Code No: R21016 **R10** SET - 1

II B. Tech I Semester, Regular Examinations, Nov – 2012
MATHEMATICS - III
(Com. to CE, CHEM, BT, PE)

Time: 3 hours Max. Marks: 75

Answer any FIVE Questions
All Questions carry Equal Marks
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1. a) Prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)\,dx = 0, if\ \alpha \neq \beta$.

   b) Prove that $\int_{-1}^{1} x^m\, p_n(x)\, dx = 0$ if m<n using Rodrigue's formula. (8M+7M)

2. a) Derive the necessary and sufficient condition for *f(z)* to be analytic in Cartesian coordinates.

   b) Find the conjugate harmonic of $u = e^{x^2 - y^2}$. Hence find *f(z)* in terms of z. (8M+7M)

3. a) Write the real and imaginary parts of tanz.

   b) Find all the values of $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{(1+i\sqrt{3})}$ (8M+7M)

4. a) Evaluate $\oint_C \frac{z-3}{z^2+2z+5}\,dz$ where C is |z+1-i | =2 using Cauchy's Integral formula.

   b) Verify Cauchy's theorem for the function $f(z) = 3z^2 + iz - 4$ if C is the square with vertices at $1 \pm i\ and\ -1 \pm i$. (7M+8M)

5. a) Find the Taylor's series expansion of *cosz* about $z = \pi i$.

   b) Expand the Laurent series of $\frac{z^2-1}{(z+2)(z+3)}\ for\ |z| > 3$. (8M+7M)

6. a) Define pole and determine the residues at each pole for f $f(z) = \frac{z^2}{(z+1)^2(z+2)}$.

   b) Expand $f(z) = \frac{e^z}{(z-1)^2}$ as a Laurent series about z = 2 and hence find the residue at that point. (7M+8M)
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7. a) State and prove Rouche's theorem.
   b) Show that the equation $z^4+4(1+i)z+1=0$ has one root in each quadrant. (7M+8M)

8. a) Define transformation. Under the transformation $\omega = \frac{1}{z}$ find the image of the circle $|z-2i| = 2$.
   b) Find the bilinear transformation which maps the points (1,i,-1) into the points $(0,1,\infty)$
   (8M+7M)

Code No: R21016 **R10** SET - 2

II B. Tech I Semester, Regular Examinations, Nov – 2012
MATHEMATICS - III
(Com. to CE, CHEM, BT, PE)

Time: 3 hours Max. Marks: 75

Answer any FIVE Questions
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1. a) Prove that $J_n(-x) = (-1)^n J_n(x)$ where 'n' is a positive or '-'ve integer.

   b) Prove that $\int_{-1}^{1} p_m(x)\, p_n(x)\, dx = 0 \ if\ m \neq n$. (8M+7M)

2. a) Show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \log\left|f^1(z)\right| = 0$ where f(z) is an analytic function.

   b) Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic find $f(z) = u + iv$. (8M+7M)

3. a) Separate the real and imaginary parts of tan hz.

   b) If tan (log (x+iy)) = a+ib where $a^2 + b^2 \neq 1$. Show that $\frac{2a}{1-a^2-b^2} = \tan\ (\log(x^2 + y^2))$.

   (7M+8M)

4. a) Verify Cauchy's theorem, for the integral of z^3 taken over the boundary of the rectangle with vertices -1, 1, 1+i, -1+i.

   b) Use Cauchy's integral formula $\oint_C \frac{z^3 - 2z + 1}{(z-i)^2} dz$ where C is the circle |z| = 2. (8M+7M)

5. a) Define circle of convergence and find the Taylor's series expansion of $f(z) = \frac{1}{z}$ about the point z = 1.

   b) Obtain the Laurent's series expansion of $f(z) = \frac{e^z}{z(1-z)}$ about z =1. (8M+7M)

6. a) Define Residue at a pole of order m.

   b) Show that $\int_0^{\pi} \frac{d\theta}{a^2 + \sin^2\theta} = \frac{\pi}{a\sqrt{1+a^2}}\ for\ a > 0$. (7M+8M)

7. a) State Rouche's theorem and use it to find the no. of zeros of the polynomial $z^8 - 4z^5 + z + 1$ that lie inside the circle |z| = 1.
   b) State and prove Liouville's theorem. (8M+7M)

8. a) Define conformal mapping.
   Find the image of the circle |z| = 2, under the transformation ω = z+3+2i.
   b) Determine the Bilinear transformation which maps z=0, 2i, -2i into $\omega = -1, 0, \infty$ (7M+8M)

SET - 3
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Code No: R21016

R10

II B. Tech I Semester, Regular Examinations, Nov – 2012
MATHEMATICS - III
(Com. to CE, CHEM, BT, PE)

Time: 3 hours Max. Marks: 75

Answer any FIVE Questions
All Questions carry Equal Marks
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1. a) Prove that $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{1}{x}\sin x - \cos x\right]$.
   b) Show that $x^3 = \frac{2}{5}p_3(x) + \frac{3}{5}p_1(x)$. (8M+7M)

2. a) Prove that if $u = x^2 - y^2$, $v = \frac{-y}{x^2+y^2}$ both u and v satisfy Laplace's equation, but $u + iv$ is not regular (analytic) function of z.
   b) Find the analytic function whose real part is $y + e^x$ cosy. (8M+7M)

3. a) Find the real part of the principal value of $i^{\log(1+i)}$.
   b) Prove that $\tan^{-1} z = \frac{i}{z}\log\left(\frac{i+z}{i-z}\right)$ (8M+7M)

4. a) Show that $\oint_C (z+1)dz = 0$ where C is the boundary of the square whose vertices at the points z = 0, z=1, z=1+i, z=i.
   b) Evaluate $\oint_C \frac{e^z dz}{(z+1)^4}$ around c: |z-1| =3. (8M+7M)

5. a) Define power series and expand $f(z) = \frac{z-1}{z+1}$ in Taylor's series about the point z =0.
   b) Define the different types of singularities. (8M+7M)

6. a) State and prove Cauchy's residue theorem.
   b) Find the residue of $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$ at z =0.5. (8M+7M)

7. a) If the real number a >e, prove by using Rouche's theorem, that the equation $e^z = a z^n$ has n roots inside the unit circle.
   b) State and prove 'Fundamental theorem of Algebra'. (8M+7M)

8. a) Find the image of |z| = 2 under the transformation $\omega = 3z$.
   b) Determine the bilinear transformation that maps the points 1-2i, 2+i, 2+3i, respectively into 2+2i, 1+3i, 4. (8M+7M)
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Code No: R21016 **R10** SET - 4

II B. Tech I Semester, Regular Examinations, Nov – 2012
MATHEMATICS - III
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Time: 3 hours Max. Marks: 75

Answer any FIVE Questions
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1. a) Prove that $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$.
   b) Prove that $(2n+1)\,p_n(x) = p_{n+1}^1(x) - p_{n-1}^1(x)$. (8M+7M)

2. a) Show that f(x) = cos z is analytic everywhere in the complex plane and find $f^1(z)$.
   b) Show that the function $u = \frac{1}{2}\log(x^2+y^2)$ is harmonic and find its conjugate. (8M+7M)

3. a) Find all the roots of the equation cos z = 2.
   b) If cosh $(u+iv) = x+iy$ then prove that:
   $$\frac{x^2}{\cosh^2 u} + \frac{y^2}{\sinh^2 u} = 1$$ (8M+7M)

4. a) Evaluate $\int_{1-i}^{2+i} (2x+1+iy)\,dz$ along the straight line joining (1, -i) and (2, i).
   b) Evaluate $\int_C \frac{e^z}{(z^2+\pi^2)^2}\,dz$ where C is |z| = 4. (8M+7M)

5. a) Find the Maclaurin's series expansion of f(z) for log (i+z).
   b) Let $f(z) = \frac{1}{(1-z)(z-2)}$, write the Laurent series expansion in |z| >2. (8M+7M)

6. a) Show that $\int_0^{\pi} \frac{d\theta}{a+b\cos\theta} = \frac{\pi}{\sqrt{a^2-b^2}} (a>b>0)$.
   b) Find the poles of the function $f(z) = \frac{1}{(z+1)(z+3)}$ and residues at these poles. (8M+7M)

7. a) Show that the equation z^4 + 4 (1+i) z+1 =0 has one root in each quadrant.
   b) State and prove argument principle. (8M+7M)

8. a) Show that the relation $\omega = \frac{5-4z}{4z-2}$ transform the circle |z| = 1 into a circle of radius unity in the $\omega-plane$.
   b) Define Bilinear transformation. Find the bilinear transformation that maps the points $(\infty, i, 0)$ in to the points $(0, i, \infty)$ (8M+7M)
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