I B.Tech II Semester Supplementary Examinations, February 2013 MATHEMATICS- II
( Common to Civil Engineering, Electrical \& Electronics Engineering, Mechanical Engineering, Electronics \& Communication Engineering, Computer Science \& Engineering, Chemical Engineering, Electronics \& Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics \& Computer Engineering, Aeronautical
Engineering, Bio-Technology, Automobile Engineering, Mining and Petroliem Technology)
Time: 3 hours
Max Marks: 75
Answer any FIVE Questions
All Questions carry equal marks

1. (a) Find the Laplace transform of $\cos 3 t \sin 5 t$
(b) Find the Laplace transform of $\frac{1-e^{-t}}{t}$
2. (a) Find $L^{-1}\left[\left(2 s^{3}+3\right) / s^{2}\left(s^{2}+1\right)\left(s^{2}+2\right)\right]$.
(b) Find $L^{-1}\left\{\frac{e^{4-3 s}}{(s+4)^{5 / 2}}\right\}$.
3. Find a fourier expansion of $f(x)=x \cos x, 0<x<2 \pi$
4. Find the fourier cosine and sine transform of $f(x)$ defined by $f(x)=e^{-a x} / x$ and hence deduce that $\int_{0}^{\infty} \frac{e^{-a x}-e^{-b x} \sin \mathrm{sx} \mathrm{dx}}{x}$
5. (a) Solve $\sqrt{p}+\sqrt{q}=x^{2}$
(b) Solve $a p+b q+c z=0$
$[8+7]$
6. (a) Solve $2 x \frac{d z}{d x}-3 y \frac{d z}{d y}=0$ by the Method of Separation of Variables
(b) Solve $\frac{d u}{d x}=2 \frac{d u}{d t}+u$, where $\mathrm{u}(\mathrm{x}, \mathrm{o})=6 \mathrm{e}^{-3 x}$ by the Method of Separation of Variables.
7. (a) Find the Z-transforms of (i) $\mathrm{e}^{-\mathrm{an}} \sin \mathrm{n} \theta$ (ii) $3 n^{2}+10 \cos \left(\frac{\mathrm{n} \pi}{2}\right)+a^{n+2}$.
(b) Find the Z-transforms of (i) $(\mathrm{n}-1)^{2}$ (ii) $5 \mathrm{e}^{-\mathrm{an}} \sin \left(\frac{\mathrm{n} \pi}{4}\right)-3 a^{4}$.
$[8+7]$
8. (a) Evaluate $\int_{0}^{1} \mathrm{x}^{3} \sqrt{1-\mathrm{x}} d x$, using beta and gamma functions.
(b) Prove that $\int_{0}^{\infty} \mathrm{x}^{2 \mathrm{n}-1} e^{-a x^{2}} d x=\frac{\Gamma(n)}{2 a^{n}}, \mathrm{a}>0, \mathrm{n}>0$.

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1. (a) Find the Laplace transform of $\mathrm{f}(\mathrm{t})=$ cost, $\mathrm{o}<\mathrm{t}<\pi \mathrm{f}(\mathrm{t})=\sin \mathrm{t}, \mathrm{t}>\pi$
(b) Using Laplace transforms, evaluate $\int_{0}^{\infty} \frac{\cos a t-\cos b t}{t} \mathrm{dt}$. $[7+8]$
2. (a) Find $L^{-1}\left\{\frac{s+1}{(s+2 s+2)^{2}}\right\}$.
(b) Find $L^{-1}\left\{\frac{1}{\left(s^{2}-1\right)\left(s^{2}+25\right)}\right\}$ using convolution theorem. [7+8]
3. Find the fourier series to represent the function $f(x)$ given by $f(x)=0$, for $-\pi<x<0$, $\mathrm{f}(\mathrm{x}) \mathrm{x}^{2}$, for $0<\mathrm{x}<\pi$
4. (a) Find the fourier cosine and sine transform of $f(x)=x e^{-a x}$
(b) Find the inverse fourier cosine transform of $(\sin \mathrm{ap}) / \mathrm{p}$
5. (a) Solve $p+q=\sin x+\sin y$
(b) Solve $\mathrm{xp}+\mathrm{yq}=\mathrm{z}$
6. A long rectangular plate of width ' $a$ ' with insulated surface has its temperature ' v ' equal to zero on both the long sides and one of the short sides so that $v(0, y)=0$, $\mathrm{v}(\mathrm{a}, \mathrm{y})=\mathrm{o} ; \mathrm{v}(\mathrm{x}, 8)=0$, and $\mathrm{v}(\mathrm{x}, 0)=\mathrm{kx}$. Find the steady state temperature in the plate.
7. (a) Find the Z-transforms of (i) $\mathrm{a}^{\mathrm{n}} \sin \mathrm{n} \alpha$ (ii) $\mathrm{a}^{\mathrm{n}} \cos \mathrm{n} \alpha, \mathrm{n}=0$.
(b) Find the Z-transforms of (i) $\mathrm{e}^{-\mathrm{an}} \cos \mathrm{n} \theta$ (ii) $2 n-5 \sin \left(\frac{\mathrm{n} \pi}{4}\right)+3 a^{4}$. $\quad[8+7]$
8. (a) Prove that $\int_{0}^{1} \frac{1}{\sqrt{1-x^{n}}} d x=\frac{\sqrt{\pi}}{n} \frac{\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}$
(b) Evaluate $\int_{0}^{\infty} \mathrm{x} e^{-x^{8}} d x \cdot \int_{0}^{\infty} x^{2} e^{-x^{4}} d x$ using beta and gamma functions. [8+7]

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Time: 3 hours
Max Marks: 75
Answer any FIVE Questions
All Questions carry equal marks

1. (a) A function is periodic in $(0,2 \mathrm{~b})$ and is defined as $\mathrm{f}(\mathrm{t})=1,0<\mathrm{t}, 0<\mathrm{b}, \mathrm{f}(\mathrm{t})=0$, $b<t<2 b$ find the Laplace transform of $f(t)$.
(b) Using Laplace transforms evaluate $\int_{0}^{\infty} \mathrm{t} \mathrm{e}^{-t} \sin \mathrm{dt}$
2. (a) Find inverse Laplace transform of $\frac{s}{(s+3)^{2}}$
(b) Find inverse Laplace transform of $\frac{1}{\left(s^{2}+5^{2}\right)^{2}}$
3. (a) Find the fourier series to represent $\mathrm{f}(\mathrm{x})=\mathrm{x}^{2}-2$ when $-2 \leq x \leq 2$
(b) Find the half range cosine series for the function $f(x)=(x-1)^{2}$ in $0<x<1$ hence deduce that $1+1 / 2^{2}+1 / 3^{2}+\ldots \ldots=\pi^{2} / 6 \quad[8+7]$
4. Find the fourier cosine and sine transform of $f(x)$ defined by $f(x)=x^{n-1}$ and $f(x)=1 /$ $v \mathrm{x}$
5. (a) Form the Partial Differential Equation by eliminating arbitrary function from $z=f(y / x)$
(b) Solve $\mathrm{p}^{2}+\mathrm{q}^{2}=\mathrm{z}^{2}$
6. A tightly stretched string with fixed end points $\mathrm{x}=0$ and $\mathrm{x}=l$ is initially at rest in its equilibrium position. If it is vibrating by giving to each of its points a velocity $\lambda x(l-x)$, find the displacement of the string at any distance ' x ' from one end at any time ' $t$ '.
7. (a) Find the inverse Z-transform of $\frac{z^{2}}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$
(b) Evaluate $Z^{-1}\left(\frac{z}{z^{2}+11 z+24}\right)$.
8. (a) Evaluate $\int_{0}^{2}\left(8-x^{3}\right)^{1 / 3} d x$ using beta and gamma functions.
(b) Prove that $\int_{0}^{\pi / 2} \sqrt{\sin \theta} d \theta \cdot \int_{0}^{\pi / 2} \frac{1}{\sqrt{\sin \theta}} d \theta=\pi$.

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1. (a) Find the Laplace transform of $\mathrm{e}^{3 t}-2 \mathrm{e}^{-2 t}+\sin 2 \mathrm{t}+\cos 3 \mathrm{t}+\sinh 3 \mathrm{t}-2 \cosh 4 \mathrm{t}$ $+9$
(b) Find the Laplace transform of $f(t)=\left\{\begin{array}{lll}1, & \text { if } 0<t<1 \\ 0, & \text { if } 1<t<2 \\ 5, & \text { if } 2<t<3 \\ 0, & \text { if } t>3\end{array}\right.$
2. (a) Find inverse Laplace transform of $\frac{s+3}{s^{2}-10 s+29}$
(b) Find inverse Laplace transform of $\frac{1}{s^{2}\left(s^{2}+a^{2}\right)}$
3. (a) Find the fourier series to represent $f(x)=x^{2}-2$ when $-2 \leq x \leq 2$
(b) Find the half range cosine series for the function $f(x)=(x-1)^{2}$ in $0<x<1$ hence deduce that $1+1 / 2^{2}+1 / 3^{2}+\ldots \ldots=\pi^{2} / 6 \quad[8+7]$
4. (a)Find the finite fourier sine and cosine transform of $f(x)=\frac{e^{-a x}-e^{-b x}}{x}$
(b) Find the finite fourier sine and cosine transform of $f(x)=x$ where $0<x<4[8+7]$
5. (a) Solve $p-q=x-y$
(b) Solve $x^{2}(y-z) p+y^{2}(z-x) q=z^{2}(x-y)$
6. A square plate is bounded by the lines $\mathrm{x}=0, \mathrm{y}=0, \mathrm{x}=l \quad \& \mathrm{y}=1$. Its tales are insulated. The temperature along the upper horizontal edge is given by $\mathrm{u}(\mathrm{x}, l)=$ $\mathrm{x}(l-\mathrm{x}), 0 \leq \mathrm{x} \leq l$, while the other three edges are kept $0^{\circ} \mathrm{C}$. Find the steady state temperature in the plate.
7. (a) Find the inverse Z-transform of $\frac{z^{2}}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$
(b) Evaluate $Z^{-1}\left(\frac{z}{z^{2}+11 z+24}\right)$.
8. (a) Prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$
(b) Evaluate $\int_{0}^{\infty} \frac{x^{8}\left(1-x^{6}\right)}{(1+x)^{24}} d x$, using beta and gamma functions.
