Code: 9ABS302

R9

B.Tech II Year I Semester (R09) Supplementary Examinations, May 2013

MATHEMATICS - III

(Common to EEE, EIE, E.Con.E, ECE and ECC)

Time: 3 hours Max Marks: 70

Answer any FIVE questions
All questions carry equal marks

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1 (a) Prove that $\Gamma(n)\,\Gamma(1-n) = \frac{\pi}{\sin n\pi}$.

(b) State and prove Rodrigne's formula.

2 (a) If $f(z)$ is a regular function of z, prove that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}|f(z)|^2 = 4\,|f^1(z)|^2$

(b) Define an analytic function. Find the analytic function $f(z) = u + iv$ given $u = a(1 + cos\theta)$.

3 (a) Find all values of z which satisfy sin $z = 2$.

(b) Find all principal values of $(1 + \mathrm{i}\sqrt{3})^{(1+\mathrm{i}\sqrt{3}}$.

4 (a) Evaluate $\int_{0,0}^{1,3} 3x^2y\ \ dx + (x^3 - 3y^2)\,dy$ along the curve (i) $y = 3x$. (ii) $y = 3x^2$.

(b) Evaluate $\int_c \frac{dz}{z^8(z+4)}$ where C is the circle $|z| = 2$.

5 (a) Obtain the Taylar series expansion of: $f(z) = \frac{e^z}{z(z+1)}$ about $z = 2$.

(b) Define singular point, expand $f(z) = \frac{e^{2z}}{(z-1)^3}$ as Laurent's series about the singular point $z = 1$.

6 (a) Evaluate $\int_c \frac{4-3z}{z(z-1)(z-2)}\,dz$ where C is the circle |z| = $\frac{3}{2}$ using residue theorem.

(b) Evaluate by contour integration $\int_0^\infty \frac{dx}{1+x^2}$

7 (a) Use Rouche's theorem to show that the equation $z^5 + 15z + 1 = 0$ has one root in the disk |z| $< \frac{3}{2}$ and four roots in the annulus $\frac{3}{2}$ <|z|< 2.

(b) State and prove fundamental theorem of algebra.

8 (a) Show that the function W = $\frac{4}{2}$ transform the straight line $x = c$ in the z −plane in to a circle in the w −plane.

(b) Find the bilinear transformation that maps the points 1, $i, -1$ in to the points $2, i, -2$.

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