Set No. 1

Code No: R10202 / R10

I B.Tech II Semester Supplementary Examinations January / February - 2012

MATHEMATICS - II

(Common to all branches)

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions

All Questions carry equal marks

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1.(a) Find the Laplace transform of $e^{-3t}(\cos 4t + 3\sin 4t)$.

(b) If $f(t) = \begin{cases} 1 & 0 \le t < 1 \\ -1 & 1 \le t < 2 \end{cases}$ is a periodic function with period 2, find its Laplace transform.

[7M + 8M]

2.(a) Find the inverse Laplace transform of $\cot^{-1}\left(\frac{s+2}{3}\right)$.

(b) Solve the initial value problem by using Laplace transform method $y'' + 7y' + 10y = 4e^{-3t}$, $y(0) = 0$ and $y'(0) = -1$

[7M + 8M]

3.(a) If $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $(0, 2\pi)$. Show that $f(x) = \frac{\pi^2}{12} + \sum_{n}^{\infty} \frac{\cos nx}{n^2}$.

Hence obtain $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$.

(b) Obtain half range cosine series $f(x) = x$ in the interval $0 \le x \le \pi$. Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$

[7M + 8M]

4.(a) State Fourier integral theorem and deduce complex form of it.

(b) Find the Fourier transform of $f(x) = \begin{cases} 1 & |x| \le a \\ 0 & |x| > a, \end{cases}$ and hence evaluate $\int_{-\infty}^{\infty} \frac{\sin ax}{x}\,dx$.

[7M + 8M]

5.(a) Form the partial differential equation by eliminating arbitrary function from $z = f(y/x)$.

(b) Solve $(x^2 - yz)\,p + (y^2 - zx)\,q = z^2 - xy$.

[7M + 8M]

6.(a) A tightly stretched string with fixed end points $x = 0$ and $x = L$ is initially at rest in its equilibrium position. If it is set to vibrate by giving each of its points a velocity $\lambda x(L - x)$, find its displacement at a later time t.

(b) Find the temperature in a laterally insulated rod of length L whose both ends are insulated and the initial temperature $u(x,0) = x$ if $0 < x < \frac{L}{2}$ and $L - x$ if $\frac{L}{2} < x < L$.

[7M + 8M]

7.(a) Find the Z-transforms of (i) $(n-1)^2$ (ii) $e^{-an}\cos n\theta$.

(b) Solve the difference equation $u_{n+2}-3u_{n+1}+2u_n=0$, given $u_0=0$ and $u_1=1$.

[7M + 8M]

8.(a) Show that $\beta(m,n)=\dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

(b) Evaluate $\int_0^1 x^2\left(\log\dfrac{1}{x}\right)^3 dx$.

[7M + 8M]

Set No. 2

Code No: R10202 / R10

I B.Tech II Semester Supplementary Examinations January / February - 2012

MATHEMATICS - II

(Common to all branches)

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions

All Questions carry equal marks

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1.(a) Using Laplace transforms evaluate $\int_0^\infty \frac{e^{-t}\sin^2 t}{t}\,dt$

(b) Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} \sin \omega t & 0 \le t < \frac{\pi}{\omega} \\ -1 & \frac{\pi}{\omega} \le t < \frac{2\pi}{\omega}, \end{cases} \text{ with period } \frac{2\pi}{\omega}$$

[7M + 8M]

2.(a) Using convolution theorem, find the inverse Laplace transform of $\frac{1}{(s^2+9)(s+1)^2}$

(b) By using Laplace transform, solve the initial value problem $y'' = t\cos 2t$, $y(0) = 0$ and $y'(0) = 0$.

[7M + 8M]

3.(a) Prove that in the interval $(-\pi, \pi)$, $x\cos x = -\frac{1}{2}\sin x + 2\sum_{2}^{\infty}\frac{(-1)^n}{n^2-1}\sin nx$.

(b) Find the half range sine series of $f(x) = (x-1)^2$ in the interval (0,1).

[7M + 8M]

4.(a) Find the Fourier cosine transform of e^{-x^2}.

(b) Using Fourier integral theorems show that $\int_0^\infty \frac{\cos\omega x}{1+\omega^2}\,d\omega = \frac{\pi}{2}e^{-x}$.

[7M + 8M]

5.(a) Form the partial differential equation by eliminating arbitrary function from $f(x^2+y^2, z-xy) = 0$.

(b) Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$.

[7M + 8M]

Code No: R10202 / R10

Set No. 2

6.(a) Use separation of variables to solve $3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0$ with $u(x,0)=4e^{-x}$

(b) A rod of length 100 cm length has its ends A and B kept at 0^0 and 100^0 centigrade until steady state conditions prevail. The temperatures at the ends are changed to 20^0C and 60^0C respectively. Find the temperature in the rod.

[7M + 8M]

7.(a) If $Z\{u_n\}=\frac{z}{z-1}+\frac{z}{z^2+1}$, find the Z-transform of u_{n+2}.

(b) Find $Z^{-1}\left\{\frac{z^3}{(z+1)(z-1)^2}\right\}$

[7M + 8M]

8.(a) Define beta function and hence prove that $\Gamma(\frac{1}{2})=\sqrt{\pi}$.

(b) Show that $\int_0^{\infty}\sqrt{x}\,e^{-x^3}dx=\frac{\sqrt{\pi}}{3}$.

[7M + 8M]

Set No. 3

Code No: R10202 / R10

I B.Tech II Semester Supplementary Examinations January / February - 2012

MATHEMATICS - II

(Common to all branches)

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks
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1.(a) Show that $\int_0^\infty t^2 e^{-4t}.\sin 2t\,dt = \frac{11}{500}$

(b) Define impulse function and finds it's Laplace transform.

[7M + 8M]

2.(a) State and apply convolution theorem to find the inverse Laplace transform of $\frac{1}{(s+2)^2(s^2+4)}$.

(b) Solve the differential equation $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+5x=e^{-t}\sin t$, $x(0)=0$ and $x'(0)=1$.

[7M + 8M]

3.(a) Find the Fourier series to represent the function $f(x)$ given by $f(x)=\begin{cases} x & 0 \le x \le \pi \\ 2\pi - x & \pi < x \le 2\pi, \end{cases}$. Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{8}$.

(b) Find the half range sine series for $f(x)=\begin{cases} \frac{1}{4}-x & 0<x<\frac{1}{2} \\ x-\frac{3}{4} & \frac{1}{2}<x<1 \end{cases}$.

[7M + 8M]

4.(a) Evaluate $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dx$.

(b) Find the Fourier sine transform of $\frac{1}{x(x^2+a^2)}$.

[7M + 8M]

5.(a) Form the differential equations of all planes which are at a constant distance 'a' from the origin.

(b) Solve $y\,p = 2\,y\,x + \log q$.

[7M + 8M]

6.(a) Solve $u_x = 2u_t + u$ where $u(x,0) = 6e^{-3x}$.

(b) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions $u(x,0) = u(x,b) = u(0,y) = 0$ and $u(a,y) = f(y)$.

[7M + 8M]

7.(a) Show that $Z\left\{\frac{1}{n+1}\right\} = z\log\frac{z}{z-1}$.

(b) Evaluate $Z^{-1}\left\{\frac{z}{z^2+11z+24}\right\}$

[7M + 8M]

8.(a) Prove that $\beta(n,n) = \frac{\sqrt{\pi}\,\Gamma(n)}{2^{2n-1}\Gamma\left(n+\frac{1}{2}\right)}$.

(b) Prove that $\int_0^1 x^5(1-x)^3\,dx = \frac{1}{504}$.

[7M + 8M]

Set No. 4

Code No: R10202 / R10

I B.Tech II Semester Supplementary Examinations January / February - 2012

MATHEMATICS - II

(Common to all branches)

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions

All Questions carry equal marks

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1.(a) Find the Laplace transform of $\frac{\sin 3t \cos t}{t}$

(b) Find the Laplace transform of $3\cos 4(t-2)\,u(t-2)$.

[7M + 8M]

2.(a) Find the Laplace transform of $\frac{2s+12}{s^2+6s+13}$.

(b) By using Laplace transform method solve $(D^2-2D+2)x=0$, given that $x=Dx=1$ at $t=0$.

[7M + 8M]

3.(a) Expand $f(x)=e^{-x}$ as a Fourier series in the interval $(-L,L)$.

(b) Obtain half range sine series for e^x in $0<x<1$.

[7M + 8M]

4.(a) Find the Fourier cosine transform of e^{-ax} and hence evaluate $\int_0^\infty \frac{\cos\lambda x}{x^2+a^2}\,dx$.

(b) Solve the integral equation $\int_0^\infty f(x)\cos\alpha x\,dx = e^{-\alpha}$.

[7M + 8M]

5.(a) Form the PDE by eliminating 'f' from $f(x+y+z, x^2+y^2+z^2)=0$.

(b) Solve $x^2p^2+y^2q^2=z^2$.

[7M + 8M]

6.(a) Solve $u_{xx}=u_y+u$ with $u(0,y)=0$ and $\frac{\partial u(0,y)}{\partial x}=1+e^{-3y}$.

(b) A bar of length L is laterally insulated with its ends A and B kept at 0^0 and 100^0 respectively until steady state condition is reached. The temperature at A is raised to 30^0 and that at B is reduced to 80^0 simultaneously. Find the temperature in the rod at a later time.

[7M + 8M]

7.(a) Find the Z-transforms of (i) $\frac{1}{(n+1)!}$ and (ii) $1+(-2)^n$.

(b) Solve $u_{n+2} - 6u_{n+1} + 9u_n = 0$

[7M + 8M]

8.(a) Prove that $2^{2n-1}\Gamma(n)\Gamma\left(n+\frac{1}{2}\right) = \Gamma(2n)\sqrt{\pi}$.

(b) By using beta and gamma functions evaluate $\int_0^1 \frac{dx}{(1-x^3)^{\frac{1}{3}}}$.

[7M + 8M]