Subject Code-: R10206/R10 **Set No - 1**

I B.Tech II Semester Regular Examinations June - 2012

MATHEMATICAL METHODS

(Common to Electronics & Communication Engineering, Information Technology, Mechanical Engineering, Chemical Engineering, Biomedical Engineering, Electronics & Computer Engineering, Petroleum Technology, & Mining)

Time: 3 hours **Max. Marks : 75**

Answer any FIVE Questions
All Questions carry equal marks
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1.(a) Solve by Gauss – Seidel method.
$6x + y + z = 105$
$4x + 8y + 3z = 155$
$5x + 4y - 10z = 65$

(b) Find two non-singular matrices P and Q such that the normal form of A is PAQ where

$A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$ hence find its rank.

[8M + 7M]

2.(a) Show that the transformation $H = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$, where $\theta = \frac{1}{2} + \tan^{-1}\frac{2h}{a-b}$, changes the matrix $C = \begin{bmatrix} a & h \\ h & b \end{bmatrix}$ to the diagonal form $D = H^{-1}CH$.

(b) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 2 & -2 & -2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

[8M + 7M]

3. Reduce the Q.F. $2x_1^2 + 4x_2^2 + 4x_3^2 + 2x_1x_2 - 2x_1x_3 + 6x_2x_3$ to canonical form and hence Find the nature, rank, index and signature of the Q.F. Also specify the matrix of transformation.

[15M]

4.(a) Apply Newton Raphson method to find a root of $x^3 - x^2 + x - 2 = 0$ correct up to four decimal places starting from $x_0 = 0$.

(b) Solve $x^3 = 2x + 5$ for a positive root by iteration method.

[8M + 7M]

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5.(a) Prove the relations

(i) $\sum_{k=0}^{n-1} \Delta^2 fk \equiv \Delta fk - \Delta fo$

(ii) $\Delta(f_i g_i) \equiv f_i \Delta g_i + g_{i+1} \Delta f_i$

(iii) $\Delta f_i^2 \equiv (f_i + f_{i+1}) \frac{\Delta f_i}{(f_i f_i + 1)}$

(b) Show that $\Delta^{10}[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)] = 24 \times 2^{10} \times 10!$ if h = 2.

[8M + 7M]

6. Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ using

(i) Trapizoidal rule

(ii) Simpson's $\frac{1}{3}$ rule and compare with the result obtained by direct integration.

[8M + 7M]

7.(a) Solve by Taylor series expansion the initial value problem $\frac{dy}{dx} = y^2 + 1$ with y (0) = 0 to find the values of y at x = 0(0.2)0.6.

(b) Solve $\frac{dy}{dx} = x^2 + y$ with y (0) = 2 by both Picard method and Taylor series method up to third degree terms. Compute y (0.2).

[8M + 7M]

8.(a) A chemical factory wish to study by effective of extraction time on the efficiency given in the table . Fit a straight line to the data by the method of least squares.

X	27	45	41	19	3	39	19	49	15	31
y	57	64	80	46	62	72	62	77	57	68

(b) Obtain a relation of the from $y = ab^x$ for the following data by the method of least squares.

X :	2	3	4	5	6
Y :	8.3	15.4	33.1	65.2	127.4

[8M + 7M]

Subject Code-: R10206/R10 **Set No - 2**

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Time: 3 hours **Max. Marks : 75**

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*** * * * ***

1.(a) Solve the system of equations,
$x + y + z = 8$
$2x + 3y + 2z = 19$
$4x + 2y + 3z = 23$ using Gauss elimination method

(b) Find the inverse of the matrix A using elementary operations

$$A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}.$$

[8M + 7M]

2.(a) Express the matrix adj $[A - \lambda I]$ as a matrix polynomial where A = $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$.

(b) Find the modal matrix P which diagonalizes the matrix A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ to diagonal form.

[8M + 7M]

3. Reduce the following quadratic form to canonical form by Diagonalization.
$6x^2 + 3y^2 + 3z^2 - 4yz - 4zx - 2xy$.

[15M]

4.(a) Find a root of the equations by bisection method $\cos 2x - x = 0$

(b) Assuming that a root of $x^3 - 9x + 1 = 0$ lies in the interval (2, 4), find that root by bisection method correct up to two decimal places.

[8M + 7M]

5.(a) If y (1) = 3, y (3) = 9, y (4) = 30 and y (6) = 132, find the four point Lagrange interpolation polynomial that takes the same values as the function y at the given points.

(b) Use Lagrange's interpolation formula to resolve the following into partial fractions.

$$\frac{t^2+6t+1}{(t+1)(t-1)(t-4)(t-6)}$$

[8M + 7M]

Subject Code-: R10206/R10 **Set No - 2**

6.(a) The following table gives the values of f(x) at equal intervals of x

x	0	0.5	1.0	1.5	2.0
y	0.399	0.352	0.242	0.129	0.054

Evaluate $\int_0^2 f(x)dx$ using Simpson's rule.

(b) Using Weddle's rule find $\int_1^7 y\,dx$ for the function tabulated below

x	1	2	3	4	5	6	7
y(x)	3.95	4.07	4.18	4.30	4.42	4.54	4.67

[8M + 7M]

7.(a) Solve $\frac{dy}{dx} = 2y + 3e^x$ with y (0) = 0 using Taylor series method to find the values of y for x = 0.1 and 0.2.

(b) Solve $\frac{dy}{dx} = x + \sqrt{y}$, y (0) = 1 by Euler modified method to find y at x = 0.2 and x = 0.4. Also find the solution y(x) at x = 0.2 and x = 0.4 by Euler method by taking h = 0.1. Compare the answers

[7M + 8M]

8.(a) Fit a parabola of the from $y = a_2x^2 + a_1x + a_0$ to the data

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

(b) Fit a straight line to the data

x	1	3	5	7	9
y	1.5	2.8	4.0	4.7	6.0

[8M + 7M]

Subject Code-: R10206/R10 **Set No - 3**

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MATHEMATICAL METHODS

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Time: 3 hours **Max. Marks : 75**

Answer any FIVE Questions
All Questions carry equal marks
*** * * * ***

1.(a) Solve the system of equations,
x + y + z =8
2x + 3y + 2z = 19
4x + 2y + 3z = 23 using Gauss – Jordan method.

(b) Find the inverse of A using ad joint method where A = $\begin{bmatrix} 1 & 0 & 9 \\ 2 & 4 & 5 \\ 1 & 2 & 6 \end{bmatrix}$.

[8M + 7M]

2.(a) Show that the matrix A = $\begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ satisfies Cayley- Hamilton theorem.

(b) Is the matrix $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ diagonalizable?

[8M + 7M]

3.(a) Find the eigen values and eigen vectors of the matrix A = $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Hence, reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy + 4xz - 2yz$ to its canonical form.

(b) Using orthogonal reduction show that the quadratic form
q = $2x_1^2 + 4x_2^2 + 4x_3^2 + 2x_1x_2 - 2x_1x_3 + 6x_2x_3$ is positive semi definite. Also specify non-zero X =(x_1, x_2, x_3) which will make q = 0.

[8M + 7M]

4.(a) Find a real root of f(x) = $x^3 - 19$ correct up to three decimal places starting with x = 1 by Newton Raphson method.

(b) Solve the equation x tan x = -1 by Regula Falsi method starting with a = 2.5 and b = 3, correct to 3 decimal places.

[8M + 7M]

Subject Code-: R10206/R10 **Set No - 3**

5.(a) Define the operations of Δ, ∇, and E, and show that

(i) $\Delta = E\nabla$

(ii) $\nabla = E^{-1}\Delta$

(iii) $E = 1 + \Delta$

(iv) $E^{-1} = 1 - \nabla$

(b) For the following data fit a polynomial

x	1	2	3	4
y	2	5	16	41

By using Newton Forward and Backward Difference Formulae.

[8M + 7M]

6.(a) A rod is rotating in a plane. The following table gives the angle ϴ (xin radians) through which the rod has turned for various values of time (seconds)

t	0.0	0.2	0.4	0.6	0.8	1.0
ϴ	0.00	0.12	0.49	1.12	2.02	3.20

Calculate the angular velocity and the angular acceleration of the rod when t = 0.3 seconds.

(b) A river is 80 meters wide. The depth d in meters at a distance x from the bank is given in the following table. Calculate the cross section of the river using Trapizoidal rule.

x	10	20	30	40	50	60	70	80
d(x)	4	7	9	12	15	14	8	3

[8M + 7M]

7.(a) Use Milne method to find y (0.8) from$y^1 = 1 + y^2, y(0) = 0$. Find the initial values y (0.2), y (0.4) and y (0.6) from Runge – Kutta method.

(b) Apply Milne Predictor Corrector method to find y (0.8), y (1.0) from the equation$\acute{y} = y - x^2$, y (0) = 1 by obtaining the starting values by Euler method.

[8M + 7M]

8.(a) Fit a least square parabola $y = a + bx + cx^2$ to the data f(-1) =-2, f(0)=1, f(1)=2, f(2)=4.

(b) Fit a straight line of the from $y = a + bx$ to the following data.

x	50	60	70	80
y	205	225	248	274

[8M + 7M]

Subject Code-: R10206/R10 **Set No - 4**

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MATHEMATICAL METHODS

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Time: 3 hours **Max. Marks : 75**

Answer any FIVE Questions
All Questions carry equal marks
*** * * * ***

1.(a) Define rank of a matrix, normal form of a matrix and echelon form of a matrix

(b) Reduce to echelon form and hence find the rank of the matrix A = $\begin{bmatrix} 1 & 2 & -4 & 5 \\ 2 & -1 & 3 & 6 \\ 8 & 1 & 9 & 7 \end{bmatrix}$

[7M + 8M]

2. Diagonalize the matrix A = $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ and find A^4 using the modal matrix.

[15M]

3.(a) If A and B are Hermitian, then prove that
(i) AB + BA is Hermitian
(ii) AB – BA is skew – Hermitian

(b) Find the orthogonal transformation which transforms the Quadratic form $x_1^2 + 3x_2^2 + 3x_3^2 + 2x_2 x_3$ to canonical form.

[8M + 7M]

4.(a) Show that Newton Raphson converges quadratically while successive iteration method converges linearly.

(b) Using Regula Falsi method find a real root of $f(x) = 2x^7 + x^5 + 1 = 0$ correct up to two decimal places using a = -1, b = 1.

[7M + 8M]

5.(a) Prove the following:-
(i) $u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + \cdots + \Delta^{n-1} u_{x-n} + \Delta^n u_{x-n-1}$
(ii) $\Delta^n y_x = y_{x+n} - C_1^n y_{x+n-1} + C_2^n y_{x+n-2} + (-1)^n y_x$
(iii) $u_1 + u_2 + \cdots + u_n = C_1^n u_0 + C_2^n \Delta u_0 + \cdots + \Delta^{n-1} u_0$.

(b) Find the Lagrange's interpolating polynomial and using it find y when x = 10, if the values of x and y are given as follows:

x	5	6	9	11
y	12	13	14	16

[8M + 7M]

Subject Code-: R10206/R10 **Set No - 4**

6.(a) Find the maximum and minimum values of y from the following table:

x	0	1	2	3	4	5
y	0	$\frac{1}{4}$	0	$\frac{9}{4}$	16	$\frac{225}{4}$

(b) The following table gives the value of f(x) at equal intervals of x.

x	0	0.5	1.0	1.5	2.0
y	0.399	0.352	0.242	0.129	0.054

Evaluate $\int_0^2 f(x)dx$ using Simpson's rule.

[8M + 7M]

7.(a) Applying Runge – Kutta fourth order method find y (0.2), y (0.4) and y (0.6) Where $y^1 = -xy^2, y(0) = 2.$ choose step size h = 0.2.

(b) Apply Milne Predictor Corrector method to find y (0.4) by obtaining the initial solution of $\frac{dy}{dx} = y + x^2$. Y (0) = 2 by Taylor series method.

[8M + 7M]

8.(a) Fit a second degree of polynomial to the following data by the method of least squares :

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.5

(b) Fit a least square parabola for the data

X	0	1	2	3	4	5	6
y	1	2	7	16	29	46	67

[8M + 7M]