# II B. Tech II Semester Regular Examinations April/May - 2013 

 PROBABILITY AND STATISTICS(Com. to CE, CHEM, PE)
Time: 3 hours
Max. Marks: 75

## Answer any FIVE Questions <br> All Questions carry Equal Marks

1. a) A Class contains 10 men and 20 women, of which half the men and half the women have brown eyes. Find the probability that a person chosen at random is a man or has brown eyes.
b) If $A$ and $B$ be events with $P(A)=\frac{1}{3}, P(B)=\frac{1}{4}$ and $P(A \cup B)=\frac{1}{2}$.

Find (i). $P(A / B)$ (ii). $P(B / A) \quad$ (iiii). $P\left(A \cap B^{c}\right) \quad$ (iv). $P\left(A / B^{c}\right)$
c) Find the probability of drawing 3 aces at random from a deck of 52 ordinary cards if the cards are replaced.
2. a) A discrete random variable X has the following probability distribution

| Value of X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $P(X=x)$ | a | 3 a | 5 a | 7 a | 9 a | 11 a | 13 a | 15 a | 17 a |

i) Find the value of ' $a$ '.
ii) Find $P(X<3), P(0<X<3), P(X \geq 3)$
iii) Find the distribution function of $X$.
b) Find the value of $c$ and the distribution function $\mathrm{f}(\mathrm{x})$ given the probability density function of a random variable X as:
$f(x)=\left\{\begin{array}{cc}\frac{c}{x^{3}}, & \text { if } \\ 0, & 1<x<\infty \\ \text { otherwise }\end{array}\right.$
3. a) If the probability of a bad reaction from a certain injection is 0.001 . Determine the probability that out of 2000 individuals more than two will get a bad reaction.
b) A sample of 100 dry battery cells tested to find the length of life produced the following results: $\bar{x}=12$ hours, $\sigma=3$ hours.
Assuming the data to be normally distributed, what percentage of battery cells are expected to have life (i) More than 15 hours (ii) Less than 6 hours (iii) Between 10 and 15 hours?
4. a) Determine the probability that $\bar{X}$ will be between 75 and 78 if a random sample of size 100 is taken from an infinite population having the mean $\mu=76$ and $\sigma^{2}=256$.
b) Find $95 \%$ confidence limits of the mean for a normally distributed population from which the following sample was taken: $19,16,15,15,14,13,12,10,9$.
5. a) Explain briefly the following
i) Type I error
ii) Type II error
iii) Critical region
b) An urban community would like to show that the incidence of breast cancer is higher than in a nearby rural area. If it is found that 20 of 200 adult women in the urban community have breast cancer and 10 of 150 adult women in the rural community have breast cancer, can we conclude at the 0.01 level of significance that breast cancer is more prevalent in the urban community?
6. Fit a Poisson distribution to the following data and test for its goodness of fit at 0.05 . level of significance

| x | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| f | 275 | 138 | 75 | 7 | 4 | 1 |

7. The following are the sample means and ranges for 10 samples each of size 5 . Calculate the control limits for the mean chart and the range chart and state whether the process is in control or not.

| Sample No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mean $(\bar{X})$ | 5.10 | 4.98 | 5.02 | 4.96 | 4.96 | 5.04 | 4.94 | 4.92 | 4.92 | 4.98 |
| $\operatorname{Range}(\mathrm{R})$ | 0.3 | 0.4 | 0.2 | 0.4 | 0.1 | 0.1 | 0.8 | 0.5 | 0.3 | 0.5 |

(Given $\mathrm{A}_{2}=0.577 ; \mathrm{D} 3=0 ; \mathrm{D}_{4}=2.115$ for sample size 5 )
8. a) Explain briefly the main characteristics of Queuing system?
b) Customers arrive at a one-man barber shop according to a Poisson process with a mean interarrival time of 20 minutes. Customers spend an average of 15 minutes in the barber chair. If an hour is used as a unit of time, then
i) What is the Probability that a customer need not wait for a haircut?
ii) What is the expected number of customers in the barber shop and in the queue?
iii) How much time can a customer expect to spend in the barber shop?
iv) Find the average time that the customers spend in the queue.

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(Com. to CE, CHEM, PE)
Time: 3 hours
Max. Marks: 75

## Answer any FIVE Questions

All Questions carry Equal Marks

1. a) In tossing a fair die, what is Probability of getting an odd number or a number less than 4 ?
b) If $A$ and $B$ be events with $P(A \cup B)=\frac{7}{8}, P(A \cap B)=\frac{1}{4}$ and $P\left(A^{c}\right)=\frac{5}{8}$.
(i). $P(A)$ (ii). $P(B)$ (iii). $P\left(A \cap B^{c}\right)$ (iv). $P\left(A^{c} \cap B\right)$ (v). $P\left(A^{c} \cup B^{c}\right)(v i) . P\left(A^{c} \cap B^{c}\right)$.
c) Find the probability of drawing 3 aces at random from a deck of 52 ordinary cards if the cards are not replaced.
2. a) A discrete random variable X has the following probability distribution given below:

| Value of X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}(\mathrm{X}=\mathrm{x})$ | 0 | a | 2 a | 2 a | 3 a | $\mathrm{a}^{2}$ | $2 \mathrm{a}^{2}$ | $7 \mathrm{a}^{2}+\mathrm{a}$ |

i) Find the value of ' $a$ '. ii) Find $P(X<6), P(0<X<4)$ and $P(X \geq 6)$
iii) Find the distribution function of X .
b) Find the value of $c$ and the distribution function $\mathrm{F}(\mathrm{x})$ given the probability density function of a random variable X as:

$$
f(x)=\left\{\begin{array}{ll}
c x & \text { if } 0 \leq x<2 \\
2 c & \text { if } 2 \leq x<4 \\
6 c-c x & \text { if } 4 \leq x<6
\end{array} .\right.
$$

3. a) If $X$ is a Poisson variate such that $P(X=2)=9 P(X=4)+90 P(X=6)$, find the standard deviation.
b) The mean height of 500 students in a certain college is 151 cm and the standard deviation is 15 cm . Assuming the heights are normally distributed, find how many students have heights between 120 and 155 cm ?
4. a) Determine the probability that $\bar{X}$ will be between 22.39 and 22.41 if a random sample of size 36 is taken from an infinite population having the mean $\mu=22.4$ and $\sigma=0.048$.
b) A random sample of 8 envelopes is taken from the letter box of a post office and their weights in grams are found to be: $12.1,11.9,12.4,12.3,11.5,11.6,12.1$, and 12.4. Find $99 \%$ confidence limits for the mean weight of the envelopes received at that post office.

1 of 2
5. a) Explain briefly the following
i) Left tailed test
ii) Right tailed test
iii) Two tailed test
b) A storekeeper wanted to buy a large quantity of bulbs from two brands A and B respectively. He bought 100 bulbs from each brand A and B and found by testing that brand A had mean life time of 1120 hrs and the S.D of 75 hrs and brand B had mean life time 1062 hrs and S.D of 82 hrs . Examine whether the difference of means is significant. Use a 0.01 level of significance.
6. In a study, a data was collected to determine whether or not the proportion of defectives produced by workers is same for the day, evening, or night shift, shown in Table.

| Shift | Day | Evening | Night |
| :--- | :---: | :---: | :---: |
| Defectives | 45 | 55 | 70 |
| Non defectives | 905 | 890 | 870 |

Use a 0.025 level of significance to determine if the proportion of defectives is the same for all three shifts.
7. The following are the sample means and ranges for 10 samples each of size 5 . Draw mean and range charts and comment on the state of control of the process.

| Sample No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mean $\left(\bar{X}_{)}\right.$ | 43 | 49 | 37 | 44 | 45 | 37 | 51 | 46 | 43 | 47 |
| Range | 5 | 6 | 5 | 7 | 7 | 4 | 8 | 6 | 4 | 6 |

(Given $\mathrm{A}_{2}=0.58 ; \mathrm{D}_{3}=0 ; \mathrm{D}_{4}=2.115$ for sample size 5)
8. a) Explain $(M / M / 1):(\infty / F C F S)$ Queuing model.
b) Patients arrive at the Government hospital for emergency service at the rate of one every hour. Currently, only one emergency case can be handled at time. Patients spend on average of 20 minutes receiving emergency care. The doctor wishes to have enough seats in the waiting room so that no more than about $1 \%$ of arriving patients will have to stand. Find
i) The probability that a patient arriving at the hospital will have to wait.
ii) The average length of the queue that forms.
iii) Average time a patient spends in the system.
iv) Average time a patient spends in the queue.

SET - 3

# II B. Tech II Semester Regular Examinations April/May - 2013 PROBABILITY AND STATISTICS 

(Com. to CE, CHEM, PE)
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Answer any FIVE Questions
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1. a) A ball is drawn at random from a box containing 6 red balls, 4 white balls, and 5 blue balls. Determine the probability that it is red or white.
b) If $A$ and $B$ be events with $P(A)=0.6, P(B)=0.3$ and $P(A \cap B)=0.2$.
(i). $P(A / B)$ and $P(B / A)$
(ii). $P(A \cup B)$
(iii). $P\left(A^{c}\right)$ and $P\left(B^{c}\right)$
(iv). $P\left(A^{c} / B^{c}\right)$ and $P\left(B^{c} / A^{c}\right)$.
c) Two cards are drawn from a well-shuffled ordinary deck of 52 cards. Find the probability that they are both aces if the first card is replaced.
2. a) A discrete random variable $X$ has the following probability distribution

| Value of X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}(\mathrm{X}=\mathrm{x})$ | 2 a | 4 a | 6 a | 8 a | 10 a | 12 a | 14 a | 4 a |

i) Find the value of ' $a$ '. $\quad$ ii) Find $P(X<3)$ and $P(X \geq 5)$
iii) Find the distribution function of $X$.
b) Find the value of $c$ and the distribution function $\mathrm{F}(\mathrm{x})$ given the probability density function of a random variable X as:
$f(x)=\frac{c}{x^{2}+1},-\infty<x<\infty$.
3. a) If a random variable has a Poisson distribution such that $P(1)=P(2)$, find
i) Mean of the distribution
ii) $P(4)$.
b) An aptitude test for selecting officers in a bank is conducted on 1000 candidates. The average score is 42 and the standard deviation of score is 24 . Assuming normal distribution for the scores, find
i) The number of candidates whose scores exceed 60 .
ii) The number of candidates whose scores lie between 30 and 60 .
4. a) Determine the probability that $\bar{X}$ will be between 66.8 and 68.3 if a random sample of size 25 is taken from an infinite population having the mean $\mu=68$ and $\sigma=3$.
b) Find $99 \%$ confidence limits for the mean of a normally distributed population from which the following sample was taken: $19,16,15,15,14,13,12,10,9$.

1 of 2
5. a) Explain briefly the following
i) Null Hypothesis, ii) Alternative hypothesis iii) Test of hypothesis
b) In a random sample of 100 tube lights produced by company A, the mean life time of tube light is 1190 hours with standard deviation of 90 hours. Also in a random sample of 75 tube lights from company B the mean life time is 1230 hours with standard deviation of 120 hours. Is there a difference between the mean lifetimes of the two brands of tube lights at a significance level of 0.05 ?
6. Referring to the data of following table, test the hypothesis that the opinions concerning the proposed abortion law the same within each political affiliation. Use a 0.05 level of significance.

| Political Affiliation |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Abortion Law | Democrat | Republican | Independent | Total |
| For | 82 | 70 | 62 | 214 |
| Against | 93 | 62 | 67 | 222 |
| Undecided | 25 | 18 | 21 | 64 |
| Total | $\mathbf{2 0 0}$ | $\mathbf{1 5 0}$ | $\mathbf{1 5 0}$ | $\mathbf{5 0 0}$ |

7. The following are the sample means and ranges for 10 samples each of size 5 . Construct a $\bar{X}$ chart and R-chart and determine whether this process is in control.

| Sample No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mean $(\bar{X})$ | 20 | 34 | 45 | 39 | 26 | 29 | 13 | 34 | 37 | 23 |
| Range | 23 | 39 | 15 | 05 | 20 | 17 | 21 | 11 | 40 | 10 |

(Given $\mathrm{A}_{2}=0.577 ; \mathrm{D} 3=0 ; \mathrm{D}_{4}=2.115$ for sample size 5 )
8. a) Derive the average number of customers in the system.

In $(M / M / 1):(\infty / F C F S)$ model.
b) Customers arrive at a box office window, being manned by a single individual, according to a Poisson input process with a mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 90 seconds. Find the average waiting time of a customer in the queue. Also determine the average number of customers in the system and average queue length.

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(Com. to CE, CHEM, PE)
Time: 3 hours
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Answer any FIVE Questions
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1. a) What is the probability of getting a total of 7 or 11 when a pair of fair dice is tossed?
b) If $A$ and $B$ are independent events with $P(A)=0.3, P(B)=0.4$.

Find: (i). $P(A \cap B)$ and $P(A \cup B) \quad$ (ii). $P(A / B)$ and $P(B / A) \quad$ (iii). $P\left(A^{c}\right)$ and $P\left(B^{c}\right)$ (iv). $P\left(A^{c} / B^{c}\right)$ and $P\left(B^{c} / A^{c}\right)$.
c) Two cards are drawn from a well-shuffled ordinary deck of 52 cards. Find the probability that both are aces if the first card is not replaced.
2. a) A discrete random variable $X$ has the following probability distribution

| Value of X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}(\mathrm{X}=\mathrm{x})$ | a | 3 a | 5 a | 7 a | 9 a | 11 a | 13 a |

i) Find the value of ' $a$ '. $\quad$ ii) Find $P(X<4), P(3<X \leq 6)$ and $P(X \geq 5)$
iii) Find the distribution function of $X$.
b) Find the value of $c$ and the distribution function $\mathrm{F}(\mathrm{x})$ given the probability density function of a random variable X as:
$f(x)=\left\{\begin{array}{ll}c(3+2 x) & \text { if } 0<x<2 \\ 0 & \text { otherwise }\end{array}\right.$.
3. a) Suppose that X has a Poisson distribution. If $P(X=2)=\frac{2}{3} P(X=1)$ find,
i) $P(X=0)$
ii) $P(X=3)$.
b) Students of a class were given a mechanical aptitude test. Their grades were found to be normally distributed with mean 60 and standard deviation 5 . What percent of students scored i) More than 60 grades? ii) Less than 56 grades? iii) Between 45 and 65 grades?
4. a) Find $P(\bar{X}>66.75)$ if a random sample of size 36 is drawn from an infinite population with mean $\mu=63$ and $\sigma=9$.
b) A random sample of 8 envelopes is taken from the letter box of a post office and their weights in grams are found to be: $12.1,11.9,12.4,12.3,11.5,11.6,12.1$, and 12.4. Find $95 \%$ confidence limits for the mean weight of the envelopes received at that post office.
5. a) Explain briefly the following
i) Level of significance
ii) Left tailed test
iii) Right tailed test
b) A manufacturer claims that the average tensile strength of thread A exceed the average tensile strength of thread B by at least 12 kilograms. To test his claim, 50 pieces of each type of thread are tested under similar conditions. Type A thread had an average tensile strength of 86.7 kilograms with known standard deviation of $\sigma_{A}=6.28$ kilograms, while type B thread had an average tensile strength of 77.8 kilograms with known standard deviation of $\sigma_{B}=5.61$ kilograms. Test the manufacturers claim at 0.01 level of significance.
6. Samples of three kinds of materials, subjected to extreme temperature changes, produced the results shown in the following table:

|  | Material A | Material B | Material C | Total |
| :--- | :--- | :--- | :--- | :--- |
| Crumbled | 41 | 27 | 22 | 90 |
| Remained intact | 79 | 53 | 78 | 210 |
| Total | 120 | 80 | 100 | 300 |

Use the 0.05 level of significance to test whether, under the stated conditions, the probability of crumbling is the same for the three kinds of materials.
7. The table below gives the sample means and ranges for ten samples, each of size 5 . Construct the control charts for mean and range and test whether the process is in control or not.

| Mean $(\bar{X})$ | 4.98 | 4.92 | 5.02 | 4.98 | 4.98 | 5.08 | 5.04 | 4.95 | 4.95 | 4.92 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\operatorname{Range}(\mathrm{R})$ | 0.3 | 0.2 | 0.4 | 0.1 | 0.4 | 0.2 | 0.7 | 0.4 | 0.4 | 0.5 |

(Given $\mathrm{A}_{2}=0.577 ; \mathrm{D}_{3}=0 ; \mathrm{D}_{4}=2.115$ for sample size 5 ).
8. a) Derive the average number of customers in the Queue.

In $(M / M / 1):(\infty / F C F S)$ model.
b) The mean arrival rate to a service centre is 3 per hour. The mean service time is found to be 10 minutes per service. Assuming Poisson arrival and exponential service time, find
i) Utilization factor for this service facility.
ii) Probability of two units in the system.
iii) Expected number of units in the queue.
iv) Expected time in minutes that a customer has to spend in the system

