^3 + x^2 - 100 = 0$

(iii) $x^3 - 2x^2 - 4 = 0$

[8M + 7M]

Subject Code-: R10107/R10 **Set No - 4**

5.(a) Using Newton Forward Difference Formula estimate y (0.12) from the following data.

x	0.10	0.15	0.20	0.25	0.30
y	0.656	0.522	0.410	0.16	0.240

(b) Evaluate $\Delta^2\left[\frac{5x+12}{x^2+5x+6}\right]$ taking the interval of differencing being one unit.

[8M + 7M]

6.(a) Evaluate $\Delta^2\left[\frac{5x+12}{x^2+5x+6}\right]$ taking the interval of differencing being one unit.

(b) Find the value of $\int_4^{5.2} \log_e x$ by dividing the interval in to 12 subintervals using Boole's and Weddle's rule.

[8M + 7M]

7.(a) Solve $\frac{dy}{dx} = x + z$ and $\frac{dz}{dx} = x - y^2$. Given that y (0) = 2 and z (0) =1. Find y (0.1), y (0.2), z(0.1) and z (0.2).

(b) Using Runge – Kutta Second order formula solve the equation $\frac{dy}{dx} = 2 + \sqrt{xy}$ with y (1) = 1 for x = 1 (0.2)1.6.

[8M + 7M]

8.(a) Fit a third degree polynomial for the following data :

x	-1	0	1	2	3
y	1	1	1	7	25

(b) Find the best of the type $y = ae^{bx}$ to the data provided in the table by using least squares method.

X:	1	5	7	9	12
Y:	10	15	12	15	21

[8M + 7M]