

Code No: X0121

R07**SET - 1****II B. Tech I Semester, Supplementary Examinations, Nov – 2012****MATHEMATICS-II**

(Com. to CE, CHEM, AE, BT)

Time: 3 hours

Max. Marks: 80

Answer any FIVE Questions

All Questions carry Equal Marks

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1. a) Find the rank by reducing the matrix to normal form and echelon form

$$\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$$

- b) Solve the following equation

$$X_1 + 2X_2 + 3X_3 = 0$$

$$2X_1 + 3X_2 + X_3 = 0$$

$$4X_1 + 5X_2 + 4X_3 = 0$$

$$X_1 + X_2 - 2X_3 = 0$$

2. a) If
- $\lambda_1, \lambda_2, \dots, \lambda_n$
- are the eigen values of A then
- $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \dots, \frac{1}{\lambda_n}$
- are the eigen values of
- $A^{-1}$

b) Diagonalize  $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

3. a) Reduce the following quadratic forms to canonical form by Diagonalization

$$x^2 + 4y^2 + 9z^2 + t^2 - 12yz + 6zx - 4xy - 2xt - 6tz$$

b) If  $A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$  find  $A^{50}$

4. a) Find the Fourier expansion
- $f(x) = \frac{(\pi - x)^2}{4}$
- in the interval
- $0 < x < 2\pi$

- b) If
- $f(x) = x$
- ,
- $0 \leq x \leq \pi$
- find the i) cosine series ii) sine series and hence deduce

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

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5. a) From the partial differential equation by eliminating the constants a and b from  
 $\log (az-1)= x+ay+b$   
b) Solve  $\sqrt{p} + \sqrt{q} = 1$
6. a) Solve by the method of separation of variables  $2x \frac{\partial z}{\partial x} - 3y \frac{\partial z}{\partial y} = 0$   
b) A tightly stretched string with fixed end points  $x=0$  and  $x=1$  is initially in a position given by  $y(x,0) = y_0 \sin \frac{\pi x}{l}$  It is released from rest from this position, find the displacement.  $y$  at any distance  $x$  from one end at any time  $t$ .
7. a) Express the function  $f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$  as Fourier integral. Hence evaluate  $\int_0^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda$   
b) Find the Fourier sine transform of  $\frac{1}{x(x^2 + a^2)}$
8. a) Find the Z- transform of  $(n-1)^2$   
b) Find the inverse Z -transform of  $\frac{z}{z^2 - 2z + 2}$

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**R07****SET - 2****II B. Tech I Semester, Supplementary Examinations, Nov – 2012****MATHEMATICS-II**

(Com. to CE, CHEM, AE, BT)

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Answer any FIVE Questions

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1. a) Find the rank by reducing the matrix to normal form and echelon form

$$\begin{bmatrix} 1 & 2 & 2 & 4 \\ 2 & 3 & 4 & 6 \\ 3 & 5 & 6 & 10 \\ -1 & 1 & -2 & -2 \end{bmatrix}$$

- b) Solve the following equations by matrix method.

$$2x_1 - x_2 + 2x_3 = 2$$

$$x_1 + 10x_2 - 3x_3 = 5$$

$$-x_1 + x_2 + x_3 = -3$$

2. a) If $\lambda_1, \lambda_2, \dots, \lambda_n$ are the eigen values of A then $\lambda_1^k, \lambda_2^k, \dots, \lambda_n^k$ are the eigen values of A^k .

- b) Find a matrix P which transform the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to diagonal form. Hence calculate

$$A^4$$

3. a) Reduce the following quadratic forms to canonical form by Diagonalization

$$2x^2 + 9y^2 + 6z^2 + 8xy + 8yz + 6zx$$

- b) Given $A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ find A^{256}

4. a) Expand $f(x) = e^{-x}$ as a Fourier series in $(-1, 1)$
 b) Find the half – range sine series for $f(x) = \cos x$ in $(0, \pi)$

5. a) From the partial differential equation by eliminating the arbitrary constants from $Z = Ae^{pt} \cos qx \sin ry$ where $p^2 = q^2 + r^2$

- b) Solve $\frac{x^2}{p} + \frac{y^2}{q} = z$

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6. a) Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by the method of separation of variables
- b) A tightly stretched flexible string has its ends fixed at $x=0$ and $x=1$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $3x(1-x)$ find the displacement.
7. a) Express the function $f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$ as Fourier integral. Hence evaluate $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$
- b) Find the Fourier sine transform of $\frac{1}{x}$
8. a) Find the Z – transform a^{n+3}
- b) Find inverse Z – transform of $\frac{2z^2 + 3Z}{(Z + 2)(Z - 4)}$

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R07**SET - 3****II B. Tech I Semester, Supplementary Examinations, Nov – 2012****MATHEMATICS-II**

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Time: 3 hours

Max. Marks: 80

Answer any FIVE Questions
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1. a) Find the rank by reducing the matrix to normal form and echelon form

$$\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$$

- b) Solve the system of equations

i)  $x+y+z=8$

ii)  $2x+3y+2z=19$

iii)  $4x+2y+3z=23$  Using elementary row transformation of the coefficient matrix.

2. a) Show that sum of the eigen values of a matrix is the trace of the matrix.

b) Verify Cayley Hamilton Theorem, for the matrix  $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$

3. a) Reduce the following quadratic forms to canonical form by Diagonalization

$$2x_1^2 + 5x_2^2 + 3x_3^2 + 4x_1x_2$$

b) Given  $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$  find  $A^{25}$

4. a) Find the Fourier series for the function

i)  $f(x) = -\pi, -\pi < x < 0$

ii)  $x, 0 < x < \pi$  and hence deduce the series  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi}{2}$

b) Find half-range cosine series for  $f(x) = \begin{cases} 1, & 0 < x < \frac{1}{2} \\ 1-x, & \frac{1}{2} < x < 1 \end{cases}$

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5. a) Form the differential equation of all planes which are at a constant distance  $d$  from the origin  
 b) Solve the equation  $z^2(p^2 + q^2) = 1$  Also find singular integral if it exists
6. a) Using the method of separation of variables, solve  $4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$  given  
 $u = 3e^{-y} - e^{-5y}$  where  $x = 0$   
 b) An insulated rod of length  $L$  has its ends A and B maintained at  $0^\circ\text{C}$  and  $100^\circ\text{C}$  respectively until steady state conditions prevail. If B is suddenly reduced to  $0^\circ\text{C}$  and maintained at  $0^\circ\text{C}$ , find the temperature at a distance  $x$  from A at time  $t$ .
7. a) Using the Fourier integrals prove that  $\int_0^\infty \frac{\cos \lambda x}{1 + \lambda^2} d\lambda = \frac{\pi}{2} e^{-x} \quad x \geq 0$   
 b) Find the fourier sine transform of  $f(x) \begin{cases} x, 0 < x < 1 \\ 2 - x, 1 < x < 2 \\ 0, x > 2 \end{cases}$
8. a) If  $Z(u_n) = \frac{z}{z-1} + \frac{z}{z^2+1}$  find the Z- transform of  $u_{n+2}$   
 b) Find the inverse Z -transform of  $\frac{z}{z^2+7z+10}$

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**R07****SET - 4****II B. Tech I Semester, Supplementary Examinations, Nov – 2012****MATHEMATICS-II**

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Time: 3 hours

Max. Marks: 80

Answer any FIVE Questions

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1. a) Find the rank by reducing the matrix to normal form and echelon form $\begin{bmatrix} 2 & 0 & -1 & 1 \\ 4 & -1 & -2 & 4 \\ 3 & 2 & 3 & -2 \\ 6 & 3 & 0 & -5 \end{bmatrix}$
- b) Solve the following system of equations
 i) $4x+2y+3w=0$ ii) $6x+3y+4z+7w=0$ iii) $2x+y+w=0$
2. a) If λ is the eigen value of A then the eigen value of $B=a_0A^2+a_1A+a_2I$ is $a_0\lambda^2+a_1\lambda+a_2$
- b) Using Cayley Hamilton theorem, find the inverse of $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$
3. a) Reduce the following quadratic forms to canonical form by Diagonalization
 $X_1^2+2X_2^2-7X_3^2-4X_1X_2+8X_1X_3$
- b) Given $A = \begin{bmatrix} 1 & 20 & 0 \\ -1 & 7 & 1 \\ 3 & 0 & -2 \end{bmatrix}$, A^5
4. a) If $f(x)=0$ for $-\pi < x < 0 = \sin x$ for $0 < x < \pi$ Show that $f(z) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi} \sum_{m=1}^{\infty} \frac{\cos 2mx}{4m^2 - 1}$ and hence deduce the series $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots \dots \dots \infty = \frac{1}{4}(\pi - 2)$
 $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots \dots \dots \infty = \frac{1}{2}$
- b) If $f(x) = \sin x$, $0 \leq x < \frac{\pi}{4} \cos x$, $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$ Find the half range sine series

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5. a) From the partial differential equation by eliminating the arbitrary constants a and b from the following equations $\log (az-1)= ax+y+b$
- b) Solve $(x + pz)^2 + (y + qz)^2 = 1$
6. a) Solve $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u; u(0, y) = e^{-5y}$
- b) The ends A and B of a rod 20 cm long have the temperature at 30°C and 80°C until steady state conditions prevail. The temperature at the ends are suddenly changed to 40°C and 60°C respectively. Find the temperature distribution in the rod at time t
7. a) Using the Fourier integrals prove that $\int_0^{\infty} \frac{\sin w \cos xw}{w} dw = \frac{\pi}{2}, 0 \leq x \leq 1$
- b) Show that Fourier transform of $\frac{-x^2}{e^2}$ is reciprocal
8. a) Show that $Z\left[\frac{1}{n+1}\right] = Z \log \left\{\frac{Z}{Z+1}\right\}$
- b) Find the Inverse transform of $\log \left(\frac{z}{z+1}\right)$ by power series method.