Set No - 1

Subject Code-: R10202/R10

I B.Tech II Semester Regular Examinations June - 2012

MATHEMATICS – II

(Common to All Branches)

Time: 3 hours **Max. Marks : 75**

Answer any FIVE Questions
All Questions carry equal marks
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1.(a) Using Laplace transform evaluate $\int_0^\infty \frac{e^{-t} - e^{-2t}}{t} dt$

(b) Find the Laplace transform of (i) $e^{-3t}(2\cos 5t - 3\sin 5t)$ (ii) $e^{3t}\sin^2 t$

[7M + 8M]

2.(a) Using the cenvolution Theorem find $L^{-1}\left[\frac{1}{S^2(S+1)^2}\right]$

(b) Solve the differential equation $\frac{d^2x}{dt^2} + 9x = \sin t$ using Laplace Transform given that $x(0) = 1, x(\pi/2) = 1$

[7M + 8M]

3.(a) Expand $f(x) = \pi x, \quad 0 < x < 1$
$= 0, \quad 1 < x < 2$
in to Fourier series.

(b) Show that in the interval (0,1), $\cos \pi x = \frac{8}{\pi}\sum_{n=1}^{\infty} \frac{n}{4n^2 - 1}\sin 2n\pi x$

[7M + 8M]

4.(a) Express the function $f(x) = 1, \ |x| \le 1$
$= 0, \ |x| > 1$
as Fourier integral. Hence evaluate $\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda} d\lambda$ and find $\int_0^\infty \frac{\sin x}{x} dx$

(b) Find the Fourier sine transform of $\frac{1}{x(x^2 + a^2)}$

[7M + 8M]

5.(a) Form the partial differential equation by eliminating the arbitrary function f from : $f(x^2 + y^2, x^2 - Z^2) = 0$

(b) Solve $(Z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$

(c) Solve $Z^2(p^2 + q^2) = x^2 + y^2$

[5M + 5M + 5M]

Subject Code-: R10202/R10 **Set No - 1**

6. A tightly stretched string with fixed end points x=0 and x=l is initially in a position given by $y = y_0 \sin^3 \frac{\pi x}{l}$. If it is released from rest from this position. Find the displacement y(x,t).

[15M]

7.(a) Show that $Z\left[\cos\left(\frac{n\pi}{2}+\theta\right)\right] = \frac{Z^2\cos\theta - Z\sin\theta}{Z^2+1}$

(b) Evaluate $Z^{-1}\left[\frac{4z^2-2z}{Z^3-5z^2+8z-4}\right]$

[7M + 8M]

8.(a) Show that $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}}dx$

(b) Prove that $\Gamma\left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}\,\Gamma(2n+1)}{z^{2n}\Gamma(n+1)}$

[7M + 8M]

Subject Code-: R10202/R10 **Set No - 2**

I B.Tech II Semester Regular Examinations June - 2012

MATHEMATICS – II

(Common to All Branches)

Time: 3 hours **Max. Marks : 75**

Answer any FIVE Questions

All Questions carry equal marks

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1.(a) Using Laplace transform, evaluate $\int_0^{\infty}\left(\frac{\cos 5t-\cos t}{t}\right)dt$

(b) Solve the D.E. $y^{11}+n^2y=a\sin(nt+2),\ y(0)=0, y^1(0)=0$ using Laplace Transform.

[7M + 8M]

2.(a) Find $L^{-1}\left(\frac{S}{S^4+4a^4}\right)$

(b) Find $L^{-1}\left[\frac{1}{(S+1)^3}\right]$

[7M + 8M]

3.(a) Find the Fourier series expansion of $f(x)=x\sin x$ in $-\pi<x<\pi$

(b) Find the half range sine series of period 2 L for $f(x)=\frac{2x}{L}, 0\le x\le L/2$

$=\frac{2}{L}(L-x), L/2\le x\le L$

[7M + 8M]

4.(a) Show that the Fourier transform of $f(x)=a-|x|,\quad |x|<a$

$=0,\quad |x|>a$

is $\sqrt{\frac{2}{\pi}}\left(\frac{1-\cos as}{S^2}\right)$. Hence deduce that $\int_0^{\infty}\left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$

(b) Find the cosine transform of $\frac{1}{1+x^2}$ and hence find the sine transform of $\frac{x}{1+x^2}$

[7M + 8M]

5.(a) Find the differential equation of all spheres of radius 8 and having their centres in the yz plane

(b) Solve the partial differential equation $z(x-y)=px^2-qy^2$

(c) Solve $\frac{x^2}{p}+\frac{y^2}{q}=z$

[5M + 5M + 5M]

Subject Code-: R10202/R10 **Set No - 2**

6. Find the temperature u(x,t) in a bar OA of length L which is perfectly insulated laterally and whose ends O and A are kept at 0^0 c, given that the initial temperature at any point P of the rod (where op=x) is given as $u(x,0) = f(x), (0 \le x \le 1)$

[15M]

7.(a) Find Z(cos h at. sin b t)

(b) If $f(z) = \frac{2z^2+3z+4}{(z-3)^3}, |z|>3$, then find the values of f (1), f (2) and f (3).

[7M + 8M]

8.(a) Prove that $\int_b^a (x-b)^{m-1}(a-x)^{n-1}dx = (a-b)^{m+n-1}\beta(m,n), m>0, n>0$

(b) Prove that $\int_0^1 \frac{x^2dx}{\sqrt{1-x^4}} x \int_0^1 \frac{dx}{\sqrt{1+x^4}} dx = \frac{\pi}{4v2}$

[7M + 8M]

Subject Code-: R10202/R10 **Set No - 3**

I B.Tech II Semester Regular Examinations June - 2012

MATHEMATICS – II

(Common to All Branches)

Time: 3 hours **Max. Marks : 75**

Answer any FIVE Questions

All Questions carry equal marks

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1.(a) Using Laplace transform evaluate $\int_0^\infty te^{-t}\sin t\,dt$.

(b) It f (x) is sectionally continuous and of exponential order and $L[f(t)]=\bar{f}(s)$, then $L[t^n f(t)]=(-1)^n \frac{d^n}{ds^n}[\bar{f}(s)]$ where n=1,2,3.

[7M + 8M]

2.(a) Find $L^{-1}\left[\frac{s+1}{(s^2+2s+2)^2}\right]$

(b) Find $L^{-1}\left[\frac{S^2}{(S^2+4)(S^2+9)}\right]$ using convolution theorem.

[7M + 8M]

3.(a) Find the Fourier series for the expansion of $f(x)=x\cos x$ in $-\pi<x<\pi$

(b) Find the half range sine series for $f(x)=1, 0<x<\frac{1}{2}$

$= 0, \frac{1}{2}<x<1$

[7M + 8M]

4.(a) Using Fourier Integral formula, show that $e^{-x}\cos x=\frac{2}{\pi}\int_0^\infty \frac{(\lambda^2+2)\cos\lambda x}{\lambda^2+4}d\lambda$

(b) Find the Fourier sine transform of $f(x)=\frac{e^{-ax}}{x}$ and deduce that

$$\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}\sin px\,dx=\tan^{-1}\frac{p}{a}-\tan^{-1}\frac{p}{b}$$

[7M + 8M]

5.(a) Find the differential equation arising from $\phi(x+y+z, x^2+y^2+Z^2)=0$

(b) Solve $(x^2-yz)p+(y^2-zx)q=z^2-xy$

(c) Solve $(x^2+y^2)(p^2+q^2)=1$

[5M + 5M + 5M]

Subject Code-: R10202/R10 **Set No - 3**

6. Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ with u (0,y) = 0 = u (x,0) and $u(x,a)=\sin\left(\frac{n\pi x}{L}\right)$

[15M]

7.(a) If $\bar{u}\ (Z)=\frac{2z^2+5z+12}{(z-1)^4}$ evaluate u_2 and u_3.

(b) Find $Z^{-1}\left[\frac{8z-Z^3}{(4-z)^3}\right]$

[7M + 8M]

8.(a) Show that $\beta(m,n)=2\int_0^{\pi/2}\sin^{2m-1}\theta\cos^{2n-1}\theta d\theta$

hence find $\int_0^{\pi/2}\sin^5\theta\cos^{7/2}\theta d\theta$

(b) Prove that $\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{2}{n}\right)\Gamma\left(\frac{3}{n}\right)........\ \Gamma\left(\frac{n-1}{n}\right)=\frac{(2n)^{\frac{n-1}{2}}}{n^{1/2}}$

[7M + 8M]

Subject Code-: R10202/R10 **Set No - 4**

I B.Tech II Semester Regular Examinations June - 2012

MATHEMATICS – II

(Common to All Branches)

Time: 3 hours **Max. Marks : 75**

Answer any FIVE Questions
All Questions carry equal marks
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1.(a) Using Laplace transform evaluate $\int_0^{\infty} te^{-t} \sin t dt$

(b) Using Laplace transform, solve
$(D^2 + 4D + 5)y = 5$, given that y(0)=0, $y^{11}(0)=1$.

[7M + 8M]

2.(a) Find $L^{-1}\left[\frac{1}{S^2(S^2+1)(S^2+4)}\right]$

(b) Solve the following differential equation using the Laplace transform,
$y^{11} - 3y^1 + 2y = 4t + e^{3t}$, y(0)=1, $y^1(0)=1$

[7M + 8M]

3.(a) Determine the Fourier series expansion of the function $f(x) = \frac{1}{12}(3x^2 - 6x\pi + 2\pi^2)$ in the inlinval $(0,2\pi)$

(b) Find the half – range sine series of the function
$f(x) = x, \quad 0 < x < \pi/2$
$= \pi - x, \quad \frac{\pi}{2} < x < \pi$

[7M + 8M]

4.(a) Show that Fourier transform of $e^{\frac{-x^2}{2}}$ is reciprocal.

(b) Find the Fourier transform of $f(x) = 1 - x^2, |x| \le 1$
$= 0, \quad |x| > 1$
and hence evaluate $\int_0^{\infty} \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2} dx.$

[7M + 8M]

5.(a) Form the partial differential equation by eliminating the arbitrary function ϕ from: $\phi(x^2 + y^2 + z^2, z^2 - 2xy) = 0$

(b) Solve $(x^3 + 3xy^2)p + (y^3 + 3x^2y)q = 2(x^2 + y^2)z$

(c) Solve $(x + pz)^2 + (y + qz)^2 = 1$

[5M + 5M + 5M]

Subject Code-: R10202/R10

Set No - 4

6. A tightly stretched string with fixed end points n=0 and x=l is initially at rest in its equilibrium position. If it is vibrating by giving to each of its points a velocity $\lambda x(l-x)$, find the displacement of the string at any distance x from one end at any time t.

[15M]

7.(a) Evaluate $Z(\cos\theta + i\sin\theta)^n$. Hence prove that

$$Z(\cos n\theta) = \frac{Z(Z-\cos\theta)}{Z^2 - 2z\cos\theta + 1} \quad \text{and} \quad Z(\sin n\theta) = \frac{Z\sin\theta}{Z^2 - 2z\cos\theta + 1}$$

(b) Find $Z^{-1}\left[\frac{1}{(z-\frac{1}{2}(z-\frac{1}{3})}\right]$ in the region

(i) $\frac{1}{3} < |z| < \frac{1}{2}$ (ii) $|z| > \frac{1}{2}$

[7M + 8M]

8.(a) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ where m>0 , n>0.

(b) Show that (i) $\beta(m,\frac{1}{2}) = 2^{2m-1}\beta(m,m)$

(ii) $\Gamma(m)\Gamma(m+\frac{1}{2}) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2^m)$

[7M + 8M]