B.Tech II Year I Semester (R07) Supplementary Examinations, May 2013

# MATHEMATICS - II <br> (Common to CE and BT) 

Time: 3 hours
Max Marks: 80

## Answer any FIVE questions

## All questions carry equal marks

1 (a) Reduce the matrix $A$ to the normal form of PAQ and hence find its rank given:

$$
A=\left[\begin{array}{cccc}
3 & 2 & -1 & 5 \\
5 & 1 & 4 & -2 \\
1 & -4 & 11 & -19
\end{array}\right]
$$

(b) Find the values of $\lambda$ for which equations:
$(\lambda-1) x+(3 \lambda+1) y+2 \lambda z=0$,
$(\lambda-1) x+(4 \lambda-2) y+(\lambda+3) z=0$,
$2 x+(3 \lambda+1) y+3(\lambda-1) z=0$,
are consistent, and find the ratios of $x: y: z$ when $\lambda$ has the smallest of these values. What happens when $\lambda$ has the greater of these values.

2 (a) If $\lambda$ is the eigen values of matrix A , then prove that eigen values of $A^{-1}$ is $\frac{1}{\lambda}$.
(b) Show that the matrix $A=\left[\begin{array}{ccc}8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1\end{array}\right]$ is diagonalizable.

3 (a) Show that the matrix $\left[\begin{array}{ccc}\cos \phi & 0 & \sin \phi \\ \sin \theta \sin \phi & \cos \theta & -\sin \theta \cos \phi \\ -\cos \theta \sin \phi & \sin \theta & \cos \theta \cos \phi\end{array}\right]$ is an orthogonal matrix.
(b) Reduce the quadratic form $3 x^{2}+3 y^{2}+3 z^{2}+2 x y+2 x z-2 y z$ to canonical form by orthogonal transformation.

4 (a) Find the Fourier series for $f(x)=e^{-x}$ in $0<x<2 \pi$.
(b) Find the half-range sine series for $f(x)=\cos x$ in $(0, \pi)$.

5 (a) Form the partial differential equation by eliminating the arbitrary function from the following:
(i) $z=x y+f\left(x^{2}+y^{2}\right)$ (ii) $l x+m y+n z=f\left(x^{2}+y^{2}+z^{2}\right)$
(b) Solve: $p x^{2}+q y^{2}=z^{2}$.

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6 (a) Solve by the method of separation of variables $2 x \frac{\partial z}{\partial x}-3 y \frac{\partial z}{\partial y}=0$.
(b) A tightly stretched string with fixed and points $x=0$ and $x=l$ is initially at rest in its equilibrium. If it is vibrating by giving to each of its points a velocity $\mu x(l-x)_{1}$ find the displacement of the string at any distance $x$ from one and at any time $f$.

7 (a) Express the function $f(x)=\left\{\begin{array}{ll}1, & |x| \leq 1 \\ 0, & |x|>1\end{array}\right.$ as Fourier integral. Hence evaluate $\int_{0}^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d \lambda$.
(b) Find the sine and cosine transform of $2 e^{-3 x}+3 e^{-2 x}$.

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(a) Find: (i) $z\left[\cos \frac{n \pi}{2}\right]$.
(ii) $z\left[\sin \frac{n \pi}{2}\right]$.
(iii) $z^{-1}\left[\frac{z+2}{z^{2}-5 z+6}\right]$.
(b) Solve: $y_{n+2}-7 y_{n+1}-8 y_{n}=2^{n} n^{2}$ by $z-$ transform.

