Set No. 1

Code No: R10102 / R10

I B.Tech I Semester Regular/Supplementary Examinations January 2012

MATHEMATICS - I

(Common to all branches)

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions

All Questions carry equal marks

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1.(a) Find the differential equations of all parabolas with x-axis as its axis and $(\alpha, 0)$ as its focus.

(b) Find the orthogonal trajectories of coaxial circles $x^2 + y^2 + 2\lambda y + c = 2$, where λ is the parameter.

[7M + 8M]

2.(a) Solve $(D^2 - 2)y = e^{-\sqrt{2}x} + \cos x$

(b) Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2\,sin\,hx$ subject to y=o and $\frac{dy}{dx} = 1$ at x=0.

[7M + 8M]

3.(a) If $u = xy + yz + zx$, $v = x^2 + y^2 + z^2$ and $w = x + y + z$, verify whether there exists a possible relationship in between u, v and w. If so find the relation.

(b) Find the minimum value of $x^2 + y^2 + z^2$ on the plane $x + y + z = 3a$

[7M + 8M]

4.(a) Trace the curve $x(x^2 + y^2) = 4(x^2 - y^2)$

(b) Trace the polar curve $r = 2 + 3\cos\theta$.

[7M + 8M]

5.(a) Find the perimeter of one loop of the curve $3a\,y^2 = x(x - a)^2$.

(b) Find the volume generated by revolving the area bounded by one loop of the curve $r = a(1 + \cos\theta)$ about the initial line.

[7M + 8M]

6.(a) Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dx.dy$ by changing the order of integration.

(b) Evaluate $\int_0^2 \int_0^{\sqrt{2x - x^2}} \frac{x}{\sqrt{x^2 + y^2}} dy dx$ by changing into polar coordinates.

[7M + 8M]

7.(a) Find the directional derivative of $\phi(x, y, z) = xy^2 + yz^3$ at the point (2,-1,1) in the direction of the vector $i + 2j + 2k$.

(b) Find $curl[r\,f(r)]$ where $r = xi + yj + zk$, $r = |r|$

[7M + 8M]

8.(a) Compute the line integral $\int (y^2\,dx - x^2\,dy)$ round the triangle whose vertices are (1,0),(0,1) and (-1,0) in the xy-plane.

(b) Evaluate the integral $I = \int\int_S x^3\,dydz + x^2 y\,dzdx + x^2 z\,dxdy$ using divergence theorem, where S is the surface consisting of the cylinder $x^2 + y^2 = a^2 (0 \le z \le b)$ and the circular disks $z=0$ and $z = b(x^2 + y^2 \le a^2)$.

[7M + 8M]

Set No. 2

Code No: R10102 / R10

I B.Tech I Semester Regular/Supplementary Examinations January 2012

MATHEMATICS - I

(Common to all branches)

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks
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1.(a) Find the solution of the differential equation $\frac{dy}{dx} = xe^{y-x^2}$ and $y(0) = 0$.

(b) A body initially at 80^0 C cools down to 50^0 C in 10 minutes, the temperature of the air being 40^0 C. What will be the temperature of the body after 20 minutes?

[7M + 8M]

2.(a) Solve $\frac{d^2y}{dx^2} + 9y = e^{2x}\, x^2$

(b) Find the general solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \sin 2x$

[7M + 8M]

3.(a) Verify whether the functions $u = sin^{-1}x + sin^{-1}y$ and $v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$ are functionally dependent. If so, find the relation between them.[7 M+8 M]

(b) Prove that the rectangular solid of maximum volume that can be inscribed into a sphere of radius 'a' is a cube.

[7M + 8M]

4.(a) Trace the parametric curve $x = a(\cos\theta + \frac{1}{2}\log tan^2(\frac{t}{2})$ and $y = a\sin t$.

(b) Trace the lemniscate $r^2 = a^2 \cos 2\theta$.

[7M + 8M]

5.(a) Find the surface area generated by revolving the arc of the curve $y = a\, cosh\,(x/c)$ from x=0 to x=c about the x-axis.

(b) Find the total length of the lamniscate $r^2 = a^2\, cos\, 2\theta$.

[7M + 8M]

6.(a) Find the area of the region which is outside the circle r=1 and inside the cordioid $r = (1+\cos\theta)$

(b) Evaluate the following integral by changing into polar coordinates $\iint \sqrt{\frac{1-(x^+ y^2)}{1+x^2+y^2}}dxdy$ over the positive coordinate of the circle $x^2 + y^2 = 1$

[7M + 8M]

7.(a) Find the directional derivative of the divergence of $F = xyi + yzj + z^2k$ at the point (2,1,2) in the direction of the outer normal to the sphere $x^2 + y^2 + z^2 = 9$.

(b) Find the value of a,b and c such that $(x + y + az)i + (bx + 2y - z)j + (-x + cy + 2z)k$ is irrotational.

[7M + 8M]

8.(a) If $f = (x^2 + y - 4)i + 3xyj + (2xz + z^2)k$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = 16$. Show by using Stokes theorem that $\int_S Curl\, f.nds = 2\pi a^3$.

(b) If S is the surface of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $ax + by + cz = 1$. Show that $\int_S r.nds = \frac{1}{2abc}$.

[7M + 8M]

Set No. 3

Code No: R10102 / R10

I B.Tech I Semester Regular/Supplementary Examinations January 2012

MATHEMATICS - I

(Common to all branches)

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions

All Questions carry equal marks

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1.(a) Solve $(x^2+y^2)\frac{dy}{dx}=xy$.

(b) A colony of bacteria is grown under ideal condition in laboratory so that the population increases exponentially with time. At the end of 3 hours there are 10000 bacteria. At the end of 5 hours there are 40000. How many bacteria were present initially?

[7M + 8M]

2.(a) Solve $(D^3-6D^2+11D-6)y=e^{-2x}+x^3$

(b) Solve $(D^2+1)y=x^2e^{2x}+x\cos x$.

[7M + 8M]

3.(a) If $u=x+y+z$, $u^2v=y+z$ and $u^3w=z$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

(b) Find the minimum and maximum distances of a point on the curve $2x^2+4xy+4y^2-8=0$.

[7M + 8M]

4.(a) Trace the parametric curve $x=a(t-\sin t)$ and $y=a(1+\cos t)$

(b) Trace the curve $y^2(x-a)=x^2(x+a)$ and $a>0$

[7M + 8M]

5.(a) Find the volume of the solid formed by revolving the area bounded by the curve $27ay^2=4(x-2a)^3$ about x-axis

(b) Find the length of the loop of the curve $r=a(1-\cos\theta)$.

[7M + 8M]

6.(a) Find the area of the loop of the curve $x^3+y^3=3axy$, by transforming it into polar coordinates.

(b) Change the order of integration and evaluate $I=\int_0^1\int_x^{\sqrt{x}} xy\,dy\,dx.$

[7M + 8M]

Code No: R10102 / R10

Set No. 3

7.(a) In what direction from the point $(1,3,2)$ is the directional derivative of $\phi = 2xz - y^2$ is maximum and what is its magnitude.

(b) Show that $\overline{F} = (y^2 cos x + z^3)i + (2y\sin x - 4)j + (3xz^2 + 2)k$ is a conservative force field and find its scalar potential.

[7M + 8M]

8.(a) Show that $F = (2xy + z^3)i + x^2 j + 3xz^2 k$ is a conservative force field. Find the scalar potential and the work done in moving an object in this field from (1,-2,1) to (3,1,4).

(b) Verify Green's theorem ,if $Mdx + Ndy$ is $(xy + y^2)dx + x^2 dy$ with c: closed curve of the region bounded by $y = x$ and $y = x^2$.

[7M + 8M]

Set No. 4

Code No: R10102 / R10

I B.Tech I Semester Regular/Supplementary Examinations January 2012

MATHEMATICS - I

(Common to all branches)

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions

All Questions carry equal marks

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1.(a) Solve $x\frac{dy}{dx} - y = x\sqrt{x^2 + y^2}$

(b) A body is heated to 110^0 C is placed in air at 10^0 C. After 1 hour its temperature is 80^0 C. When will the temperature be 30^0 C?

[7M + 8M]

2.(a) Solve $(D^2 + 3D + 2)\, y = \sin x \sin 2x$

(b) Solve $(D^2 + 2D - 3)\, y = x^3 e^{-2x}$.

[7M + 8M]

3.(a) Verify whether the functions $u = \frac{x-y}{x+z}$ and $v = \frac{x+z}{y+z}$ are functionally dependent. If so, find the relation in between them.

(b) The temperature T at any point (x, y, z) in the space is given as $T = 400\, x^2 yz$. Find the highest temperature on the surface of the sphere $x^2 + y^2 + z^2 = 1$

[7M + 8M].

4.(a) Trace the curve $x^3 + y^3 = 3axy$

(b) Trace the polar curve $r = a\,(1 - \sin\theta)$.

[7M + 8M]

5 (a) Find the surface area generated by revolving the arc $x^{2/3} + y^{2/3} = a^{2/3}$ about x-axis.

(b) Find the volume of the solid generated by revolving the cardioid $r = a\,(1 + \cos\theta)$ about the initial line.

[7M + 8M]

6.(a) Find the area of a plate in the form of a quadrant of an ellipse $x^2 / a^2 + y^2 / b^2 = 1$ by changing into polar coordinates.

(b) By changing the order of integration, evaluate the integral $\int_0^{4a} \int_{\frac{y^2}{4a}}^{2\sqrt{ay}} dx\, dy$.

[7M + 8M]

Code No: R10102 / R10

Set No. 4

7.(a) Find the constants a and b so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2 y + z^3 = 4$ at the point $(1,-1,2)$.

(b) Determine the constant b such that $\overline{A} = (bx^2 y + yz)i + (xy^2 - xz^2)j + (2xyz - 2x^2 y^2)k$ has zero divergence.

[7M + 8M]

8.(a) Evaluate $\int_c \overline{f}.d\overline{r}$ where $\overline{f} = x^2 i + y^2 j$ and curve c is the arc of the parabola $y=x^2$ in the xy-plane from (0,0) to (1,1).

(b) Evaluate by Stokes theorem $\oint_C (x+y)dx + (2x-z)dy + (y+z)dz$, where C is the boundary of the triangle vertices (0,0,0), (1,0,0) and (1,1,0).

[7M + 8M]