# II B. Tech I Semester, Regular Examinations, Nov - 2012 <br> PROBABILITY AND STATISTICS 

(Com. to CSE, IT)
Time: 3 hours
Max. Marks: 75

Answer any FIVE Questions<br>All Questions carry Equal Marks

1. a) Four cards are drawn from a pack of cards. Find the probability that i) all are diamonds
ii) there is one card of each suit, and (iii) there are two spades and two hearts.
b) A bag $X$ contains 2 white and 3 red balls and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y.
2. a) A random variable $X$ has the following probability function :

Values of X,

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}(\mathrm{x})$ | 0 | $k$ | $2 k$ | $2 k$ | $3 k$ | $k^{2}$ | $2 k^{2}$ | $7 k^{2}+k$ |

i) Find $k$, ii) Evaluate $p(X<6), p(X \geq 6), p(3<X \leq 6)$
iii) Find the minimum value of x so that $p(X \leq x)>\frac{1}{2}$.
b) Let X be a continuous random variable with distribution :

$$
\begin{aligned}
& \qquad f(x)= \begin{cases}\frac{1}{6} x+k, & \text { if } 0 \leq x \leq 3 \\
0 & \text { elsewhere }\end{cases} \\
& \text { i) Evaluate } \mathrm{k} \\
& \text { ii) Find } P(1 \leq X \leq 2) .
\end{aligned}
$$

3. a) Assume that on the average one telephone number out of fifteen called between 2 P.M. and 3 P.M. on week-days is busy. What is the probability that if 6 randomly selected telephone numbers are called i) not more than three ii) at least three of them will be busy?
b) In a normal distribution, $31 \%$ of the items are under 45 and $8 \%$ are over 64 . Find the mean and standard deviation of the distribution?
4. a) A population consists of the four numbers $1,5,6,8$. Consider all possible samples of size two that can be drawn without replacement from this population. Find i) The population mean, ii) The population standard deviation, iii) The mean of the sampling distribution of means, iv) The standard deviation of the sampling distribution of means.
b) Determine a $95 \%$ confidence interval for the mean of a normal distribution with variance $\sigma^{2}=9$, using a sample of $n=100$ values with mean $\bar{x}=5$.

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5. a) In a random sample of 100 tube lights produced by company A, the mean life time of tube light is 1190 hours with standard deviation of 90 hours. Also in a random sample of 75 tube lights from company B the mean life time is 1230 hours with standard deviation of 120 hours. Is there a difference between the mean lifetimes of the two brands of tube lights at a significance level of 0.01 ?
b) An urban community would like to show that the incidence of breast cancer is higher than in a nearby rural area. If it is found that 20 of 200 adult women in the urban community have breast cancer and 10 of 150 adult women in the rural community have breast cancer, can we conclude at the 0.05 level of significance that breast cancer is more prevalent in the urban community?
6. a) The mean life of 10 electric motors was found to be 1450 hrs with a S.D. of 423 hrs . A second sample of 17 motors chosen from a different batch showed a mean life of 1280 hrs with a S.D. of 398 hrs . Is there a significant difference between the means of the two samples? Use a 0.01 level of significance.
b) In two independent samples of sizes 8 and 10 the sum of squares of deviations of the sample values from the respective sample means were 84.4 and 102.6. Test whether the difference of variances of the populations is significant or not. Use a 0.05 level of significance.
7. a) The following data show the values of sample mean $\bar{X}$ and the range R for the sample of size 5 each. Calculate the values for central line and control limits for mean-chart and range chart and determine whether the process is in control

| Sample No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mean $(\bar{X})$ | 43 | 49 | 37 | 44 | 45 | 47 | 51 | 46 | 43 | 47 |
| Range $(\mathrm{R})$ | 05 | 06 | 05 | 07 | 07 | 04 | 08 | 06 | 04 | 06 |

(Given $n=5, A_{2}=0.577, D_{3}=0, D_{4}=2.115$ )
8. a) Explain briefly the main characteristics of Queuing system?
b) The arrival rate of customers at a counter in a bank follows Poisson distribution with a mean of $45 /$ hour; service rate of the clerk follows Poisson distribution with a mean of $60 /$ hour. Find the probability of having $0,5,10$ customers in the system. Find $L_{s}, L_{q}, W_{s}$ and $W_{q}$.

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1. a) In a given race, the odds in favor of four horses $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ are $1: 3,1: 4,1: 5,1: 6$ respectively. Assuming that a dead heat is impossible; find the chance that one of them wins the race.
b) In a bolt factory, machines A, B and C manufacture respectively $25 \%, 35 \%$ and $40 \%$ of the total. Of their output 5, 4 and 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B ?
2. a) A random variable X has the following probability function :
Values of X,

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}(\mathrm{x})$ | a | 3 a | 5 a | 7 a | 9 a | 11 a | 13 a | 15 a | 17 a |

i) Determine the value of a,
ii) Evaluate $P(X<3), P(X \geq 3), P(2 \leq X<5)$.
b) Let X be a continuous random variable with distribution :
$f(x)=\left\{\begin{array}{lc}\frac{1}{8} & \text { if } 0 \leq x \leq 8 \\ 0 & \text { elsewhere }\end{array}\right.$
Find i) $P(2 \leq X \leq 5) \quad$ ii) $P(3 \leq X \leq 7) \quad$ iii) $P(X \leq 6)$ iv) Determine and plot the graph of the cumulative distribution function F of X .
3. a) Out of 800 families with 4 children each, how many families would be expected to have (i) 2 boys and 2 girls (ii) at least one boy (iii) no girl (iv) at most two girls? Assume equal probabilities for boys and girls.
b) In a normal distribution, $7 \%$ of the items are under 35 and $89 \%$ are under 63 . What are the mean and standard deviation of the distribution?
4. a) A population consists of the four numbers $1,5,6,8$. Consider all possible samples of size two that can be drawn with replacement from this population. Find i) The population mean,
ii) The population standard deviation, iii) The mean of the sampling distribution of means,
iv) The standard deviation of the sampling distribution of means
b) Determine a $99 \%$ confidence interval for the mean of a normal distribution with variance $\sigma^{2}=4$, using a sample of $n=200$ values with mean $\bar{x}=10$
5. a) A manufacturer claims that the average tensile strength of thread A exceed the average tensile strength of thread B by at least 12 kilograms. To test his claim, 50 pieces of each type of thread are tested under similar conditions. Type A thread had an average tensile strength of 86.7 kilograms with known standard deviation of $\sigma_{A}=6.28$ kilograms, while type B thread had an average tensile strength of 77.8 kilograms with known standard deviation of $\sigma_{B}=5.61$ kilograms. Test the manufacturers claim at 0.05 level of significance.
b) In a study to estimate the proportion of residents in a certain city and its suburbs who favor the construction of a nuclear power plant, it is found that 63 of 100 urban residents favor the construction while only 59 of 125 suburban residents are in favor. Is there a significant difference between the proportion of urban and suburban residents who favor construction of the nuclear plant? Use a 0.05 level of significance.
6. a) Two samples of sodium vapor bulbs were tested for length of life and the following results were returned :

|  | Size | Sample mean | Sample S.D. |
| :--- | :--- | :--- | :--- |
| Type I | 8 | 1234 hrs | 36 hrs |
| Type II | 7 | 1036 hrs | 40 hrs |

Is the difference in the means significant to generalize that type I is superior to type II regarding length of life? Use a 0.05 level of significance.
b) Two independent samples of size 9 and 8 had the following values of the variables:

| Sample I | 17 | 27 | 18 | 25 | 27 | 29 | 27 | 23 | 17 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Sample II | 16 | 16 | 20 | 16 | 20 | 17 | 15 | 21 |  |

Do the estimates of the population variance differ significantly? Use a 0.05 level of significance.
7. The following data show the values of sample mean $\bar{X}$ and the range R for The sample of size 5 each. Calculate the values for central line and control limits for mean-chart and range chart and determine whether the process is in control

| Sample <br> No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mean $(\bar{X})$ | 11.2 | 11.8 | 10.8 | 11.6 | 11.0 | 9.6 | 10.4 | 9.6 | 10.6 | 10.0 |
| Range(R) | 07 | 04 | 08 | 05 | 07 | 04 | 08 | 04 | 07 | 09 |

(Given $n=5, A_{2}=0.577, D_{3}=0, D_{4}=2.115$ )
8. a) Explain (M/ M/ 1): ( $\infty /$ FCFS) Queuing model.
b) Assume that both arrival rate and service rate following Poisson distribution.

The arrival rate and service rate are 25 and 35 customers/hour respectively, at a single window in RTC reservation counter. Find $L_{s}, L_{q} W_{s}$ and $W_{q}$.

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1. a) A card is drawn from a well-shuffled pack of playing cards. What is the probability that it is either a spade or an ace?
b) The contents of bags I, II and III are as follows:

1 white, 2 black and 3 red balls,
2 white, 1 black and 1 red balls, and
4 white, 5 black and 3 red balls.
One bag is chosen at random and two balls drawn. They happen to be white and red. What is the probability that they come from bags I, II or III?
2. a) A random variable X has the following probability function :

Values of X,

| X | -3 | -1 | 2 | 5 |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}(\mathrm{X}=\mathrm{x})$ | $\frac{2 k-3}{10}$ | $\frac{k+1}{10}$ | $\frac{k-1}{10}$ | $\frac{k-2}{10}$ |

i) Find the value of $k$ (ii) Determine the distribution of X .
b) Let X be a continuous random variable with distribution:
$f(x)= \begin{cases}k x & \text { if } 0 \leq x \leq 5 \\ 0 & \text { elsewhere }\end{cases}$
i) Evaluate k .
ii) Find: $P(1 \leq X \leq 3), P(2 \leq X \leq 4)$ and $P(X \leq 3)$.
3. a) The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that
i) exactly two will be defective
ii) at least two will be defective
iii) none will be defective.
b) A sample of 100 dry battery cells tested to find the length of life produced the following results: $\bar{x}=12$ hours, $\sigma=3$ hours.
Assuming the data to be normally distributed, what percentage of battery cells are expected to have life i) more than 15 hours ii) less than 6 hours iii) between 10 and 14 hours?
4. a) A population consists of the four numbers $3,7,11,15$. Consider all possible samples of size two that can be drawn without replacement from this population.
Find: i) The population mean, ii) The population standard deviation,
iii) The mean of the sampling distribution of means,
iv) The standard deviation of the sampling distribution of means
b) Determine a $95 \%$ confidence interval for the mean of a normal distribution with variance $\sigma^{2}=0.25$, using a sample of $n=100$ values with mean $\bar{x}=212.3$.
5. a) A random sample of 64 bags of white cheddar popcorn weighed, on average, 5.23 ounces with a standard deviation of 0.24 ounces. Test the hypothesis that $\mu=5.5$ ounces against the alternative hypothesis, $\mu<5.5$ ounces at the 0.05 level of significance.
b) If 57 out of 150 patients suffering with certain disease are cured by allopathy and 33 out of 100 patients with same disease are cured by homeopathy, is there reason to believe that allopathy is better than homeopathy at 0.05 level of significance.
6. a) Samples of sizes 10 and 14 were taken from two normal populations with S.D. 3.5 and 5.2. The sample means were found to be 20.3 and 18.6. Test whether the means of the two populations are the same at $5 \%$ level.
b) The two random samples reveal the following data :

| Sample no. | Size | Mean | Variance |
| :--- | :--- | :--- | :--- |
| I | 16 | 440 | 40 |
| II | 25 | 460 | 42 |

Test whether the samples come the same normal population.
7. A machine is set to deliver packets of a given weight. 10 samples of size 5 each were recorded. Below are given the relevant data

| Sample No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mean $(\bar{X})$ | 15 | 17 | 15 | 18 | 17 | 14 | 18 | 15 | 17 | 16 |
| $\operatorname{Range}(\mathrm{R})$ | 07 | 07 | 04 | 09 | 08 | 07 | 12 | 04 | 11 | 05 |

Calculate the values of the central line and the control limits for the mean chart and range chart and then comment on the state of control.
(Given $n=5, A_{2}=0.577, D_{3}=0, D_{4}=2.115$ )
8. a) Derive the average number of customers in the queue. In (M/M/1): ( $\infty / \mathrm{FCFS}$ ) model.
b) The containers from railway goods wagons are unloaded at a single platform of a railway goods yard. The arrival rate of wagons is 8 wagons per day and service rate is 14 wagons per day. Assuming the arrival rate and service rate to follow Poisson distribution, determine the following: (i) Ls (ii) Lq (iii) Ws (iv) Wq.

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1. a) A problem in statistics is given to five students. Their chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{4}$ and $\frac{1}{5}$. What is the probability that the problem will be solved?
b) A factory has two machines A and B. Past records shows that machine A produced $60 \%$ of the items of output and machine B produced $40 \%$ of the items. Further, $2 \%$ of the items produced by machine A were defective and $1 \%$ produced by machine B was defective. If a defective item is drawn at random, what is the probability that it was produced by machine A?
2. a) A random variable X has the following probability function :

Values of X,

| X | -3 | 2 | 4 | 7 |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}(\mathrm{X}=\mathrm{x})$ | $\frac{k+1}{10}$ | $\frac{2 k-2}{10}$ | $\frac{3 k-5}{10}$ | $\frac{k+2}{10}$ |

i) Find the value of $k \quad$ ii) Determine the distribution of $X$.
b) Let X have the density function $f(x)=\left\{\begin{array}{cc}0.75\left(1-x^{2}\right), & -1 \leq x \leq 1 \\ 0 & \text {,otherwise }\end{array}\right.$

Find the distribution function. Find the Probabilities $P\left(-\frac{1}{2} \leq X \leq \frac{1}{2}\right)$ and. $P\left(\frac{1}{4} \leq X \leq 2\right)$
3. a) In 800 families with 5 children each, how many families would be expected to have (i) 3 boys and 2 girls (ii) 2 boys and 3 girls (iii) no girl (iv) at the most two girls.
b) An aptitude test for selecting offers in a bank is conducted on 1000 candidates. The average score is 42 and the standard deviation of score is 24 . Assuming normal distribution for the scores, find: i) The number of candidates whose scores exceed 60 ii) The number of candidates whose scores lie between 30 and 60 .
4. a) A population consists of the four numbers $3,7,11,15$. Consider all possible samples of size two that can be drawn with replacement from this population.
Find
i) The population mean,
ii) The population standard deviation,
iii) The mean of the sampling distribution of means, iv) The standard deviation of the sampling distribution of means
b) Determine a $99 \%$ confidence interval for the mean of a normal distribution with variance $\sigma^{2}=9$, using a sample of $n=100$ values with mean $\bar{x}=5$.
5. a) A random sample of 100 recorded deaths in the United States during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this seem to indicate that the mean life span today is greater than 70 years? Use a 0.05 level of significance.
b) A commonly prescribed drug for relieving nervous tension is believed to be only $60 \%$ effective. Experimental results with a new drug administered to a random sample of 100 adults who were suffering from nervous tension show that 70 received relief. Is this sufficient evidence to conclude that the new drug is superior to the one commonly prescribed? Use a 0.05 level of significance.
6. a) The height of 6 randomly chosen sailors is, in inches $63,65,68,69,71$ and 72 . Those of 9 randomly chosen soldiers are $61,62,65,66,69,70,71,72$ and 73 . Test whether the sailors are on the average taller than soldiers. Use a 0.05 level of significance.
b) Two independent samples of size 8 and 9 had the following values of the variables:

| Sample I | 20 | 30 | 23 | 25 | 21 | 22 | 23 | 24 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Sample II | 30 | 31 | 32 | 34 | 35 | 29 | 28 | 27 | 26 |

Do the estimates of the population variance differ significantly? Use a 0.05 level of significance.
7. a) The following data show the values of sample mean $\bar{X}$ and the range R for The sample of size 5 each. Calculate the values for central line and control limits for mean-chart and range chart and determine whether the process is in control

| Sample No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Sample Mean | 12.8 | 13.1 | 13.5 | 12.9 | 13.2 | 14.1 | 12.1 | 15.5 | 13.9 | 14.2 |
| Sample Range | 2.1 | 3.1 | 3.9 | 2.1 | 1.9 | 3.0 | 2.5 | 2.8 | 2.5 | 2.0 |

( Given $n=5, A_{2}=0.577, D_{3}=0, D_{4}=2.115$ )
8. a) Derive the average number of customers in the system, In (M/M/1): ( $\infty$ /FCFS) model.
b) A self-service store employs one cashier at its counter. Nine customers arrive on an average every 5 minutes while the cashier can serve 10 customers in 5 minutes. Assuming Poisson distribution for arrival rate and exponential distribution for service rate, find
a) Average number of customers in the system.
b) Average number of customers in queue or average queue length.
c) Average time a customer spends in the system.
d) Average time a customer waits before being served.

