

Code No: X0221

**R07****SET - 1****II B. Tech I Semester Supplementary Examinations Nov – 2012****MATHEMATICS - III**

(Com. to EEE, ECE, EIE, ECC)

Time: 3 hours

Max. Marks: 80

Answer any FIVE Questions  
All Questions carry Equal Marks

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1. a) Evaluate  $\int_0^1 \left\{ \sqrt[3]{x \ln\left(\frac{1}{x}\right)} \right\} dx$ .
- b) Evaluate  $\int_0^{\frac{\pi}{2}} \sin^6 \theta \cos^7 \theta d\theta$ .
- c) Show that  $\frac{d}{dx} [x^n J_n(x)] = x^n J_{n-1}(x)$ .
2. a) If  $f(z)$  is an analytic function, Show that  $\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4|f'(z)|^2$
- b) Find the analytic function whose imaginary part is  $e^x (x \sin y + y \cos y)$ .
3. a) Find the all the roots of the equation  $e^z = -2$ .
- b) Separate into real and imaginary parts of  $f(z) = \cot(x + iy)$ .
- c) Find the real part of the principal value of  $(1 + i)^{1-i}$ .
4. a) Evaluate  $\int_0^{3+i} z^2 dz$  along the path the real axis to 3 and then vertically to  $3+i$ .
- b) Use Cauchy's integral formula to evaluate  $\int_C \frac{1}{z^2 + 9} dz$  Where  $C$  is the circle
- i)  $|Z - 3i| = 4$       ii)  $|Z + 3i| = 2$ .
5. a) Find Taylor's expansion of  $f(z) = \sin z$  about the point  $z = \frac{\pi}{2}$ .
- b) Find the Laurent series of  $f(z) = \frac{(z-2)(z+2)}{(z+1)(z+4)}$ , for  $1 < |z| < 4$ .
- c) What type of singularity has the function  $f(z) = \frac{1 - e^{2z}}{z^4}$

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**R07****SET - 1**

6. a) Determine the poles of the function  $f(z) = \frac{z^2 - 2x}{(z+1)^2 (z^2 + 1)}$  and find residue at each pole.

b) Evaluate  $\oint_C \frac{\tan z}{(z^2 - 1)} dz$  where  $C$  is  $|z| = \frac{3}{2}$ .

c) Use residue theorem to evaluate  $\int_0^{2\pi} \frac{d\theta}{\sqrt{2 - \cos \theta}}$

7. a) State and prove Argument principle.

b) Use Rouché's theorem to determine the number of zeros of the polynomial

$$p(z) = e^z - 4z^n + 1 \text{ which lie inside the circle } |z| = 1.$$

8. a) Determine the image of the region  $|z - 2i| = 2$  under the transformation  $w = \frac{1}{z}$ .

b) Determine the bilinear transformation that maps the points  $z_1 = 0, z_2 = i, z_3 = \infty$  into the points  $w_1 = 0, w_2 = \frac{1}{2}, w_3 = \infty$  respectively.

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**R07****SET - 2****II B. Tech I Semester Supplementary Examinations Nov – 2012****MATHEMATICS - III**

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1. a) Evaluate  $\int_0^2 \{x^3 \sqrt{8-x^3}\} dx$ .  
 b) Evaluate  $\int_0^{\frac{\pi}{2}} \frac{\sqrt[3]{\sin 8x}}{\sqrt{\cos x}} dx$   
 c) Show that  $\frac{d}{dx} \left[ x^{-n} J_n(x) \right] = -x^n J_{n+1}(x)$
2. a) If  $f(z)$  is an analytic function,  
 Show that  $\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^n = n^2 |f(z)|^{n-2} |f'(z)|^2$   
 b) Find the analytic function whose real part is  $e^{2x}(x \cos 2y - y \sin 2y)$ .
3. a) Find the all Solutions of the equation  $e^{2z-1}=1$ .  
 b) Separate into real and imaginary parts of  $f(z) = \tan(x + iy)$ .  
 c) Find the real part of the principal value of  $i^{\log(1+i)}$ .
4. a) Evaluate  $\int_0^{2+i} z^{-2} dz$  along the path of real axis to 2 and then vertically to 2+i.  
 b) Use Cauchy's integral formula to evaluate  $\int_C \frac{e^z}{z^2 + 1} dz$  where C is the circle  
 i)  $|z - i| = 1$ .    ii)  $|z + i| = 1$ .
5. a) Find Taylor's expansion of  $f(z) = \cos z$  about the point  $z = \frac{\pi}{2}$ .  
 b) Find the Laurent series of  $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$  for  $|z| > 3$ .  
 c) What type of singularity has the function  $f(z) = \frac{z - \sin z}{z^2}$ .

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**R07****SET - 2**

6. a) Determine the poles of the function  $f(z) = \frac{z^2 + 1}{z^2 - z}$  and find the residue at each pole.
- b) Evaluate  $\oint_C \frac{\cosh z}{z^3 - 3iz} dz$  where  $C$  is  $|z| = 1$ .
- c) Use residue theorem to evaluate  $\int_0^{2\pi} \frac{d\theta}{2 + \cos \theta}$ .
7. a) State and prove Rouché's theorem.
- b) Use Rouché's theorem to determine the number of zeros of the polynomial  $P(z) = z^4 - 8z + 10$  that lie within the annulus region  $1 < |z| < 3$ .
8. a) Find and plot the rectangular region  $0 \leq x \leq 2, 0 \leq y \leq 1$  under the transformation  $w = \sqrt{2} e^{\frac{i\pi}{4} z}$ .
- b) Determine the bilinear transformation that maps the points  $z_1 = -2, z_2 = 0, z_3 = 2$  into the points  $w_1 = \infty, w_2 = \frac{1}{2}, w_3 = \frac{3}{4}$  respectively.

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**R07****SET - 3****II B. Tech I Semester Supplementary Examinations Nov – 2012****MATHEMATICS - III**

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1. a) Evaluate  $\int_0^a \left\{ x^4 \sqrt{a^2 - x^2} \right\} dx$ .
- b) Evaluate  $\int_0^{\frac{\pi}{2}} \sqrt{\cot \theta} d\theta$
- c) Prove that  $(n+1) P_{n+1}(x) = (2n+1) x P_n(x) - n P_{n-1}(x)$ .
2. a) If  $f(z)$  is an analytic function,  
Show that  $\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |Re(f(z))|^2 = 2 |f'(z)|^2$ .
- b) Find the analytic function whose imaginary part is  $\frac{2 \sin x \sin y}{(\cos 2x + \cosh 2y)}$ .
3. a) Find the all Solutions of the equation  $e^z = 3+4i$ .  
b) Separate into real and imaginary parts of  $f(z) = \cos(x + iy)$ .  
c) Find the principal value of  $(1+i)^{2-i}$ .
4. a) Evaluate  $\int_{1-i}^{2+i} (2x + iy + 1) dz$  along the path the straight line joining  $1-i$  and  $2+i$ .  
b) Use Cauchy's integral formula to evaluate  $\int_C \frac{\cos z}{(z - \pi i)^2} dz$  where  $C$  is the circle  $|z| = 5$ .
5. a) Find Taylor's expansion of  $f(z) = \log(1+z)$  about the point  $z=0$ .  
b) Find the Laurent's expansion of  $f(z) = \frac{7z - 2}{(z + 1)z(z - 2)}$  in the region  $1 < |z + 1| < 3$ .  
c) What type of singularity has the function  $f(z) = \frac{e^{2z}}{(z - 1)^4}$

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**R07****SET - 3**

6. a) Determine the poles of the function  $f(z) = \frac{z^2}{(z-1)^2(z+2)}$  and the residue at each pole.
- b) Evaluate  $\int_C \frac{dz}{\sinh z}$  Where  $C$  is the circle  $|z| = 4$ , using residue theorem
- c) Use residue theorem to evaluate  $\int_0^{2\pi} \frac{d\theta}{\frac{5}{4} + \sin \theta}$
7. a) State and prove Fundamental theorem of Algebra.
- b) Use Rouché's theorem to determine the number of zeros of the polynomial  $P(z) = z^7 - z^3 + 12$  lie within the annulus region  $1 < |z| < 2$ .
8. a) Determine and plot the image of the region  $1 \leq |z| \leq \frac{3}{2}$  and  $\frac{\pi}{6} \leq |\theta| \leq \frac{\pi}{3}$  under  $w = z^2$ .
- b) Determine the bilinear transformation that maps the points  $z_1 = -1, z_2 = i, z_3 = 1$  into the points  $w_1 = 0, w_2 = i, w_3 = \infty$  respectively.

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**R07****SET - 4****II B. Tech I Semester Supplementary Examinations Nov – 2012****MATHEMATICS - III**

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1. a) Evaluate  $\int_0^1 x^4 \left[ \ln\left(\frac{1}{x}\right) \right]^3 dx$ .

b) Show that  $\int_0^{\frac{\pi}{2}} \sqrt{\tan \theta} d\theta$ .

c) Show that  ${}^n P_n(x) = x p_n'(x) - P_{n-1}'(x)$ .

2. a) If  $f(z)$  is an analytic function, show that  $\left[ \frac{\partial}{\partial x} |f| \right]^2 + \left[ \frac{\partial}{\partial y} |f| \right]^2 = (f')^2$

b) Find the analytic function whose real part is  $\frac{\sin 2x}{(\cosh 2y - \cos 2x)}$ .

3. a) Find the all Solutions of the equation  $e^{2z-1} = 1+i$ .

b) Separate into real and imaginary parts of  $f(z) = \sin(x + iy)$ .

c) Find the principal value of  $(1+i)^i$ .

4. a) Evaluate  $\int_0^{1+i} (x^2 + iy) dz$  along the paths  $y=x$  and  $y=x^2$ .

b) Use Cauchy's integral formula to evaluate  $\int_C \frac{e^{2z}}{(z+1)^4} dz$

Where  $C$  is the circle  $|z| = 3$ .

5. a) Find Taylor's expansion of  $f(z) = \frac{2z^3 + 1}{z^2 + z}$  about the point  $z = i$ .

b) Find the Laurent's expansion of  $f(z) = \frac{z^2 - 6z - 1}{(z-1)(z-3)(z+2)}$  in the region  $3 < |z+2| < 5$ .

c) What type of singularity has the function  $f(z) = z^2 e^{\frac{1}{z}}$ .

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**R07****SET - 4**

6. a) Find the residue of  $f(z) = \frac{ze^z}{(z-1)^3}$  at its pole.
- b) Evaluate  $\oint_C \frac{\cosh 5z}{z^2 + 4}$  where  $C$  is  $|z - i| = 2$ .
- c) Use residue theorem to evaluate  $\int_0^{2\pi} \frac{d\theta}{7 + 6 \cos \theta}$ .
7. a) State and prove Liouville's theorem.
- b) Use Rouché's theorem to determine the number of zeros of the polynomial  $p(z) = z^4 - 5z + 1$  that lie within the annulus region  $1 < |z| < 2$ .
8. a) Determine and plot the image of the region  $-1 \leq x \leq 1$  and  $-\pi \leq y \leq \pi$  under  $w = e^z$ .
- b) Determine the bilinear transformation that maps the points  $z_1=0$ ,  $z_2=1$ ,  $z_3=\infty$  into the points  $w_1=-1$ ,  $w_2=-i$ ,  $w_3=1$  respectively.